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Operational Study of a Standby System Incorporating the Concept of Delay in Time with Critical Human Error and Environmental Failure

[*1]Sachin Kumar, [2]Anand Tyagi and [3]Sanjay Goel

[1]Associate Professor of Mathematics in AS&H Department,
Krishna Institute of Engineering & Technology, Ghaziabad, India
[2]Professor of Mathematics in AS&H Department,
Raj Kumar Goel Engineering College, Pikakhua, Ghaziabad, India
[3]Associate Professor of Mathematics in AS&H Department,
Krishna Institute of Engineering and Technology, Ghaziabad, India
**Corresponding Author E-mail: sachin17@kiet.edu,*
sachin_aryabhatt@rediffmail.com

Abstract

This paper presents a complex system having two subsystems A& B in series (1-out-of-2: F). The system fails if any of the two subsystems A&B fails. The subsystem A consists of three identical units. Initially only two units work and third unit is in standby. The standby unit is operated when any of the two units fails. It is assumed that the switching mechanism is not automatic and standby unit does not come into operation instantaneously. The operator takes some time in changeover & meanwhile system works in degraded state where only one unit works. The system may also fail due to critical human error & environmental failure. The failure and repair time for the system follows exponential and general distributions respectively. Using Laplace transforms various state probabilities are evaluated along with cost function. Numerical examples have been added to highlight the important results.

Keywords: Standby redundancy, Environmental failure and Human error.

Introduction

Gaurov & Utkin [2] considered a system in which they assumed the switch over to be imperfect & non instantaneous. Further [1, 3, 4, 5] consider the possibility of failure of the switching mechanism. Yet a study with delay in time for standby to operate with critical human error and environmental failure seems unavoidable in respect of customer supply reliability for a system.

Keeping these facts in view, the authors have considered a complex system consisting of two subsystems A& B in series (1-out-of-2: F). The power system fails if any of the two subsystems fails. The subsystem A consists of three identical units. Initially only two units work and third unit is in standby. The standby unit is operated when any of the two units fails. It is assumed that the switching mechanism is not automatic and standby unit does not come into operation instantaneously. The operator takes some time in changeover & meanwhile system works in degraded state where only one unit works. The power system may also fail due to critical human error & environmental failure. The failure and repair time for the system follows exponential and general distributions respectively. Using Laplace transform various state probabilities have

Notations

$P_i(t)$	Probability that at time t system is in operable state for i=0,1,2,3
$P_4(x,t)/P_5(y,t)$	Probability that at time t system is in failed state and elapsed repair time lies in interval $(x, x+\Delta)/(y, y+\Delta)$
$P_6(t)$	Probability that at time t system is in failed state due to environmental failure.
λ_A/λ_B	Constant failure rate for each unit of subsystem A/ subsystem B.
λ_H/λ_E	Constant failure rate for critical human error /environmental failure
$\phi_F(x)/\phi_H(y)$	General repair from state 4/5
μ/η	Constant repair rate from state 2 and 3/6
W	Constant waiting rate for the standby to start operation

Assumptions

- Initially the system is good.
- Switching mechanism is not automatic and the operator takes some time in the changeover.
- Subsystem A consists of three units out of which two operate & one remains in standby.
- Subsystem B is a single unit.
- The repaired unit works like new and, repair does not damage anything.
- After repair, if any unit of the subsystem A is in spare then it becomes the standby unit.
- Repair time follows general distribution for states 4 and 5 and remains constant for other states.

Formulation of Mathematical Model

By elementary probability & continuity arguments the difference differential equations for stochastic process which is continuous in time and discrete in space are:

$$\left(\frac{d}{dt}+2\lambda+\lambda_H+\lambda_B+\lambda_E\right)P_0(t)=\mu P_1(t)+\mu P_2(t)+\int_0^\infty P_4(x,t)\phi_F(x)dx$$

$$+\int_0^\infty P_5(y,t)\phi_H(y)dy+\eta P_6(t) \tag{1}$$

$$\left(\frac{d}{dt}+\lambda+\lambda_H+\lambda_B+\lambda_E+w+\mu\right)P_1(t)=2\lambda P_0(t) \tag{2}$$

$$\left(\frac{d}{dt}+2\lambda+\lambda_H+\lambda_B+\lambda_E+\mu\right)P_2(t)=wP_1(t)+\mu P_3(t) \tag{3}$$

$$\left(\frac{d}{dt}+\lambda+\lambda_H+\lambda_B+\lambda_E+\mu)\right)P_3(t)=2\lambda P_2(t) \tag{4}$$

