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A Constrained Stochastic Inventory Model: Fuzzy Geometric Programming and Intutionistic Fuzzy Geometric Programming Approach

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Abstract

A stochastic inventory model with deterministic constraint is analyzed here. First time we introduce the application of intuitionistic fuzzy geometric programming technique to solve this multi-objective inventory problem with uniform lead-time demand. Intuitionistic fuzzy geometric programming technique minimizes the expected annual cost more than the fuzzy geometric programming technique. Then this model is solved with fuzzy constraint. In this case fuzzy geometric programming technique perform better than fuzzy non-linear programming technique. Finally, all the numerical results are compared and analyzed.

Keywords: *Fuzzy optimization, Intuitionistic Fuzzy optimization, Geometric programming, Fuzzy Geometric programming, Intuitionistic Fuzzy Geometric programming.*

Mathematics Subject Classification Code: *90C15, 90C29, 90C70*.

Introduction

Geometric Programming (GP) is an effective method to solve a non-linear programming problem. It has certain advantages over the other optimization methods. Here, the advantages are that is usually much simpler to work with the dual than primal. Degree of Difficulty plays a significant role for solving a non-linear programming problem by GP method. Since late 1960, GP has been known and used in various fields (like OR, Engineering Sciences etc.). Duffin, Petersen and Zener (1966) discussed the basic theories with engineering applications in their books.

Another famous book on GP and its application appeared in Beightler and Philips (1976). There are many references on application and the methods of GP in the survey papers (like Eckar (1980), Beightler et.al. (1979), Zener (1971). Hariri et. al. (1997) discussed the multi-item production lot-size inventory model with varying order cost under a restriction Jung and Klain (2001) developed single item inventory problems and solved by GP method. Ata Fragany and Wakeel (2003) considered some inventory problems solved by GP technique. Zadeh (1965) first gave the concept of fuzzy set theory. Later on Bellman and Zadeh (1970) used the fuzzy set theory to the decision making problem Tanaka (1974) introduced the objective as fuzzy goal over the α-cut of a fuzzy constraint set and Zimmerman (1978) gave the concept to an inventory and production problem. Banerjee and Roy (2008) discussed the single and multi-objective stochastic inventory model in fuzzy environment. Constrained and unconstrained Stochastic Inventory Model with Fuzzy cost components and Fuzzy random variable was analyzed by Banerjee and Roy (2010). Cao (1993) and his recent book (2002) discussed fuzzy geometric programming with zero degree of difficulty. Das et. al. (2000) developed a multi-item inventory model with quantity dependent inventory costs and demand dependent unit cost under imprecise objective function and constraint and solved by GP technique. Roy and Maiti (1997) solved single objective fuzzy EOQ model by GP technique. Recently Mondal et. al. (2005) developed a multi-objective inventory model and solved it by GP method. A multi-objective fuzzy economic production quantity model is solved using GP approach by Islam and Roy (2004).

Intuitionistic Fuzzy Set (IFS) was introduced by K. Atanassov (1986) and seems to be applicable to real world problems. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in case where available information is not sufficient for the definition of an imprecise concept by means of a conventional fuzzy set. Thus it is expected that, IFS can be used to simulate human decision-making process and any activitities requiring human expertise and knowledge that are inevitably imprecise or totally reliable. Here the degree of rejection and satisfaction are considered so that the sum of both values is always less than unity (1986). Atanossov also analyzed Intuitionistic fuzzy sets in a more explicit way. Atanassov(1989) discussed an Open problems in intuitionistic fuzzy sets theory. An Interval valued intuitionistic fuzzy sets was analyzed by Atanassov and Gargov(1999). Atanassov and Kreinovich(1999) implemented Intuitionistic fuzzy interpretation of interval data. The temporal intuitionistic fuzzy sets are discussed also by Atanossov[1999]. Intuitionistic fuzzy soft sets are considered by Maji Biswas and Roy(2001). Nikolova, Nikolov, Cornelis and Deschrijver(2002) presented a Survey of the research on intuitionistic fuzzy sets. Rough intuitionistic fuzzy sets are analyzed by Rizvi, Naqvi and Nadeem(2002). Angelov (1997) implemented the Optimization in an intuitionistic fuzzy environment. He (1995) also contributed in his another two important papers, based on Intuitionistic fuzzy optimization. Pramanik and Roy (2005) solved a vector optimization problem using an Intuitionistic Fuzzy goal programming. A transportation model is solved by Jana and Roy (2007) using multi-objective intuitionistic fuzzy linear programming. Banerjee and Roy (2009) considered application of the Intuitionistic Fuzzy Optimization in the Constrained

