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Fuzzy Soft Rings on Fuzzy Lattices

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Abstract

Molodtsov [1999] initiated the concept of soft set theory as a new approach for modeling uncertainities. Then Maji et. al [2001] expanded this theory to fuzzy soft set theory. The algebraic structures of soft set theory have been studied increasingly in recent years. Aktas and Ca'gman [2007] defined the notion of soft groups. Feng et.al [2008] initiated the study of soft semi rings and finally soft rings are defined by Acar et.al [2010]. In this study we introduce, fuzzy soft ring, which is a generalisation of soft rings introduced by Acar et. al. and we study some of their properties.

Keywords: Soft ring, Fuzzy Soft ring, imaginable, pre-image, soft homomorphism, fuzzy soft isomorphism.

Introduction

Most of the existing mathematical tools for formal modeling, reasoning and computing are crisp, deterministic and precise in character. But in real life situation, the problems in economics, engineering, environment, social science, medical science etc do not always involve crisp data. Consequently, we cannot successfully by using the traditional classical methods because of various types of uncertainties in this problem. There are several theories, for example, theory of fuzzy sets [27], theory of intuitionist fuzzy sets [5], vague sets [13], interval mathematics [26], and rough sets [23], which can be considered as mathematical tools for dealing with uncertainties. But all these theories have this inherents difficulties as what where point out by Molodtsov in [21]. The reason for these difficulties is possibly the inadequacy of the parameterization tool of the theories.

In 1999 Molodtsov [21], initiated the Novel concept of soft set theory which is completely new approach for modeling vagueness and uncertainties. Soft set theory has a rich potential for applications in several directions, few of which had been shown by Molodtsov in [21] After Molodtsov work, some different applications of soft set theory where studied in [1].

Furthermore Maji, Biswas and Roy worked on soft set theory in [18]. Also Maji et. al [17] presented the definition of fuzzy soft set and Roy et. al presented some applications of there notion to decision making problems. The algebraic structures of set theories dealing with uncertainities has also been studied by some authors. Rosenfield [25] proposed the concept of fuzzy groups in order to establish the algebraic structures of fuzzy sets. Rough groups were defined by Biswas and some authors have studied the algebraic properties of rough sets as well. Recently the many authors discuss the soft set research on the soft set theory is progressing rapidly. For example, the concept of soft semi ring, soft group, soft BCK/BCI algebras, soft BL-algebras and fuzzy soft groups. This paper begins by introducing the basic concept of fuzzy soft set theory, then we introduce the basic version of fuzzy soft ring theory, which extends the notion of the ring to the algebraic structure of fuzzy soft set.

In this paper, we study Molodtsov motion of soft sets and fuzzy soft set considering the fact that the parameters are mostly fuzzy hedges or fuzzy parameters. We discuss fuzzy soft sets algebraic structure and given the definition of fuzzy soft ring. We define operations on fuzzy soft rings soft set, research on the soft set theory is progressing rapidly. For example, the concept of soft semi ring, soft group, soft BCK/BCI algebras, soft BL-algebras and fuzzy soft groups. This paper begins by introducing the basic concept of fuzzy soft set theory, then we introduce the basic version of fuzzy soft ring theory, which extends the notion of the ring to the algebraic structure of fuzzy soft set.

In this paper, we study Molodtsov motion of soft sets and fuzzy soft set considering the fact that the parameters are mostly fuzzy hedges or fuzzy parameters. We discuss fuzzy soft set algebraic structure and given the definition of fuzzy soft ring. We define operations on fuzzy soft rings and prove some results on them. Finally we present image, pre-image, fuzzy soft homomorphisms and discussed their properties.

Preliminaries

Through out this paper R denotes a commutative ring and all fuzzy soft sets are considered over R.

Definition 2.1

A pair (f, A), is called a soft set over the lattice L, If $f:A \to P(L)$. Here L be the initial universe and E be the set of parameters. Let P(L) denotes the power set of L and I^L denotes the set of all fuzzy sets on L.

Definition 2.2

A pair (f, A is called a fuzzy soft set over L, where $f:A \to I^L$, ie. for each $a \in A$, $f_a: L \to I$ is a fuzzy set in L.

Definition 2.3
Let (f, A) be a non-null soft set over a ring R. Then (f, A) is said to be a soft ring over R if and only if f(a) is sub ring of R for each $a \in A$.

Definition 2.4
Let (f, A) be a non Null fuzzy soft set over a ring R. Then (f, A) is called a fuzzy soft ring over R If and only if $f(a) = f_a$ is a fuzzy sub ring of R. for each $a \in A$.

(FSR1) $f_a(x + y) \geq T\{f_a(x), f_a(y)\}$

(FSR2) $f_a(-x) \geq f_a(x)$

(FSR3) $f_a(xy) \geq T\{f_a(x), f_a(y)\}$

for all $x, y \in R$

Definition 2.5
Let (f, A) be a fuzzy soft set over L. The soft set $(f, A)_\alpha = \{(f_a)_\alpha / a \in A\}$ for each $\alpha \in [0, 1]$ is called α–level soft set.

