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Intuitionistic Fuzzy Linear Transformations

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Abstract

In this paper, we introduce the concept of linear transformation on intuitionistic fuzzy vector space and prove that the set of all linear transformations between two intuitionistic fuzzy vector spaces forms a vector space under the intuitionistic fuzzy operations. By using the standard intuitionistic fuzzy linear combation of a vector, one to one correspondence between the set of all linear transformations on a finitely generated subspace of V_n and the set of all intuitionistic fuzzy matrices $(IF)_n$ is obtained.

Keywords: Fuzzy matrix, Intuitionistic fuzzy matrix, Intuitionistic fuzzy vector space, Intuitionistic fuzzy linear transformation, Standard basis.

Introduction

We deal with fuzzy matrices that is, matrices over the fuzzy algebra $\mathcal{F}^M$ and $\mathcal{F}^N$ with support [0,1] and fuzzy operations {+,.} defined as a + b = max {a, b}, a.b = min{a,b} for all a,b $\in \mathcal{F}^M$ and a + b = min {a, b}, a.b = max{a, b} for all a, b $\in \mathcal{F}^N$. Let $\mathcal{F}^M_{mxn}$ be the set of all mxn Fuzzy matrices over $\mathcal{F}$. A matrix A $\in \mathcal{F}^M_{mxn}$ is said to be regular if there exists X $\in \mathcal{F}^M_{nxm}$ such that AXA = A, X is called a generalized inverse (g-inverse) of A. In [4], Kim and Roush have developed the theory of fuzzy matrices, under max min composition analogous to that of Boolean matrices. Cho [3] has discussed the consistency of fuzzy matrix equations, if A is regular with a g-inverse X, then b.X is a solution of x A=b. Further every invertible matrix is regular. For more details on fuzzy matrices one may refer [5].In[6], Meenakshi and inbam have introduced the concept of linear transformation on fuzzy vector spaces. The new concept of fuzzy linear transformation on vector spaces and fuzzy vector spaces are discussed in [11]. Atanassov has introduced and developed the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [1,2].If A = $(a_{ij}) \in (IF)_{mxn}$, then A = $\left(\left\langle a_{ij\mu}, a_{ij\nu}\right\rangle\right)$, where $a_{ij\mu}$ and $a_{ij\nu}$ are the membership values and non membership values of a_{ij} in A respectively with respect to the fuzzy sets μ and ν, maintaining the condition 0 ≤ $a_{ij\mu}$+ $a_{ij\nu}$ ≤ 1. In [7], we have studied the structure of row space and column space of intuitionistic fuzzy matrices. The maximum and minimum solutions of fuzzy relational equations involving membership and non-membership matrix of the intuitionistic fuzzy matrix are determined in [8].In[9],we prove that any finitely generated subspace of V_n over the intuitionistic fuzzy algebra (ƗƑ) has a unique standard basis. The concept of semiring of intuitionistic fuzzy matrices are studied in [10].

In this paper, we define a linear transformation and prove that set of all linear transformations between two intuitionistic fuzzy vector spaces forms a vector space.

Preliminaries

Let $(IF)_{mxn}$ be the set of all intuitionistic fuzzy matrices of order mxn Let $(IF)_{mxn}$ be the set of all intuitionistic fuzzy matrices of order mxn. First we shall represent A $\in (IF)_{mxn}$ as Cartesian product of fuzzy matrices. The Cartesian product of any two matrices A=$\left(a_{ij}\right)_{mxn}$ and B=$\left(b_{ij}\right)_{mxn}$, denoted as $\langle A,B\rangle$ is defined as the matrix whose ij[th] entry is the ordered pair $\langle A,B\rangle$= $\left(\left\langle a_{ij}, b_{ij}\right\rangle\right)$. For $A=\left(a_{ij}\right)_{mxn}$ = $\left(\left\langle a_{ij\mu}, a_{ij\nu}\right\rangle\right)$. We define $A_\mu = \left(a_{ij\mu}\right) \in \mathcal{F}^M_{mxn}$ as the membership part of A and $A_\nu = \left(a_{ij\nu}\right) \in \mathcal{F}^N_{mxn}$ as the non membership part of A. Thus A is the Cartesian product of A_μ and A_ν written as $A = \left\langle A_\mu, A_\nu\right\rangle$ with $A_\mu \in \mathcal{F}^M_{mxn}$, $A_\nu \in \mathcal{F}^N_{mxn}$.