Multi-Objective Stochastic Inventory Model. Banerjee and Roy (2010) also discussed the solution of Single and Multi-Objective Stochastic Inventory Models with Fuzzy Cost Components by Intuitionistic Fuzzy Optimization Technique.

A stochastic inventory model with deterministic and then with fuzzy constraint is analyzed here. We solve this multi-objective inventory problem with uniform lead-time demand by intuitionistic fuzzy geometric programming technique. We also compare the results solved by Fuzzy Geometric programming technique and it is observed that our Intuitionistic Fuzzy Geometric programming always performs better than the Fuzzy Geometric programming.

Mathematical Model

Backorder Case: Stockout Cost Per Unit

Here the policy is to order a lot size Q when the inventory level drops to a reorder point r ant it is supposed that the inventory position of an item is monitored after every transaction. The demand in any given interval of time is a random variable and the expected value of demand in a unit of time, say a year, is D. We let x denote the demand during the lead time and f(x) denote its probability distribution.

With backorders, there is no loss of sales, since the customer awaits the arrival of the order if stock is not available. The expected safety stock is defined as

$$S = \int_0^\infty (r-x) f(x)dx = r\int_0^\infty f(x)dx - \int_0^\infty xf(x)dx = r - \bar{x}$$

The number of backorders per lead time is zero if x – r < 0 and x – r if x – r > 0. The expected number of backorders per lead time is

$$E(x > r) = \int_r^\infty (x-r) f(x)dx$$

Here, annual safety stock cost = holding cost + stock out cost

i.e. $$TC = SH + \frac{KD}{Q}\int_r^\infty (x-r) f(x)dx$$

$$= H(r-\bar{x}) + \frac{KD}{Q}\int_r^\infty (x-r) f(x)dx$$

The following mathematical notations are used:

For the ith item:-

r_i = reorder point in units,

S_i = safety stock in units,

H_i = holding cost per unit of inventory per year,

K_i = backordering cost per unit,

x = lead time demand in units (a random variable),

$\bar{x}$ = average lead time demand in units,

$x - r_i$ = size of stock out in units

p_i = purchasing price of each product

TC = expected annual cost of safety stock, B=total budget

Multi Objective Stochastic Inventory Model with Deterministic Constraint

$$MinTC_i(Q_1,Q_2,......,Q_n,r_1,r_2,......,r_n) = S_iH_i + \frac{K_iD_i}{Q_i}\int_{r_i}^{\infty}(x-r_i)\,f_i(x)dx$$

subject to the constraints

$$\sum_{i=1}^{n} p_iQ_i \le B \quad l_{Q_i} \le Q_i \le u_{Q_i}, \qquad l_{r_i} \le r_i \le u_{r_i} \tag{2.1}$$

Multi Objective Stochastic Inventory Model with Fuzzy Constraint

$$Mi\tilde{n}TC_i(Q_1,Q_2,......,Q_n,r_1,r_2,......,r_n) = S_iH_i + \frac{K_iD_i}{Q_i}\int_{r_i}^{\infty}(x-r_i)\,f_i(x)dx$$

subject to the constraints

$$\sum_{i=1}^{n} p_iQ_i \le \tilde{B} \quad Q_i, r_i > 0 \forall i = 1,2,......., n. \tag{2.2}$$

(Here wavy bar '~' indicates "fuzzification" of the parameters).

Mathematical Analysis

Geometric Programming Problem

Geometric programming (GP) can be considered to be an innovative modus operandi to solve a nonlinear problem in comparison with other nonlinear techniques. It was originally developed to design engineering problems. It has become a very popular technique since its inception in solving nonlinear problems. The advantages of this method is that, this technique provides us with a systematic approach for solving a class of nonlinear optimization problems by finding the optimal value of the objective function and then the optimal values of the design variables are derived. Also. This method often reduces a complex nonlinear optimization problem to a set of simultaneous equations and this approach is more amenable to the digital computers.