Definition 2.6
Let f_a be a fuzzy soft ring in R. Let $\theta: R \to R'$ be a map and define

$$f_a^{\theta}(x) = f_a(\theta x) \text{ by defining } f_a^{\theta}: R \to [0, 1].$$

Definition 2.7
Let $\phi: X \to Y$ and $\Psi: A \to B$ be two functions, where A and B are parameter sets for the crisp sets X and Y respectively. Then the pair (ϕ, Ψ) is called a fuzzy soft function from X to Y.

Definition 2.8
The pre-image of (g, B) under the fuzzy soft function (ϕ, Ψ) denoted by $(\phi, \Psi)^{-1}$ is the fuzzy soft set defined by $(\phi, \Psi)^{-1}(g, B) = (\phi^{-1}(g), \Psi^{-1}(B))$.

Definition 2.9
Let $(\phi, \Psi): X \to Y$ is a fuzzy soft function, if ϕ is a homomorphism from $X \to Y$ then (ϕ, Ψ) is said to be fuzzy soft homomorphism. If ϕ is a isomorphism from $X \to Y$ and Ψ is 1 – 1 mapping from A on to B then (ϕ, Ψ) is said to be fuzzy soft isomorphism.

Properties of Fuzzy Soft Ring

Preposition 3.1
Let f_a be a fuzzy soft ring of R then

$f_a(x) \leq f_a(0)$ for all $x \in R$, the subset

$Rf_a = \{x \in R / f_a(x) = f_a(0)\}$ is a fuzzy soft ring of R.

Proof: Let $x \in G$, then

(FSR1) $f_a(x + y) = T\{ f_a(x), f_a(y)\}$

$= T\{ f_a(x), f_a(-x)\}$

$\leq f_a((x) + (-x))$

$\leq f_a(0)$

This implies subdivision (i). To verify subdivision (ii), it follows that $x \in R\, f_a$ and $R\, f_a \neq \phi$.

Now let $x, y \in R$, then

(FSR2) $fa(x + (-y)) \geq T\{f_a(x), f_a(-y)\}$

$= T\{f_a(x), f_a(y)\}$

$= T\{f_a(0), f_a(0)\}$

$= f_a(0)$

But, from subdivision (i), $f_a(x) \leq f_a(0)$ for all $x, y \in R$ and so $f_a(x + (-y)) = f_a(0)$. Which then $x + (-y) \in Rf_a$ is fuzzy soft ring of R.

(FSR3) Let $x, y \in Rf_a$ and setting $y = x$, then

$f_a(xy) \geq T\{f_a(x), f_a(y)\}$

$= T\{f_a(x), f_a(x)\}$

$= T\{f_a(0), f_a(0)\}$

$= f_a(0)$

$\therefore$ f_a is a fuzzy soft ring over R.

Corollary: 3.2
Let R be a finite ring and f_a be a fuzzy soft ring of R. Consider the subset H of R given by

$$H = \{x \in R \,/\, f_a(x) = f_a(0) \}$$

Then H is called crisp subring of R.

Proposition 3.3
Let f_a and f_b be two fuzzy soft rings of R then $f_a \cap f_b$ is fuzzy soft ring of R.

Proof: Let $x, y \in R$

(FSR1) $(f_a \cap f_b)\,(x + y) = T\{f_a(x + y), f_b(x + y)\}$

$\geq T\{T\, f_a(x), f_a(y)\}, T\{ f_b(x), f_b(y) \}$

$\geq T\{T\{ f_a(x), f_a(y), f_b(x), f_b(y)\}\}$

$\geq T\{T\{f_a(x), f_b(y)\}, T\{f_a(y), f_b(y)\}\}$

$\geq T\{(f_a \cap f_b)(x), (f_a \cap f_b)(y)\}$

FSR1 satisfied in R

(FSR2) $(f_a \cap f_b)(-x) = T\{f_a(-x), f_b(-x)\}$

$\geq T\{f_a(x), f_b(x)\}$

$\geq \{(f_a \cap f_b)(x)$

(FSR3) $(f_a \cap f_b)(xy) = T\{f_a(xy), f_b(xy)\}$

$T\{T\{f_a(x), f_a(y)\}, T\{f_b(x), f_b(y)\}\}$

$T\{T\{f_a(x), f_a(y)\}, f_b(x), f_b(y)\}\}$

$T\{T\{f_a(x), f_b(x)\}, T\{f_a(y), f_b(y)\}\}$

$T\{(f_a \cap f_b)(x), (f_a \cap f_b)(y)\}$

Hence $f_a \cap f_b$ is fuzzy soft ring of R

Preposition 3.4
If f_a is fuzzy soft ring of R then the non-empty level subset $U(f_a; t)$ is fuzzy soft ring for all $t \in I_m(f_a)$.