We shall follow the matrix operations on intuitionistic fuzzy matrices as defined in our earlier work [7].

For A, B $\in \mathcal{F}^M_{mxn}$, if A=$\left(a_{ij}\right)$ and B = $\left(b_{ij}\right)$ then

(2.1) $\quad$ A+B = $\left(\max\left\{a_{ij}, b_{ij}\right\}\right)$

(2.2) $\quad$ A B = $\left(\max_k \min\left\{a_{ik}, b_{kj}\right\}\right)$

The nxn identity matrix I_n^M in $\mathcal{F}_n^M$ is the matrix defined as $\left(I_n^M\right)_{ij}$ =1 if i=j and $\left(I_n^M\right)_{ij}$ = 0 if i ≠ j and the nxn zero matrix O is the matrix all of whose entries are zero satisfying the following properties:

(2.3) $\quad$ A I_n^M = I_n^M A = A, for all A $\in \mathcal{F}_n^M$

(2.4) $\quad$ O A = A O = O.

For A, B $\in \mathcal{F}^{N}_{mxn}$, if A = (a_{ij}) and B= (b_{ij}) then

(2.5) $\quad A + B = \left(\min\{a_{ij}, b_{ij}\}\right)$

(2.6) $\quad A\,B = \left(\min_{k}\max\{a_{ik}, b_{kj}\}\right)$

The nxn identity matrix I^{N}_{n} in $\mathcal{F}^{N}_{n}$ is the matrix defined as $\left(I^{N}_{n}\right)_{ij}$ =0 if i=j and $\left(I^{N}_{n}\right)_{ij}$ =1 if i $\neq$ j. The nxn zero matrix J in $\mathcal{F}^{N}_{n}$ is the matrix all of whose entries are 1 satisfying the following properties:

(2.7) $\quad A\,I^{N}_{n} = I^{N}_{n}\,A = A$, for all $A \in \mathcal{F}^{N}_{n}$

(2.8) $\quad$ A J = J A = J.

For $A, B \in (IF)_{mxn}$, if A = $\left(\langle a_{ij\mu}, a_{ij\nu}\rangle\right)$ and B = $\left(\langle b_{ij\mu}, b_{ij\nu}\rangle\right)$ then

(2.9) $\quad A+B = \left(\langle \max\{a_{ij\mu}, b_{ij\mu}\}, \min\{a_{ij\nu}, b_{ij\nu}\}\rangle\right)$

(2.10) $\quad A\,B = \left(\langle \max_{k}\min\{a_{ik\mu}, b_{kj\mu}\}, \min_{k}\max\{a_{ik\nu}, b_{kj\nu}\}\rangle\right)$

Let us define the order relation on $(IF)_{mxn}$ as,

(2.11) $\quad A \leq B \Leftrightarrow a_{ij\mu} \leq b_{ij\mu}$ *and* $a_{ij\nu} \geq b_{ij\nu}$, for all i and j

Definition 2.1[7]
An $A \in (IF)_{mxn}$ is said to be regular if there exists $X \in (IF)_{nxm}$ satisfying AXA = A and X is called a generalized inverse (g-inverse) of A which is denoted by A^{-}. Let A{1} be the set of all g-inverses of A.

Definition 2.2[10]
A square intuitionistic fuzzy matrix is called intuitionistic fuzzy permutation matrix,if every row and column contains exactly one <1,0> and all other entries are <0,1>. Let P_n be the set of all nxn permutation matrix in $(IF)_n$.

Lemma 2.1[9]

Any two basis for a finitely generated subspace of the intuitionistic fuzzy algebra (ɫF) = < $\mathcal{F}^{M}$, $\mathcal{F}^{N}$ > have the same cardinality and any finitely generated subspace over (ɫF) has a unique standard basis.