GP is an optimization problem of the form:

$$Min\, g_0(t) \tag{3.1}$$

subject to

$g_j(t) \le 1$,

j = 1, 2, ………, m.

$h_k(t) = 1$, k=1, 2, ……….., p

$t_i > 0$, I = 1, 2, ………., n

where, $g_j(t)$ (j = 1, 2, ………, m) are posynomial or signomial functions and $h_k(t)$ k=1, 2, ……….., p) are monomials t_i (i = 1, 2, ………., n) are decision variable vector of n components t_i (i = 1, 2, ………., n).

The problem (3.1) can be written as:

$$Min\, g_0(t)$$

subject to

$g'_j(t) \le 1$, j = 1, 2, ………, m.

t > 0, [since $g_j(t) \le 1$, $h_k(t) = 1 \Rightarrow g'_j(t) \le 1$ where $g'_j(t)(= g_j(t)/h_k(t))$ be a posynomial(j=1, 2, ………, m ; k=1, 2, ………, p)].

Posynomial Geometric Programming Problem

A Primal problem

$$Min\ g_0(t) \tag{3.1.1}$$

subject to

$g_j(t) \le 1$, j = 1, 2, ………, m.

$t_i > 0$, (i =1, 2, …………..,n)

where $g_j(t) = \sum_{k=1}^{N_j} c_{jk} \prod_{i=1}^{n} t_i^{\alpha_{jki}}$

here, $c_{jk} > 0$ and α_{jki} (i=1, 2, ………,n ; k=1, 2, ………, N_j ; j=0, 1, ………,m) are real numbers.

$T=(t_1, t_2, ………., t_n)^T$.

It is a constrained posynomial primal geometric problem (PGP). The number of inequality constraints in the problem (3.1.1) is m. The number of terms in each posynomial constraint function varies and is denoted by N_j for each j=0, 1, 2, ……, m.

The degree of difficulty (DD) of a GP is defined as (number of terms in a PGP) – (number of variables in PGP)-1.

Dual Problem

The dual problem of (3.1.1) is as follows:

Max $d(w) = \prod_{j=0}^{m} \prod_{k=1}^{N_j} \left(\frac{c_{jk} w_{j0}}{w_{jk}} \right)^{w_{jk}}$

Subject to

$\sum_{k=1}^{N_0} w_{0k} = 1$ (normality condition)

$\sum_{j=0}^{m} \sum_{k=1}^{N_j} \alpha_{jki} w_{jk} = 0$, (i=1, 2, ………, n) (orthogonality condition)

$w_{j0} = \sum_{k=1}^{N_0} w_{jk} \ge 0$, $w_{jk} \ge 0$, (i=1, 2, ………,n ; k=1, 2, ………, N_j), $w_{00} = 1$.

There are n+1 independent dual constraint equalities and $N = \sum_{j=1}^{m} N_j$ independent dual variables for each term of primal problem. In this case DD=N-n-1.

Signomial Geometric Programming Problem

Primal problem

$$Min\, g_0(t) \tag{3.1.2}$$

subject to

$g_j(t) \le \delta_j$, j = 1, 2, ………, m.

$t_i > 0$, (i =1, 2, …………..,n)

where $g_j(t) = \sum_{k=1}^{N_j} \delta_{jk} c_{jk} \prod_{i=1}^{n} t_i^{\alpha_{jki}}$

here, $c_{jk} > 0$ and α_{jki} $\delta_j = \pm 1$ (j = 2, …..,m)

$\delta_{jk} = \pm 1$ (k=1, 2, ………, N_j ; j= 1, ………,m) are real numbers.

$T=(t_1, t_2, ………., t_n)^T$.