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\phi_F(x)\right)P_4(x,t)=0 \tag{5}$$

$$\left(\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\phi_H(y)\right)P_5(y,t)=0 \tag{6}$$

$$\left(\frac{d}{dt}+\eta\right)P_6(t)=\lambda_E[P_0(t)+P_1(t)+P_2(t)+P_3(t)] \tag{7}$$

Boundary Conditions:

$$P_4(0,t)=\lambda\{P_1(t)+P_3(t)\}+\lambda_B\{P_0(t))+P_1(t)+P_2(t)+P_3(t)\} \tag{8}$$

$$P_5(0,t)=\lambda_H\{P_0(t)+P_1(t)+P_2(t)+P_3(t)\} \tag{9}$$

Initial Condition:

$$P_i(0)=\begin{cases}1 & \text{when} \quad i=0\\ 0 & \text{otherwise}\end{cases} \tag{10}$$

By taking Laplace Transforms of equations (1) to (9) & then solving with the help of equation (10) one may obtain

$$P_{up}(s)=\frac{G(s)}{A(s)} \tag{11}$$

$$P_{down}(s)=\frac{1}{A(s)}\left[\{\lambda F(s)+\lambda_B G(s)\}r_F(s)+\lambda_H G(s)r_H(s)+\frac{\lambda_E}{s+\eta}G(s)\right] \tag{12}$$

When repair follows exponential distribution, setting

$S_F(s)=\dfrac{\phi_F}{s+\phi_F}$ & $S_H(s)=\dfrac{\phi_H}{s+\phi_H}$, one may get

$$P_{up}(s) = \frac{G(s)}{A_1(s)}$$

Where

$$A_1(s) = s + I_1 - \left[\frac{2\mu\lambda}{(s + I_1 + w)} + \frac{2\mu w\lambda(s + I_1)}{\{(s + I_1)(s + I_2) - 2\mu\lambda\}\{s + I_1 + w)\}} \right.$$

$$\left. + \{\lambda F(s) + \lambda_B G(s)\} \frac{\phi_F}{s + \phi_F} + \lambda_H G(s) \frac{\phi_H}{s + \phi_H} + \frac{\eta\lambda_E}{s + \eta} G(s) \right] \tag{13}$$

To obtain expression for reliability, taking all repair rate equal to zero, by inversion process one may obtain

$$R(t) = a * e^{-a_1 t} + b * te^{-a_2 t} - c * e^{-a_2 t} + d * e^{-a_3 t} \tag{14}$$

Cost function:

If K_1 & K_2 are the revenue & service cost per unit time, then expected cost is obtained by

$$H(t) = K_1 \int_0^t R(t)dt - K_2 t$$

Or

$$H(t) = K_1 \left[\frac{a^*}{a_1}(1 - e^{-a_1 t}) + \frac{b*}{a_2^2}\{1 - e^{-a_2 t}(1 + a_2 t)\} + \frac{d*}{a_3}(1 - e^{-a_3 t}) - \frac{c*}{a_2}(1 - e^{-a_2 t}) \right] - . \tag{15}$$

Variance of time to failure:

It is obtained by

$$\sigma^2 = -2 \lim_{s \to 0} \frac{dR(s)}{ds} - (M.T.T.F)^2 \tag{16}$$

Numerical computation:

To study the effect of waiting rate over system, following computations are made

(i) Reliability Analysis: Fixing $\lambda = 0.01, \lambda_B = 0.02, \lambda_H = 0.05$ & $\lambda_E = 0.03$ one obtain graph 2.1.

(ii) Cost function:

Setting $K_1 = 1, w = 0.2, \lambda = 0.01, \lambda_B = 0.02, \lambda_H = 0.05, \lambda_E = 0.03$, one obtain graph 2.2.