Lemma 2.2[9]
Let S be a finitely generated subspace of V_n and let c = $\{c_1, c_2, \ldots, c_n\}$ be the standard basis for S. Then any vector x $\in$ S can be expressed uniquely as a linear combination of the standard basis.

Lemma 2.3[9]
Let $\{c_1,c_2,\dots,c_n\}$ be the standard basis of the subspace W in V_n. In the standard fuzzy linear combination of the basis vector c_i, the i^{th} coefficient of c_i is <1,0>.

Intuitionistic Fuzzy Linear Transformations (IFLT)

In this section, we introduce the concept of linear transformation on intuitionistic fuzzy vector spaces and prove that the set of all linear transformations between two intuitionistic fuzzy vector spaces forms a vector space under the intuitionistic fuzzy operations.

Definition 3.1

Let V and V' be vector spaces over the intuitionistic fuzzy algebra(ƗƑ).A mapping T of V into V' is called a intuitionistic fuzzy linear transformation if for any x, y $\in$ V and $\alpha \in$(ƗƑ).

i. $T(x+y) = T(x) + T(y)$

ii. $T(\alpha x) = \alpha\, T(x)$

Theorem 3.1

Let V and V' be vector spaces over the intuitionistic fuzzy algebra(ƗƑ). Then $L(V,V')$, the set of all intuitionistic fuzzy linear transformation from V into V' is a intuitionistic fuzzy vector space over the intuitionistic fuzzy algebra(ƗƑ) under addition and multiplication defined by, $(T_1+T_2)x = T_1(x) + T_2(x)$ and $(\alpha\, T_1)x = \alpha\, T_1(x)$ for all T_1, $T_2 \in L(V,V')$, $x \in V$ and $\alpha \in$(ƗƑ).

Proof
Let $L(V,V')$ is closed with respect to '+' and '.'

For any $x,y \in V$.

$$\begin{aligned}(T_1+T_2)(x+y) &= T_1(x+y) + T_2(x+y)\\ &= T_1(x) + T_1(y) + T_2(x) + T_2(y)\\ &= (T_1+T_2)x + (T_1+T_2)y \text{ for every } T_1,T_2 \in L(V,V')\end{aligned}$$

$$\begin{aligned}\text{For } x \in V,\ (T_1+T_2)(\alpha x) &= T_1(\alpha x) + T_2(\alpha x)\\ &= \alpha\, T_1(x) + \alpha\, T_2(x)\\ &= \alpha\, [T_1(x) + T_2(x)]\\ &= \alpha[(T_1+T_2)(x)] \text{ for every } T_1,T_2 \in L(V,V') \text{ and } \alpha \in (\text{ƗƑ}).\end{aligned}$$

Thus $T_1+T_2 \in L(V,V')$ for all $T_1,T_2 \in L(V,V')$.

For $\alpha \in$(ƗƑ) and T$\in$ L(V,V$^{'}$).

$$
\begin{aligned}
(\alpha T)(x+y) &= \alpha[T(x+y)] \\
&= \alpha[T(x)+ T(y)] \\
&= \alpha T(x)+ \alpha\, T(y). \\
(\alpha T)(\beta x) &= \alpha[T(\beta x)] \\
&= \alpha[\beta T(x)] \\
&= \beta[(\alpha T)(x)]
\end{aligned}
$$

Hence $\alpha T \in$ L(V,V$^{'}$).

For T_1, T_2, $T_3 \in$ L(V,V$^{'}$) and α, $\beta \in$(ƗƑ) the following hold;

i. $T_1+T_2 = T_2 + T_1$

ii. $(T_1+T_2)+ T_3 = T_1+(T_2+ T_3)$

iii. $(\alpha\beta)\, T_1 = \alpha(\beta\, T_1)$

iv. $(\alpha+\beta)\, T_1 = \alpha\, T_1+\beta\, T_1$

v. $\alpha(T_1+T_2) = \alpha\, T_1+ \alpha\, T_1$

vi. $_{1.}\, T_1 = T_1$

vii. $T_1+ 0 = 0 + T_1 = T_1$

viii. $0\, T_1 = T_1\, 0 =0$

Proof of (i) to (viii) can be easily verified. Hence L(V,V$^{'}$) is a intuitionistic fuzzy vector space over (ƗƑ). ■

Theorem 3.2

For a intuitionistic fuzzy vector space V over (ƗƑ), L(V) is an algebra under multiplication defined by T_1T_2 (x) = T_1 (T_2 (x)) for all T_1, $T_2 \in$ L(V).