Dual Problem

The dual problem of (8.1.1) is as follows:

$$\text{Max}\, d(w) = \delta_0 (\prod_{j=0}^{m} \prod_{k=1}^{N_j} \left(\frac{c_{jk} w_{j0}}{w_{jk}} \right)^{\alpha_{jk} w_{jk}})^{\delta_0} \tag{3.1.3}$$

Subject to

$\sum_{k=1}^{N_0} \delta_{0k} w_{0k} = \delta_0$ (normality condition)

$\sum_{j=0}^{m} \sum_{k=1}^{N_j} \delta_{jk} \alpha_{jki} w_{jk} = 0$, (i=1, 2, ………, n) (orthogonality condition)

$\delta_j = \pm 1$ (j = 2, …..,m) $\delta_0 = +1, -1$.

$\delta_{jk} = \pm 1$ (k=1, 2, ………, N_j ; j= 1, ………,m) are real numbers.

$w_{j0} = \delta_j \sum_{k=1}^{N_0} \delta_{jk} w_{jk} \ge 0$, $w_{jk} \ge 0$, (j=1, 2, ………,m ; k=1, 2, ………, N_j), $w_{00} = 1$.

Functional Substitution

When a non-linear programming problem (NLP) is of the following form:

$$Min y(x) = f(x) + (q(x))^n h(x) \qquad x > 0, \quad n > 0.$$

Where, $f(x)$, $q(x)$ and $h(x)$ are single or multi-term functionals of posynomial or signomial form. This generalized formulation is not directly solvable using geometric programming; however, under a simple transformation it can be changed into standard geometric programming form. Let $P = q(x)$ and replace the above problem with the following one:

$$Min \bar{y}(x) = f(x) + P^n h(x)$$

subject to

$$P^{-1}(q(x)) \le 1$$

$$x, P > 0.$$

The rationale used in constructing the equivalent problem with an inequality constraint is based on the following logic. Since $y(x)$ is to be minimized, if $q(x)$ is replaced by P, then it is correct to say that $P \geq q(x)$, realizing that in the minimization process P will remain as small as possible. Hence $P = q(x)$ at optimality. Note that $h(x)$ and/or $q(x)$ are permitted to be multiple term expressions and that the optimal (minimizing) solution to $\bar{y}(x)$ is obviously the same as the optimal solution to $y(x)$.

Fuzzy Non-linear Programming (FNLP) Technique to Solve Multi-Objective Non-Linear Programming Problem (MONLP)

A Multi-Objective Non-Linear Programming (MONLP) or Vector Minimization problem (VMP) may be taken in the following form:

$$M\inf(x) = (f_1(x), f_2(x), \ldots\ldots.., f_k(x))^T$$

Subject to $x \in X = \{x \in R^n : g_j(x) \leq or = or \geq b_j \; forj = 1,2,\ldots\ldots, m\}$ (3.2.1)

and $l_i \leq x \leq u_i \, (i = 1,2,\ldots., n)$

Zimmermann (1978) showed that fuzzy programming technique could be used to solve the multi-objective programming problem.

To solve MONLP problem, following steps are used:

Step 1: Solve the MONLP of equation (3.2.1) as a single objective non-linear programming problem using only one objective at a time and ignoring the others, these solutions are known as ideal solution.

Step 2: From the result of step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{array}{c c} & \begin{array}{cccc} f_1(x) & f_2(x) & \ldots\ldots.. & f_k(x) \end{array} \\ \begin{array}{c} x^1 \\ x^2 \\ \ldots. \\ x^k \end{array} & \begin{bmatrix} f_1^*(x^1) & f_2(x^1) & \ldots. & f_k(x^1) \\ f_1(x^2) & f_2^*(x^2) & \ldots. & f_k(x^2) \\ \ldots. & \ldots. & \ldots. & \ldots. \\ f_1(x^k) & f_2(x^k) & \ldots. & f_k^*(x^k) \end{bmatrix} \end{array}$$

Here $x^1, x^2, \ldots\ldots, x^k$ are the ideal solutions of the objective functions $f_1(x), f_2(x), \ldots\ldots., f_k(x)$ respectively.