Proof:
Let f_a be a fuzzy soft ring and $t \in I_m(f_a)$.

Now $x, y \in U(f_a; t)$, we have $f_a(x) \geq t$, $f_a(y) \geq t$.

(FSR1) $f_a(x + y) \geq T\{f_a(x), f_a(y)\}$

$\geq T\{t, t\}$

$\geq t$

So, which implies $x + y \in U(f_a; t)$.

(FSR2) $f_a(-x) \geq f_a(x)$

$\geq t$

Therefore $-x \in U(f_a; t)$.

(FSR3) $f_a(xy) \geq T\{f_a(x), f_a(y)\}$

$\geq T\{t, t\}$

$\geq t$

Therefore $xy \in U(f_a; t)$.

Hence, U (f_a; t) is a fuzzy soft ring of R.

Proposition 3.5
If f_a is a fuzzy soft ring of R and defined by $f_a^+(x) = f_a(x) + 1 – f_a(0)$, for all $x \in R$, then f_a^+ (x) is normal fuzzy soft ring of R which contains f_a.

Proof:
For any $x, y \in R$. It follows that $f_a^+(x) = f_a(x) + 1 – f_a(0)$

(FSR1) $f_a^+ (x + y) = f_a (x + y) + 1 – f_a(0)$

$\geq T \{\{ f_a(x), f_a(y)\}+ 1 – f_a(0)\}$

$\geq T \{\{ f_a(x) + 1 – f_a(0), f_a(y) + 1 – f_a(0),\}$

$\geq T \{ f_a^+(x), f_a^+(y)\}$

(FSR2) $f_a^+ (–x) = f_a (–x) + 1 – f_a(0)$

$\geq f_a(x) + 1 – f_a(0)$

$\geq f_a^+(x)$

(FSR3) $f_a^+ (x y) = f_a (x y) + 1 – f_a(0)$

$\geq T \{\{ f_a(x), f_a(y)\}+ 1 – f_a(0)\}$

$\geq T \{\{ f_a(x) + 1 – f_a(0), f_a(y) + 1 – f_a(0)\}$

$\geq T \{ f_a^+(x), f_a^+(y)\}$

Therefore, f_a^+ is normal fuzzy soft ring of R.

Proposition 3.6
If $\{ f_{a_i} \}_{i \in f_a}$ is a family of fuzzy soft rings of R, then $\cap f_{a_i}$ is fuzzy soft ring of R, whose $\cap f_{a_i} = \{(x, \Lambda f_{a_i} (x) / x \in R\}$, where $i \in f_a$.

Proof:
Let $x, y \in R$, then for $i \in f_a$ It follows that

(FSR1) $\cap f_{a_i} (x + y) = \Lambda f_{a_i} (x + y)$

$\geq \Lambda T \{ f_{a_i} (x), f_{a_i} (y)\}$

$\geq T \{\Lambda f_{a_i} (x), \Lambda f_{a_i} (y)\}$

$\geq T \{\cap f_{a_i} (x), \cap f_{a_i} (y)\}$

(FSR2) $\cap f_{a_i} (–x) = \Lambda f_{a_i} (–x)$

$\geq \Lambda\ f_{a_i}(x)$

$\geq \cap\ f_{a_i}(x)$

(FSR3) $\cap\ f_{a_i}(xy) = \Lambda f_{a_i}(xy)$

$\geq \Lambda\ T\{f_{a_i}(x),\ f_{a_i}(y)\}$

$\geq T\{\Lambda\ f_{a_i}(x), \Lambda\ f_{a_i}(y)\}$

$\geq T\{\cap\ f_{a_i}(x), \cap\ f_{a_i}(y)\}$

Therefore, $\cap\ f_{a_i}$ is fuzzy soft ring of R.

Preposition 3.7

If f_a is a fuzzy soft ring of R then f_a^c is a fuzzy soft ring of R.

Proof:

Let x, y $\in$ R. then

(FSR1) $f_a^c(x + y) = 1 - f_a(x + y)$

$\leq 1 - T\{f_a(x), f_a(y)\}$

$\leq S\{1 - f_a(x), 1 - f_a(y)\}$

$\leq S\{f_a^c(x),\ f_a^c(y)\}$

(FSR2) $f_a^c(-x) = 1 - f_a(-x)$

$\leq 1 - f_a(x)$

$\leq f_a^c(x)$

(FSR3) $f_a^c(xy) = 1 - f_a(xy)$

$\leq 1 - T\{f_a(x), f_a(y)\}$

$\leq S\{1 - f_a(x), 1 - f_a(y)\}$

$\leq S\{f_a^c(x),\ f_a^c(y)\}$

Therefore, f_a^c is fuzzy soft ring of R.

Preposition 3.8

Let f_a and f_b be two fuzzy soft ring of R then f_a U f_b is fuzzy soft ring of R.