Proof

L(V) is a intuitionistic fuzzy vector space follows from theorem(3.1).We will prove that $T_1\, T_2 \in$ L(V), for T_1, $T_2 \in$ L(V).

$$
\begin{aligned}
(T_1T_2)\,(x+y) &= T_1\,(T_2\,(x+y)) \\
&= T_1\,(T_2\,(x)+ T_2\,(y)) \\
&= T_1\,(T_2\,(x)\,) + T_1\,(T_2\,(y)) \\
&= (T_1\,T_2\,)(x) + (T_1\,T_2\,)(y) \\
(T_1T_2)\,(\alpha\, x) &= T_1\,(T_2\,(\alpha\, x)) \\
&= T_1\,(\alpha\, T_2(x))
\end{aligned}
$$

$= \alpha\, T_1\,(T_2(x))$

$= \alpha\,(T_1\,T_2\,)(x).$

Thus $T_1\,T_2 \in L(V)$. Hence the proof. ■

Definition 3.2

Let L and $L^{'}$ be vector spaces over the intuitionistic fuzzy algebra (ƗƑ). L and $L^{'}$ are said to be intuitionistic fuzzy isomorphic if there exists a one to one and onto linear transformation between them, that is, f is one to one,onto map satisfying

i. $f(x+y) = f(x) + f(y)$,for $x, y\in L$

ii. $f(\alpha x) = \alpha\, f(x)$, for $\alpha \in$(ƗƑ).

Theorem 3.3

Let L and $L^{'}$ be finite dimensional vector spaces over the intuitionistic fuzzy algebra(ƗƑ). L and $L^{'}$ are said to be intuitionistic fuzzy isomorphic if and only if $\dim(L) = \dim(L^{'})$.

Proof

Let $f: L \rightarrow L^{'}$ be a fuzzy isomorphism. If $\{x_1,x_2, \ldots,x_k\}$ is a standard basis of L.

Then it can be verified that $\{ f(x_1),f(x_2), \ldots,f(x_k)\}$ is a standard basis of $L^{'}$. Hence $\dim(L) = \dim(L^{'})$.

Conversely, $\dim(L) = \dim(L^{'})$,then L and $L^{'}$ will have same cardinality. Let $\{x_1,x_2, \ldots,x_k\}$ and $\{y_1,y_2, \ldots,y_k\}$ be standard basis of L and $L^{'}$ respectively. By lemma (2.2), any $x \in L$ has a unique standard linear combination of basis vectors of the form $x = \alpha_1x_1+ \alpha_2x_2+\ldots+ \alpha_kx_k$. Define $f(x) = y$, where $y= \alpha_1y_1+ \alpha_2y_2+\ldots+ \alpha_ky_k$.it can be verified that f satisfies the definition of a intuitionistic fuzzy isomorphism. Hence L and $L^{'}$ are intuitionistic fuzzy isomorphic. ■

Intuitionistic fuzzy matrices associated with IFLT

Let $B=\{c_1,c_2,\ldots,c_n\}$ be an ordered standard basis for a subspace W of intuitionistic fuzzy vector space V_n. Then by lemma(2.2), each vector in $W= \langle W_\mu,W_\nu \rangle$ is uniquely expressible as a standard fuzzy linear combination of $c_i^{'}$s. For each i, $c_i = \langle c_{i\mu}, c_{i\nu} \rangle$. where $c_{i\mu} \in \mathcal{F}^M$ and $c_{i\nu} \in \mathcal{F}^N$ and $B_\mu=\{c_{1\mu},c_{2\mu},\ldots,c_{n\mu}\}$ and $B_\nu =\{c_{1\nu},c_{2\nu},\ldots,c_{n\nu}\}$ are the standard basis for W_μ and W_ν respectively.