So $U_r = \max\{f_r(x_1), f_r(x_2), \ldots\ldots. f_r(x_k)\}$

and $L_r = \min\{f_r(x_1), f_r(x_2), \ldots\ldots. f_r(x_k)\}$

[L_r and U_r be lower and upper bounds of the r^{th} objective functions $f_r(x)$ $r = 1,2,\ldots.., k)$]

Step 3: Using aspiration level of each objective of the MONLP of equation (3.2.1) may be written as follows:

Find x so as to satisfy

$f_r(x) \tilde{\leq} L_r \qquad (r = 1,2,........,k)$
$x \in X$

Here objective functions of equation (3.2.1) are considered as fuzzy constraints. These type of fuzzy constraints can be quantified by eliciting a corresponding membership function:

$\mu_r(f_r(x) = 0$ or $\to 0$ if $f_r(x) \geq U_r$
$= \mu_r(f_r(x)$ if $L_r \leq f_r(x) \leq U_r \quad (r = 1,2,........,k)$
$= 1$ if $f_r(x) \leq L_r$ (3.2.2)

Having elicited the membership functions (as in equation (3.2.2)) $\mu_r(f_r(x)$ for r = 1, 2, …… , k, introduce a general aggregation function

$\mu_{\tilde{D}}(x) = G(\mu_1(f_1(x)), \mu_2(f_2(x)), ..., \mu_k(f_k(x))).$

So a fuzzy multi-objective decision making problem can be defined as

Max $\mu_{\tilde{D}}(x)$

subject to $x \in X$ (3.2.3)

Here we adopt the fuzzy decision as:

Fuzzy decision based on minimum operator (like Zimmermann's approach (1978). In this case equation (3.2.3) is known as $FNLP_M$.

Then the problem of equation (3.2.3), using the membership function as in equation (3.2.2), (according to addition operator)

Max $\sum_{r=1}^{k} \mu_r[f_r(x)]$

Subject to

$x \in X, \quad 0 \leq \mu_r[f_r(x)] \leq 1, \quad r = 1, 2, ..., k$ (3.2.4)

Step 4: Solve the equation (3.2.4) to get optimal solution.

We apply Fuzzy Programming Technique to solve MOSIM of section 4 and thus according to step 2 Pay-off matrix is formulated as follows:

$$\begin{array}{cc} & \begin{array}{cc} TC_1(Q_1,r_1) & TC_2(Q_2,r_2) \end{array} \\ \begin{array}{c} Q^1 \\ Q^2 \end{array} & \begin{bmatrix} TC_1*(Q_1^{\,1},r_1^{\,1}) & TC_2(Q_2^{\,1},r_2^{\,1}) \\ TC_1(Q_1^{\,2},r_1^{\,2}) & TC_2*(Q_2^{\,2},r_2^{\,2}) \end{bmatrix} \end{array}$$

Now, U_1, L_1, U_2, L_2 (where $L_1 \leq TC_1(Q_1,r_1) \leq U_1$ and $L_2 \leq TC_2(Q_2,r_2) \leq U_2$) are identified and $Q^1 = (Q_1^{\,1}, r_1^{\,1})$, $Q^2 = (Q_2^{\,2}, r_2^{\,2})$ are the ideal solutions of the objective functions $TC_1(Q_1,r_1)$ and $TC_2(Q_2,r_2)$.

Here, for simplicity, fuzzy linear membership functions μ_{TC_1} and μ_{TC_2} for the objective functions $TC_1(Q_1,r_1)$ and $TC_2(Q_2,r_2)$ respectively are identified as follows:

$$\mu_{TC_i}(Q_i, r_i) = \begin{cases} 0 & \text{for } \mathrm{TC_i}(Q_1, r) \le \mathrm{L_i} \\ \dfrac{U_i - \mathrm{TC_i}(Q_i, r_i)}{U_i - L_i} & \text{for } \mathrm{L_i} \le \mathrm{TC_i}\ (Q_i, r_i) \le \mathrm{U_i} \\ 1 & \text{for } \mathrm{TC_i}(Q_i, r_i) \ge \mathrm{U_i} \end{cases}$$

$\forall \mathrm{i} = 1,2.$

Weights in FNLP

Here, positive weights w_r reflect the decision maker's preferences regarding the relative importance of each objective goal $f_r(x)$ for r = 1, 2, ….. , k. These weights can be normalized by taking $\sum w_i = 1$. In the fuzzy non-linear programming the decision maker assigns different weights as coefficients of the individual terms in simple additive/ product achievement function to reflect their relative importance.