Proof:
Let x, y $\in$ R, then

(FSR1) $(f_a \text{ U } f_b)(x + y) = S\{ f_a(x + y), f_b(x + y)\}$

$\geq S\{ T\{ f_a(x), f_b(y)\}, T\{\{ f_b(x), f_b(y)\}\}$

$\geq T\{ S\{ f_a(x), f_a(y), f_b(x), f_b(y)\}\}$

$\geq T\{ S\{ f_a(x), f_b(x)\}, S\{f_a(y), f_b(y)\}\}$

$\geq T\{ (f_a \text{ U } f_b)(x), (f_a \text{ U } f_b)(y)\}$

(FSR2) $(f_a \text{ U } f_b)(-x) = S\{ f_a(-x), f_b(-x)\}$

$\geq S\{ f_a(x), f_b(x)\}$

$\geq (f_a \text{ U } f_b)(x)$

(FSR3) $(f_a \text{ U } f_b)(xy) = S\{ f_a(xy), f_b(xy)\}$

$\geq S\{ T\{ f_a(x), f_a(y)\}, T\{ f_b(x), f_b(y)\}\}$

$\geq T\{S(f_a(x), f_a(y), f_b(x), f_b(y)\}$

$\geq T\{ S\{ f_a(x), f_b(x)\}, S\{f_a(y), f_b(y)\}\}$

$\geq T\{ (f_a \text{ U } f_b)(x), (f_a \text{ U } f_b)(y)\}$

Therefore, f_a U f_b is fuzzy soft ring of R.

Preposition 3.9
Let f_a and f_b be two fuzzy soft rings of R then $f_a \times f_b$ is fuzzy soft ring of R.

Proof:
Let x, y $\in$ R. then

(FSR1) $(f_a \times f_b)(x + y) = T\{ f_a(x + y), f_b(x + y)\}$

$\geq T\{ T\{ f_a(x), f_a(y)\}, T\{\{ f_b(x), f_b(y)\}\}$

$\geq T\{ T\{ f_a(x), f_a(y), f_b(x), f_b(y)\}\}$

$\geq T\{ T\{ f_a(x), f_b(x)\}, T\{f_a(y), f_b(y)\}\}$

$\geq T\{ (f_a \times f_b)(x), (f_a \times f_b)(y)\}$

(FSR2) $(f_a \times f_b)(-x) = T\{ f_a(-x), f_b(-x)\}$

$\geq T\{ f_a(x), f_b(x)\}$

$\geq (f_a \times f_b)(x)$

(FSR3) $(f_a \times f_b)(xy) = T\{ f_a(x y), f_b(x y)\}$

$\geq T\ \{\ T\ \{\ f_a\ (x),\ f_a\ (y)\},\ T\ \{\{\ f_b\ (x),\ f_b\ (y)\}\}$

$\geq T\ \{\ T\ \{\ f_a\ (x),\ f_a\ (y),\ f_b\ (x),\ f_b\ (y)\}\}$

$\geq T\ \{\ T\ \{\ f_a\ (x),\ f_b\ (x)\},\ T\ \{f_a\ (y),\ f_b\ (y)\}\}$

$\geq T\ \{\ (f_a \times f_b)\ (x),\ (f_a \times f_b\ )\ (y)\}$

Therefore, $f_a \times f_b$ is fuzzy soft ring of R.

Preposition 3.10
If $f_{a_1}, f_{a_2}, \ldots, f_{a_n}$ be fuzzy soft ring of the rings $R_1, R_2, \ldots, R_n$ respectively then $f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n}$ is fuzzy soft ring of $R_1 \times R_2 \times \ldots \times R_n$.

Proof:
Let $X = (x_1, x_2, \ldots, x_n), Y = (y_1, y_2, \ldots, y_n) \in R$

(FSR1) $(f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n})(X + Y)$

$= (f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n})\ ((x_1, x_2, \ldots, x_n) + (y_1, y_2, \ldots, y_n))$

$= (f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n})\ (x_1 + y_1, x_2 + y_2, \ldots, x_n + y_n)$

$= T\ \{\ f_{a_1}(x_1 + y_1),\ f_{a_2}(x_2 + y_2), \ldots, f_{a_n}(x_n + y_n)\}$

$\geq T\ \{\ T\ \{f_{a_1}(x_1),\ f_{a_1}(y_1)\},\ T\ \{f_{a_2}(x_2),\ f_{a_2}(y_2)\}, \ldots,$

$T\ \{f_{a_n}(x_n),\ f_{a_n}(y_n)\}\}$

$\geq T\ \{\ T\ \{f_{a_1}(x_1),\ f_{a_2}(x_2) \ldots, f_{a_n}(x_n),$

$f_{a_1}(y_1),\ f_{a_2}(y_2) \ldots, f_{a_n}(y_n)\}\}$

$\geq T\ \{\ T\ \{f_{a_1}(x_1),\ f_{a_2}(x_2) \ldots, f_{a_n}(x_n)\},$

$T\{f_{a_1}(y_1),\ f_{a_2}(y_2) \ldots, f_{a_n}(y_n)\}\}$

$\geq T\ \{\ (f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n})\ (x_1, x_2, \ldots, x_n),$

$(f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n})\ (y_1, y_2, \ldots, y_n)\}$

FSR1 satisfied in $R_1 \times R_2 \times \ldots \times R_n$.