If T is a linear transformation on W to itself, then for each $c_j = \langle c_{j\mu}, c_{j\nu} \rangle$ the vector $T(c_j)= T(\langle c_{j\mu}, c_{j\nu} \rangle)$ is in W.

Let $T(c_j)= T(\langle c_{j\mu}, c_{j\nu} \rangle) = \sum_{i=1}^{n} \langle \alpha_{ij\mu}, \alpha_{ij\nu}\rangle \langle c_{i\mu}, c_{i\nu} \rangle$ be the standard fuzzy linear combination in terms of the standard basis vectors.

Construct the matrix [T]= < $[T_\mu]$, $[T_\nu]$>,where $[T_\mu] \in \mathcal{F}^M$ and $[T_\nu] \in \mathcal{F}^N$,Whose j^{th} column is the transpose of the intuitionistic row vector (< $a_{1j\mu}, a_{1j\nu}$ >,< $a_{2j\mu}, a_{2j\nu}$ >,…,< $a_{nj\mu}$, $a_{nj\nu}$ >).Since a_{ij} = < $a_{ij\mu}, a_{ij\nu}$> s are uniquely determined by $T(c_j)$= T(< $c_{j\mu}$, $c_{j\nu}$ >) for each j,[T] is the unique intuitionistic fuzzy matrix corresponding to the linear transformation T on L(W) with respect to the unique standard ordered basis B of the subspace W. Let us denote it as $[T]_{B'}$.

Theorem 4.1

Let B={$c_1, c_2, \ldots, c_n$} be an ordered standard basis for a subspace W of V_n. Then the mapping T → T', which assigns to each linear transformation T its matrix relative to B is an isomorphism of the algebra L(W) onto the matrix algebra $(\mathcal{IF})_n$.

Proof

By using the unique standard basis B={$c_1, c_2, \ldots, c_n$} we have assigned a matrix

[T]=[< $a_{ij\mu}, a_{ij\nu}$>] to each linear transformation T on W.

Claim: T → [T] is one to one

Let $[T_1]$ =[< $\alpha_{ij\mu}$, $\alpha_{ij\nu}$>] and $[T_2]$ =[< $\gamma_{ij\mu}$, $\gamma_{ij\nu}$>]. Then

$$T_1 (c_j) = T_1 (< c_{j\mu}, c_{j\nu} >) = \sum_{i=1}^{n} < \alpha_{ij\mu}, \alpha_{ij\nu}> < c_{i\mu}, c_{i\nu} > \text{ and}$$

$$T_2(c_j) = T_2 (< c_{j\mu}, c_{j\nu} >) = \sum_{i=1}^{n} < \gamma_{ij\mu}, \gamma_{ij\nu}> < c_{i\mu}, c_{i\nu} >.$$

Let $x= \sum_{j=1}^{n} \beta_j < c_{j\mu}, c_{j\nu} >$ be the standard fuzzy linear combination of x in terms of the standard basis vectors {$c_1, c_2, \ldots, c_n$}.

Suppose $[T_1] = [T_2]$ then $\alpha_{ij\mu} = \gamma_{ij\mu}$ and $\alpha_{ij\nu} = \gamma_{ij\nu}$ for all i,j=1,2,…,n.

$$T_1(x) = T_1[\sum_{j=1}^{n} \beta_j < c_{j\mu}, c_{j\nu} >] = \sum_{j=1}^{n} \beta_j T_1(< c_{j\mu}, c_{j\nu} >)$$

$$= \sum_{j=1}^{n} \beta_j \left(\sum_{i=1}^{n} < \alpha_{ij\mu}, \alpha_{ij\nu}> < c_{i\mu}, c_{i\nu} >\right)$$

$$= \sum_{j=1}^{n} \beta_j \left(\sum_{i=1}^{n} < \gamma_{ij\mu}, \gamma_{ij\nu}> < c_{i\mu}, c_{i\nu} >\right)$$

$$= \sum_{j=1}^{n} \beta_j T_2(< c_{i\mu}, c_{i\nu} >)$$

$$= T_2 \sum_{j=1}^{n} \beta_j (< c_{i\mu}, c_{iv} >) = T_2(x)$$

Hence T → [T] is one to one.