To achieve the same objective, suitable inverse weights are assigned to different membership functions in the fuzzy non- linear programming $FNLP_M$ method. So introducing normalized weights in FNLP, using additive operator (3.2.4) becomes,

$$\text{Max } \sum_{r=1}^{k} w_r \mu_r (f_r(x))$$

subject to

$x \in X$, $0 \le \mu_r(f_r(x)) \le 1$ for r = 1, 2, ……, k

where $\sum_{r=1}^{k} w_r = 1$, $0 < w_r < 1$ (for r = 1, 2, …, k)

Fuzzy Geometric Programming Problem

Multi-objective geometric programming (MOGP) is a special type of a class of MONLP problems. Biswal (1992) and Verma(1990) developed a fuzzy geometric programming technique to solve a MOGP problem. Here, we have discussed a fuzzy geometric programming technique based on max-min and max-convex combination operators to solve a MOGP.

To solve the MOGP we use the Zimmerman's technique. The procedure consists of the following steps.

Step 1. Solve the MOGP as a single GP problem using only one objective at a time and ignoring the others. These solutions are known as ideal solutions. Repeat the process k times for k different objectives. Let $x^1, x^2, \ldots\ldots\ldots, x^k$ be the ideal solutions for the respective objective functions, where

$$x^r = (x_1^r, x_2^r, \ldots\ldots\ldots\ldots\ldots\ldots, x_n^r)$$

Step 2. From the ideal solutions of Step1, determine the corresponding values for every objective at each solution derived. With the values of all objectives at each solution, the pay-off matrix of size (k x k) can be formulated as follows:

$$\begin{array}{c|cccc} & f_1(x) & f_2(x) & \cdots\cdots & f_k(x) \\ \hline x^1 & f_1^*(x^1) & f_2(x^1) & \cdots & f_k(x^1) \\ x^2 & f_1(x^2) & f_2^*(x^2) & \cdots & f_k(x^2) \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ x^k & f_1(x^k) & f_2(x^k) & \cdots & f_k^*(x^k) \end{array}$$

Step 3. From the Step 2, find the desired goal L_r and worst tolerable value U_r of $f_r(x)$, r = 1, 2, …….., k as follows:

$L_r \le f_r \le U_r$, r = 1, 2, …….., k

Where, $U_r = \max \{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$

$L_r = \min \{f_r(x^1), f_r(x^2), \ldots\ldots, f_r(x^k)\}$

Step 4. Define a fuzzy linear or non-linear membership function $\mu_r\,[f_r(x)]$ for the r-th objective function $f_r(x)$, r = 1, 2, …….., k

$\mu_r\,[f_r(x)] = 0$ or $\rightarrow 0$ if $f_r(x) \ge U_r$

$= d_r(x)$ if $L_r \le f_r(x) \le U_r$ (r = 1, 2, …… , k)

$= 1$ or $\rightarrow 1$ if $f_r(x) \le L_r$

Here $d_r(x)$ is a strictly monotonic decreasing function with respect to $f_r(x)$.

Step 5. At this stage, either a max-min operator or a max-convex combination operator can be used to formulate the corresponding single objective optimization problem.

Formulation of Intuitionistic Fuzzy Optimization [IFO]

When the degree of rejection (non-membership) is defined simultaneously with degree of acceptance (membership) of the objectives and when both of these degrees are not complementary to each other, then IF sets can be used as a more general tool for describing uncertainty.

To maximize the degree of acceptance of IF objectives and constraints and to minimize the degree of rejection of IF objectives and constraints, we can write:

$$\max \mu_{\mathrm{i}}(\overline{X}), \overline{X}\varepsilon R, \mathrm{i} = 1,2,\ldots\ldots,\mathrm{K}+\mathrm{n}$$

$$\min \upsilon_{\mathrm{i}}(\overline{X}), \overline{X}\varepsilon R, \mathrm{i} = 1,2,\ldots\ldots,\mathrm{K}+\mathrm{n}$$

Subject to

$$\upsilon_i(\overline{X}) \ge 0,$$

$$\mu_i(\overline{X}) \ge \upsilon_i(\overline{X})$$

$$\mu_i(\overline{X}) + \upsilon_i(\overline{X}) < 1$$

$$\overline{X} \ge 0$$

Where $\mu_i(\overline{X})$ denotes the degree of membership function of $(\overline{X})$ to the i^{th} IF sets and $\nu_i(\overline{X})$ denotes the degree of non-membership (rejection) of $(\overline{X})$ from the i^{th} IF sets.