(FSR2) $(f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n})\ (-X)$

$= (f_{a_1} \times f_{a_2} \times \ldots \times f_{a_n})$

$(-(x_1, x_2, \ldots, x_n))$

$= (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})$

$(-x_1, -x_2, \dots, -x_n)$

$= T\{f_{a_1}(-x_1), f_{a_2}(-x_2), \dots, f_{a_n}(-x_n)\}$

$\geq T\{f_{a_1}(x_1), f_{a_2}(x_2), \dots, f_{a_n}(x_n)\}$

$\geq (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(x_1, x_2, \dots, x_n)$

$\geq (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(X)$

(FSR3) $(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(XY)$

$= (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})$

$((x_1, x_2, \dots, x_n)(y_1, y_2, \dots, y_n))$

$= (f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})$

$(x_1 y_1, x_2 y_2, \dots, x_n y_n)$

$\geq T\{f_{a_1}(x_1 y_1), f_{a_2}(x_2 y_2), \dots,$

$f_{a_n}(x_n y_n)\}$

$\geq T\{T\{f_{a_1}(x_1), f_{a_1}(y_1)\}, T\{f_{a_2}(x_2), f_{a_2}(y_2)\}, \dots,$

$T\{f_{a_n}(x_n), f_{a_n}(y_n)\}\}$

$\geq T\{T\{f_{a_1}(x_1), f_{a_2}(x_2) \dots, f_{a_n}(x_n),$

$f_{a_1}(y_1), f_{a_2}(y_2) \dots, f_{a_n}(y_n)\}\}$

$\geq T\{T\{f_{a_1}(x_1), f_{a_2}(x_2) \dots, f_{a_n}(x_n)\},$

$T\{f_{a_1}(y_1), f_{a_2}(y_2) \dots, f_{a_n}(y_n)\}\}$

$\geq T\{(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(x_1, x_2, \dots, x_n),$

$(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(y_1, y_2, \dots, y_n)\}$

$\geq T\{(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(X),$

$(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n})(Y)\}$

Therefore, $(f_{a_1} \times f_{a_2} \times \dots \times f_{a_n}$ is fuzzy soft ring of $R_1 \times R_2 \times \dots \times R_n$.

Preposition 3.11

Let R and R´ be two ring and $\theta : R \to R'$ be a soft homomorphism. If f_b is a fuzzy soft ring of R then the pre-image $\theta^{-1}(f_b)$ is fuzzy soft ring of R.

Proof:
Assume that f_b is fuzzy soft ring of R´. Let x, y ∈ R.

(FSR1) $\mu_{\theta^{-1}(f_b)}(x + y) = \mu_{f_b}\ \theta(x + y)$

$= \mu_{f_b}(\theta x + \theta y)$

$\geq T\{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\}$

$\geq T\{\mu_{\theta^{-1}(f_b)}(x), \mu_{\theta^{-1}(f_b)}(y)\}$

(FSR2) $\mu_{\theta^{-1}(f_b)}(-x) = \mu_{f_b}\ \theta(-x)$

$\geq \mu_{f_b}\theta(x)$

$\geq \mu_{\theta^{-1}(f_b)}(x)$

(FSR3) $\mu_{\theta^{-1}(f_b)}(x\ y) = \mu_{f_b}\ \theta(x\ y)$

$= \mu_{f_b}((\theta x)(\theta y))$

$\geq T\{\mu_{f_b}\theta(x), \mu_{f_b}\theta(y)\}$

$\geq T\{\mu_{\theta^{-1}(f_b)}(x), \mu_{\theta^{-1}(f_b)}(y)\}$

Therefore $\theta^{-1}(f_b)$ is fuzzy soft ring of R´.

Preposition 3.12
Let θ: R → R´ be an epimorphism and f_b be fuzzy soft set in R´. If $\theta^{-1}(f_b)$ is fuzzy soft ring of R´ then f_b is fuzzy soft ring of R.

Proof
Let x, y ∈ R. Then there exist a, b ∈ R such that θ (a) = x, θ (b) = y. It follows that.