Claim: T → [T] is onto.

Let $[T] = [< \alpha_{ij\mu}, \alpha_{ijv} >]$ be an element in $(\mathcal{F})_n$. Now define a map T on the standard basis vectors $B=\{c_1,c_2,\ldots,c_n\}$ as

$$T(x) = T[\sum_{j=1}^{n} \beta_j < c_{j\mu}, c_{jv} >] = \sum_{j=1}^{n} \beta_j T(< c_{j\mu}, c_{jv} >)$$

$$= \sum_{j=1}^{n} \beta_j \left(\sum_{i=1}^{n} < \alpha_{ij\mu}, \alpha_{ijv} > < c_{i\mu}, c_{iv} >\right)$$

$$= \sum_{i,j=1}^{n} \beta_j \left(< \alpha_{ij\mu}, \alpha_{ijv} >\right) < c_{i\mu}, c_{iv} > \text{ is well defined.}$$

We can show that T is a linear transformation on W.

Let $x = \sum_{j=1}^{n} \beta_j < c_{j\mu}, c_{jv} >$ and $y = \sum_{j=1}^{n} \gamma_j < c_{j\mu}, c_{jv} >$. Then $x+y = \sum_{j=1}^{n} (\beta_j + \gamma_j) < c_{j\mu}, c_{jv} >$.

(4.1) $$T(x+y) = \sum_{i,j=1}^{n} [(\beta_j + \gamma_j) < \alpha_{ij\mu}, \alpha_{ijv} >] < c_{j\mu}, c_{jv} >$$

(4.2) $$T(x) + T(y) = \sum_{i,j=1}^{n} [< \alpha_{ij\mu}, \alpha_{ijv} > (\beta_j + \gamma_j)] < c_{j\mu}, c_{jv} >$$

From (4.1) and (4.2) it follows that, T(x+y) = T (x)+ T (y).

$$T(\alpha x) = T\left(\sum_{j=1}^{n} \alpha \beta_j < c_{j\mu}, c_{jv} >\right) = \sum_{i,j=1}^{n} (< \alpha_{ij\mu}, \alpha_{ijv} > \alpha \beta_j) < c_{j\mu}, c_{jv} >.$$

$$= \alpha \sum_{i,j=1}^{n} (< \alpha_{ij\mu}, \alpha_{ijv} > \beta_j) < c_{j\mu}, c_{jv} > = \alpha T(x)$$

Thus T is a linear transformation on W.

Now, $(T_1 + T_2) < c_{j\mu}, c_{jv} > = (T_1 < c_{j\mu}, c_{jv} > + T_2 < c_{j\mu}, c_{jv} >)$

$$= \sum_{i=1}^{n} < \alpha_{ij\mu}, \alpha_{ijv} > < c_{i\mu}, c_{iv} > + \sum_{i=1}^{n} < \gamma_{ij\mu}, \gamma_{ijv} > < c_{i\mu}, c_{iv} >$$

$$= \sum_{i=1}^{n} (< \alpha_{ij\mu}, \alpha_{ijv} > + < \gamma_{ij\mu}, \gamma_{ijv} >) < c_{i\mu}, c_{iv} >.$$

If we define addition in $(\mathcal{F})_n$ by, $[<\alpha_{ij\mu}, \alpha_{ijv}>]+[< \beta_{ij\mu}, \beta_{ijv}>]=[\max\{ \alpha_{ij\mu},$

$\beta_{ij\mu}\}$,min$\{\alpha_{ij\nu},\beta_{ij\nu}\}$] and $[\alpha<\alpha_{ij\mu}, \alpha_{ij\nu}>] = [\min\{\alpha, \alpha_{ij\mu}\}, \max\{1-\alpha, \alpha_{ij\nu}\}]$.

(4.3) $[T_1+T_2] = [T_1]+[T_2]$ and

(4.4) $[\alpha T_1] = \alpha [T_1]$, for any $\alpha\in(\text{ƗƑ})_n$.