(iii) Variance of time to failure:

Taking $\lambda = 0.01, \lambda_B = 0.02, \lambda_H = 0.05, \lambda_E = 0.03$ one gets graph 2.3.

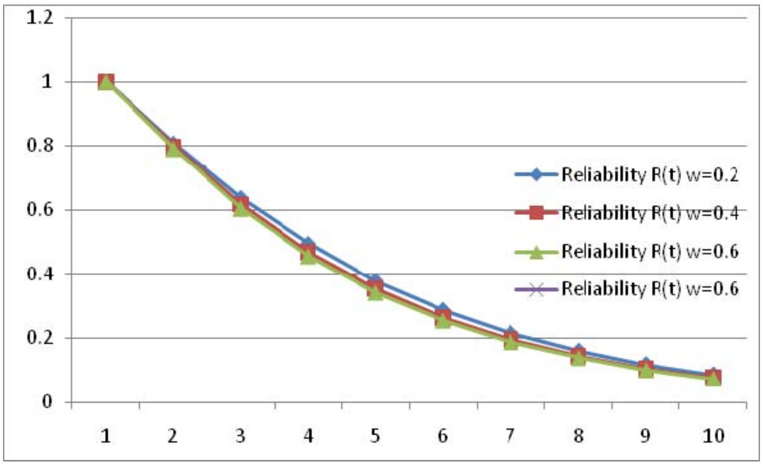

Figure 2.1

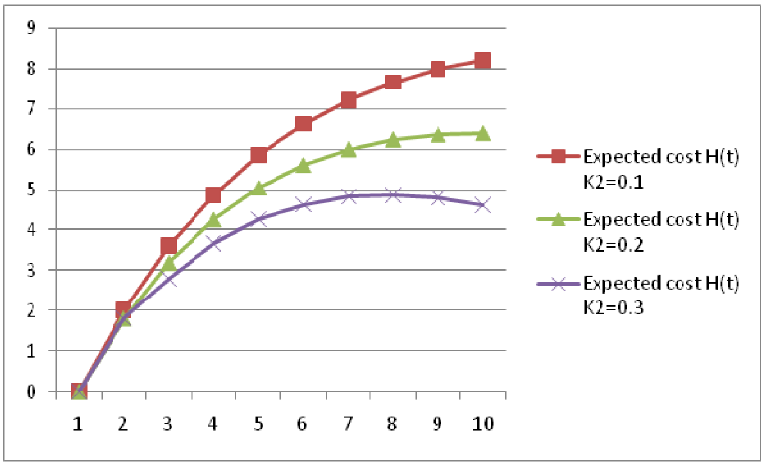

Figure 2.2

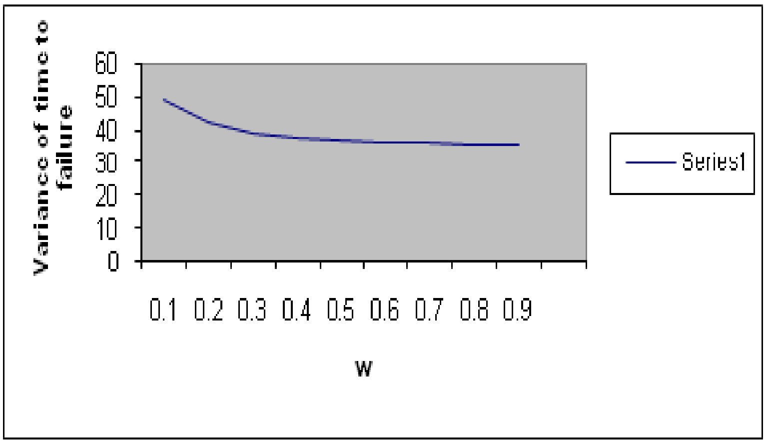

Figure 2.3

Interpretation of Results

An observation of graph Reliability V/S time reveals the fact that reliability of the system decreases with increase of time for various values of waiting rate.

A critical examination of graph, Expected cost V/S time discloses the fact that cost function initially increases and at last becomes steady.

The graph, Variance of time to failure & waiting rate indicates that variance of time to failure decreases with increase in waiting rate.

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