(FSR1) $\mu_{\theta(f_b)}(x + y) = \mu_{f_b}\ \theta(x + y)$

$= \mu_{f_b}(\theta x + \theta y)$

$\geq T\{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\}$

$\geq T\{\mu_{\theta(f_b)}(x), \mu_{\theta(f_b)}(y)\}$

(FSR2) $\mu_{\theta(f_b)}(-x) = \mu_{f_b}\,\theta(-x)$
$\geq \mu_{f_b}\,\theta(x)$
$\geq \mu_{\theta(f_b)}(x)$

(FSR3) $\mu_{\theta(f_b)}(x\,y) = \mu_{f_b}\,\theta(x\,y)$
$= \mu_{f_b}((\theta x)(\theta y))$
$\geq T\{\mu_{f_b}(\theta x), \mu_{f_b}(\theta y)\}$
$\geq T\{\mu_{f_b}\,\theta(x), \mu_{f_b}\,\theta(y)\}$
$\geq T\{\mu_{\theta(f_b)}(x), \mu_{\theta(f_b)}(y)\}$

Therefore $\theta(f_b)$ is fuzzy soft ring of R.

Proposition 3.13
If f_a is fuzzy soft ring of R and $\theta:R \to R'$ be a soft homomorphism of R then the fuzzy soft set

$$f_a^{\theta} = \{(x, f_a^{\theta}(x)) / x \in R\} \text{ is fuzzy soft ring of R.}$$

Proof
Let $x, y \in R$.

(FSR1) $f_a^{\theta}(x + y) = f_a\,\theta(x + y)$
$= f_a(\theta x + \theta y)$
$\geq T\{f_a(\theta x), f_a(\theta y)\}$
$\geq T\{f_a^{\theta}(x), f_a^{\theta}(y)\}$

(FSR2) $f_a^{\theta}(-x) = f_a(\theta(-x))$
$\geq f_a(\theta(x))$
$\geq f_a^{\theta}(x)$

(FSR3) $f_a^{\theta}(x\,y) = f_a(\theta(x\,y))$
$= f_a((\theta x)(\theta y))$
$\geq T\{f_a(\theta x), f_a(\theta y)\}$
$\geq T\{f_a^{\theta}(x), f_a^{\theta}(y)\}$

Therefore, f_a^{θ} is fuzzy soft ring of R.

Proposition 3.14
Let f_a be a fuzzy soft set over L, then f_a is fuzzy soft ring over L if and only if for all a

$\in$ A and for arbitrary $\alpha \in [0, 1]$ with $(f_a)_\alpha \neq 0$, then α– level soft set $(f_a)_\alpha$ is fuzzy soft ring over L.

Proof

Let f_a be fuzzy soft ring over L. Then for each $a \in A$, f_a is a fuzzy sub ring of L. For arbitrary $\alpha \in [0, 1]$, $(f_a)_\alpha \neq 0$. Let $x, y \in (f_a)_\alpha$. Then $f_a(x) \geq \alpha$ and $f_a(y) \geq \alpha$.

(FSR1) $\quad (f_a)_\alpha (x + y) \geq T \{(f_a)_\alpha (x), (f_a)_\alpha (y)\}$

$\geq T \{f_a(x), f_a(y) \}$

$\geq T \{\alpha, \alpha\}$

$\geq \alpha$

Therefore, $x + y \in (f_a)_\alpha$

(FSR2) $\quad (f_a)_\alpha (-x) \geq \{(f_a)_\alpha (x)$

$\geq (f_a)_\alpha (x)$

$\geq \alpha$

Therefore, $- x \in (f_a)_\alpha$

(FSR3) $\quad (f_a)_\alpha (x\, y) \geq T \{(f_a)_\alpha (x), (f_a)_\alpha (y)\}$

$\geq T \{f_a (x), f_a(y) \}$

$\geq T \{\alpha, \alpha\}$

Therefore, $xy \in (f_a)_\alpha$

Therefore $(f_a)_\alpha$ is a fuzzy soft ring of R.

Proposition 3.15

Every imaginable fuzzy soft ring μ of R is fuzzy soft ring of R.

Proof

Assume that μ is imaginable fuzzy soft ring of R. Then we have

$\mu (x + y) \geq T \{ \mu (x), \mu (y)\}$ and

$\mu (-x) \geq \mu (x)$, $\mu (xy) \geq T\{ \mu (x), \mu (y)\}$ for all $x, y \in R$.

Since μ is imaginable, we have.

$\min \{ \mu (x), \mu (y)\}= T \{ \min \{ \mu (x), \mu (y)\}, \min \{ \mu (x), \mu (y)\}\}$

$\leq T \{ \mu (x), \mu (y)\}$

$\leq \min \{ \mu (x), \mu (y)\}$

and so

$$T\{\mu(x), \mu(y)\} = \min\{\mu(x), \mu(y)\}$$

It follows that $\mu(x + y) \geq T\{\mu(x), \mu(y)\}$ for all $x, y \in R$.
Hence μ is fuzzy soft ring of R.

Proposition 3.16
If μ is fuzzy soft ring of R and θ is endomorphism of R then $\mu^{[\theta]}$ is fuzzy soft ring of R.