Finally, $(T_1T_2)\, c_j = (T_1T_2) < c_{j\mu}, c_{j\nu} > = T_1(T_2 < c_{j\mu}, c_{j\nu} >)$

$$= T_1 \sum_{k=1}^{n} (< \beta_{kj\mu}, \beta_{kj\nu}>< c_{k\mu}, c_{k\nu} >).$$

$$= \sum_{k=1}^{n} < \beta_{kj\mu}, \beta_{kj\nu}> (T_1< c_{k\mu}, c_{k\nu} >).$$

$$= \sum_{k=1}^{n} < \beta_{kj\mu}, \beta_{kj\nu}> (\sum_{i=1}^{n} < \alpha_{ik\mu}, \alpha_{ikj\nu}> < c_{k\mu}, c_{k\nu} >).$$

$$[<\alpha_{ij\mu}, \alpha_{ij\nu}>] [< \beta_{ij\mu}, \beta_{ij\nu}>] = \sum_{i,k=1}^{n} (< \alpha_{ik\mu}, \alpha_{ikj\nu}>< \beta_{kj\mu}, \beta_{kj\nu}>) < c_{k\mu}, c_{k\nu} >.$$

(4.5) Therefore $[T_1][T_2] = [T_1 T_2]$.

The operations defined by (4.3),(4.4) and (4.5) are the standard max-min,min-max operations for intuitionistic fuzzy matrices. ■

Remark 4.1
The image of the zero linear transformation under the mapping T → [T] is the zero matrix, all of whose entries are <0,1>.

Remark 4.2
In particular for the identity transformation on W, since $I(c_i) = c_i$ for each basis vector c_i in the standard basis $B=\{c_1,c_2,\dots,c_n\}$ of W. From lemma(2.3),the diagonal entries of I is <1,0>.

Example 4.1
Let W be the subspace of W_3.The identity mapping I: W→W.Since I(x) = x for all x∈W, the standard fuzzy linear combination of the standard basis vector

(<0.5,0 >,<0.5,0.5>,<0.5,0>) is obtained in example(3.2)[9] as,

(<0.5,0>,<0.5,0.5>,<0.5,0>) = (<1, 0 > (<0.5,0>,<0.5,0.5>,<0.5,0>) + <0.5, 0.5 > (<0,1>,<1,0>,<0.5,0.5>)+ <0.5, 0 >(<0,1>,<0.5,0.5>,<1,0>).

Similarly, the standard fuzzy linear combination of the other basis vectors are obtained as,

(<0,0.5>,<1,0>,<0,0.5>) = (<0, 0.5 > (<0.5,0>,<0.5,0.5>,<0.5,0.0>) + <1,0 > (<0,1>,<1,0>,<0.5,0.5>)+ <0, 0.5 >(<0,1>,<0.5,0.5>,<1,0>) and

(<0,1>,< 0.5,0.5>,<1,0>) = (<0, 1 > (<0.5,0>,<0.5,0.5>,<0.5,0>) + <0.5,0.5> (<0,1>,<1,0>,<0.5,0.5>)+ <1, 0 >(<0,1>,<0.5,0.5>,<1,0>).

Hence the matrix [I] corresponding to the identity linear transformation with respect to the standard basis {(<0.5,0>,<0.5,0.5>,<0.5,0>),(<0,1>,<1,0>,<0.5,0.5>), (<0,1>,<0.5,0.5>, <1,0>) } is,

$$[I]_B = \begin{pmatrix} \langle 1,0\rangle & \langle 0,0.5\rangle & \langle 0,1\rangle \\ \langle 0.5,0.5\rangle & \langle 1,0\rangle & \langle 0.5,0.5\rangle \\ \langle 0.5,0\rangle & \langle 0,0.5\rangle & \langle 1,0\rangle \end{pmatrix}$$

Corollary 4.1

For any $T\in L(W)$ and the identity transformation $I\in L(W)$ the corresponding matrices [T] and [I] satisfy [T]. [I]= [I]. [T]= [T] under the max-min, min-max compositions of intuitionistic fuzzy matrices.

Proof

Since T I=I T=T, by using (4.5) we get the matrix identity [T]. [I]= [I]. [T]= [T]. ■

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