Proof
For any $x, y \in R$, we have

(FSR1) $\mu^{[\theta]}(x + y)) = \mu(\theta(x + y))$

$= \mu(\theta x + \theta y)$

$\geq T\{\mu(\theta x), \mu(\theta y)\}$

$\geq T\{\mu^{[\theta]}(x), \mu^{[\theta]}(y)\}$

(FSR2) $\mu^{[\theta]}(-x) = \mu(\theta(-x)$

$\geq \mu\,\theta(x)$

$\geq \mu^{[\theta]}(x)$

(FSR3) $\mu^{[\theta]}(xy) = \mu\,\theta(x\,y)$

$= \mu((\theta x)(\theta y))$

$\geq T\{\mu(\theta x), \mu(\theta y)\}$

$\geq T\{\mu\theta(x), \mu\,\theta(y)\}$

$\geq T\{\mu^{[\theta]}(x), \mu^{[\theta]}(y)\}$

Therefore, $\mu^{[\theta]}$ is fuzzy soft ring of R.

Proposition 3.17
Let T be a continuous t-norms and let f be a soft homomorphism on R. If μ is fuzzy soft ring of R then μ^f is fuzzy soft ring of f(R).

Proof:
Let $A_1 = f^{-1}(y_1)$ and $A_2 = f^{-1}(y_2)$ and

$A_{12} = f^{-1}(y_1 + y_2)$. Where y_1 , y_2 f(R).

Consider the set.

$$A_1 + A_2 = \{x \in R \,/\, x = a_1 + a_2\}, \text{ for some } a_1 \in A_1 \text{ and } a_2 \in A_2$$

If $x \in A_1 + A_2$ then $x = x_1 + x_2$ for some $x_1 \in A_1$ and $x_2 \in A_2$. So that we have

$$f(x) = f(x_1) + f(x_2)$$
$$= y_1 + y_2$$

Therefore, $x \in f^{-1}(y_1 + y_2) = A_{12}$
Thus $A_1 + A_2 \in A_{12}$
It follows that

(FSR1) $\mu^f(y_1 + y_2) = \sup\{\mu(x) \,/\, x \in f^{-1}(y_1 + y_2)\}$

$$= \sup\{\mu(x) \,/\, x \in A_{12}\}$$

$$= \sup\{\mu(x) \,/\, x \in A_1 + A_2\}$$

$$\geq \sup\{\mu(x_1 + x_2) \,/\, x_1 \in A_{1\,,}\, x_2 \in A_2\}$$

Since T is continuous, therefore for every $\in > 0$, we see that if
Sup $\{\{\mu(x_1) / x_1 \in A_1\} + x_1^* \leq \delta$ and
Sup $\{\{\mu(x_2) / x_2 \in A_2\} + x_2^* \leq \delta$
$T\{\text{ Sup }\{\{\mu(x_1) / x_1 \in A_1\}, \text{Sup }\{\{\mu(x_2) / x_2 \in A_2\}\} + T(x_1^*, x_2^*) \leq \in$

Choose $a_1 \in A_1$ and $a_2 \in A_2$ such that
Sup $\{\{\mu(x_1) / x_1 \in A_1\} + \mu(a_1) \leq \delta$ and
Sup $\{\{\mu(x_2) / x_2 \in A_2\} + \mu(a_2) \leq \delta$
$T\{\text{ Sup }\{\{\mu(x_1) / x_1 \in A_1\}, \text{Sup }\{\{\mu(x_2) / x_2 \in A_2\}\} + T(\mu(a_1), \mu(a_2)) \leq \in$
Consequently, we have
$\mu^f(y_1 + y_2) \geq \sup\{T\{\mu(x_1), \mu(x_2) / x_1 \in A_1, x_2 \in A_2\}\}$
$\geq T\{\sup\{\mu(x_1)/ x_1 \in A_1\}, \sup\{\mu(x_2) / x_2 \in A_2\}\}$
$\geq T\{\mu^f(y_1), \mu^f(y_2)\}$

Similarly, we can show that, $\mu^f(-x) \geq \mu^f(x)$, and

$$\mu^f(xy) \geq T\{\mu^f(x), \mu^f(y)\}$$

Hence μ^f is fuzzy soft ring of f(R).

Proposition 3.18
Onto homomorphic image of fuzzy soft ring with sup-property is fuzzy soft ring of R.

Proof
Let $f: R \to R'$ be an onto homomorphism of rings and μ be a sup property of fuzzy soft ring of R.

Let $x', y' \in R'$ and $x_0\, f^{-1}(x')$ and $y_0 \in f^{-1}(y')$ be such that

$$\mu(x_0) = \sup_{h \in f^{-1}(x')} \mu(h) \text{ and } \mu(y_0) = \sup_{h \in f^{-1}(y')} \mu(h)$$

respectively, then we can deduce that

(FSR1) $$\mu^f(x' + y') = \sup_{z \in f^{-1}(x' + y')} \mu(z)$$
$$\geq T\{\mu(x_0), \mu(y_0)\}$$
$$= T\{\sup_{h \in f^{-1}(x')} \mu(h), \sup_{h \in f^{-1}(y')} \mu(h)\}$$
$$\geq T\{\mu^f(x'), \mu^f(y')\}$$

(FSR2) $$\mu^f(-x') = \sup_{z \in f^{-1}(-x')} \mu(z)$$
$$\geq \mu(x_0)$$
$$= \sup_{h \in f^{-1}(x')} \mu(h)$$
$$\geq \mu^f(x')$$

(FSR3) $$\mu^f(x'y') = \sup_{z \in f^{-1}(x'y')} \mu(z)$$
$$\geq T\{\mu(x_0), \mu(y_0)\}$$
$$= T\{\sup_{h \in f^{-1}(x')} \mu(h), \sup_{h \in f^{-1}(y')} \mu(h)\}$$
$$\geq T\{\mu^f(x'), \mu^f(y')\}$$

Therefore μ^f is fuzzy soft ring of R´.

Proposition 3.19

Let f_a be a fuzzy soft ring over R and (ϕ, Ψ) be a fuzzy soft homomorphism from R to R´. Then $(\phi, \Psi)f_a$ is fuzzy soft ring over R´.

Proof:

Let $k \in (\Psi)f_a$ and $y_1, y_2 \in Y$. If $\phi^{-1}(y_1) = \phi$ or $\phi^{-1}(y_2) = \phi$.
The proof is straight forward.

Let us assume that, there exist $x_1, x_2 \in X$ such that

$$\phi(x_1) = y_1 \text{ and } \phi(x_2) = y_2.$$

Now

$\phi\ (f_a)_k\ (y_1 + y_2) = \vee\ \vee\ f_a(t)$

$\phi\ (t) = y_1 + y_2\ \Psi\ (a) = k$

$\geq \vee\ f_a(x_1 + x_2)$

$\Psi\ (a) = k$

$\geq \vee\ \{T\ \{f_a(x_1),\ f_a(x_2)\}$

$\Psi\ (a) = k$

$\geq T\ \{\ \vee\ \{f_a(x_1),\ \vee\ f_a(x_2)\}$

$\Psi\ (a) = k\ \Psi\ (a) = k$

This inequality is satisfy for each x_1, $x_2 \in X$, which satisfy

$\phi\ (x_1) = y_1,\ \phi\ (x_2) = y_2,$

then we have

(FSR1) $\phi\ (f_a)_k\ (y_1 + y_2) \geq T\ \{\ (\ \vee\ ,\ \vee\ f_{a_1}\ (t_1)\ )\ ,$

$\phi\ (t_1) = y_1\ \Psi\ (a) = k$

$(\ \vee\ ,\ \vee\ f_{a_2}\ (t_2)\ )\ \}$

$\phi\ (t_2) = y_2\ \Psi\ (a) = k$

$T\ \{\phi\ (f_a)_k\ (y_1),\ \phi\ (f_a)_k\ (y_2)\}$

And similarly, we can have

$\phi\ (f_a)_k\ (-y) \geq \phi\ (f_a)_k\ (y),$

$\phi\ (f_a)_k\ (y_1\ y_2) \geq T\ \{\phi\ (f_a)_k\ (y_1),\ \phi\ (f_a)_k\ (y_2)\}$

Therefore $(\phi, \Psi)f_a$ is fuzzy soft ring of R´.

Proposition 3.20

Let g_b be a fuzzy soft ring over R´ and (ϕ, Ψ) be a fuzzy soft homomorphism from R to R´, then $(\phi, \psi)^{-1}$ g_b is fuzzy soft ring over R.

Proof

Let $a \in \Psi^{-1}(B)$ and x_1, $x_2 \in X$.

(FSR1) $\phi^{-1}\ (g_a)\ (x_1 + x_2) = g_{\Psi\ (a)}\ (\phi\ (x_1\ .\ x_2)$

$= g_{\Psi\ (a)}\ (\phi\ (x_1)\ .\ (x_2))$

$\geq T\ \{\ g_{\Psi\ (a)}\ \phi(x_1)\ .\ g_{\Psi\ (a)}\ \phi\ (x_2)\}$

$$\geq T\{\phi^{-1}(g_a)(x_1), \phi^{-1}(g_a)(x_2)\}$$

and similarly, we have

$$\phi^{-1}(g_a)(-x) \geq \phi^{-1}(g_a)(x) \text{ and}$$

$$\phi^{-1}(g_a)(x_1 x_2) \geq \{\phi^{-1}(g_a)(x_1), \phi^{-1}(g_a)(x_2)\}$$

Therefore, $(\phi,\psi)^{-1} g_b$ is fuzzy soft ring over R.

Conclusions

This paper summarized the basic concept of fuzzy soft set. We then presented a detailed theoretical study of fuzzy soft set, which led to the definition of new algebraic structures in ring structures. This work focused on fuzzy soft rings, homomorphism of fuzzy soft rings, and pre-image of fuzzy soft rings. To extend this work one could study the properties of fuzzy soft sets in other algebraic structures such as near rings, Groups, ideals, fields and G-modules.

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