An Intuitionistic Fuzzy Approach for Solving MOIP with Linear Membership and Non-Membership Functions

To define the membership function of MOIM problem, let L_k^{acc} and U_k^{acc} be the lower and upper bounds of the k^{th} objective function. These values are determined as follows: Calculate the individual minimum value of each objective function as a single objective IP subject to the given set of constraints. Let $\overline{X}_1^*, \overline{X}_2^*, ...\overline{X}_k^*$ be the respective optimal solution for the k different objective and evaluate each objective function at all these k optimal solution. It is assumed here that at least two of these solutions are different for which the k^{th} objective function has different bounded values. For each objective, find lower bound (minimum value) L_k^{acc} and the upper bound (maximum value) U_k^{acc}. But in intuitionistic fuzzy optimization (IFO), the degree of rejection (non-membership) and degree of acceptance (membership) are considered so that the sum of both values is less than one. To define membership function of MOIM problem, let L_k^{rej} and U_k^{rej} be the lower and upper bound of the objective function $Z_k(\overline{X})$ where $L_k^{acc} \le L_k^{rej} \le U_k^{rej} \le U_k^{acc}$. These values are defined as follows:

The linear membership function for the objective $Z_k(\overline{X})$ is defined as:

$$\mu_k(Z_k(\overline{X})) = \begin{cases} 1 & \text{if } Z_k(\overline{X}) \le L_k^{acc} \\ \dfrac{U_k^{acc} - Z_k(\overline{X})}{U_k^{acc} - L_k^{acc}} & \text{if } L_k^{acc} \le Z_k(\overline{X}) \le U_k^{acc} \\ 0 & \text{if } Z_k(\overline{X}) \ge U_k^{acc} \end{cases} \tag{3.5.1}$$

$$\nu_k(Z_k(\overline{X})) = \begin{cases} 1 & \text{if } Z_k(\overline{X}) \ge U_k^{rej} \\ \dfrac{Z_k(\overline{X}) - L_k^{rej}}{U_k^{rej} - L_k^{rej}} & \text{if } L_k^{rej} \le Z_k(\overline{X}) \le U_k^{rej} \\ 0 & \text{if } Z_k(\overline{X}) \le L_k^{rej} \end{cases} \tag{3.5.2}$$

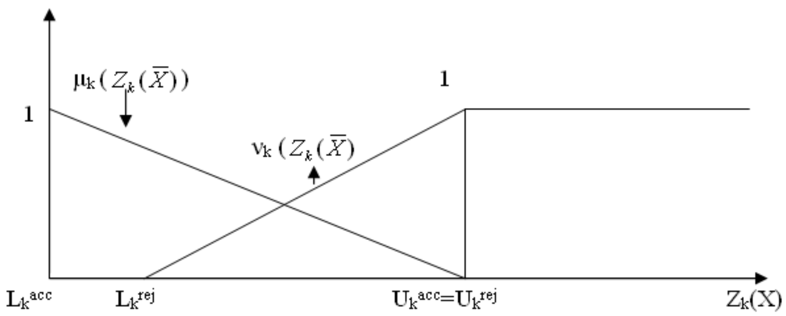

Figure-1 : Membership and non-membership functions of the objective goal

Lemma: In case of minimization problem, the lower bound for non-membership function (rejection)) is always greater than that of the membership function (acceptance).

Now, we take new lower and upper bound for the non-membership function as follows:

$$L_k^{rej} = L_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ where } 0 < t < 1$$
$$U_k^{rej} = U_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ for } t = 0$$

Following the fuzzy decision of Bellman-Zadeh [2] together with linear membership function and non-membership functions of (3.5.1) and (3.5.2), an intuitionistic fuzzy optimization model of MOIM problem can be written as:

$$\begin{aligned} &\max \mu_k(\overline{X}), \overline{X}\varepsilon R, \text{k} = 1,2,\ldots\ldots,\text{K} \\ &\min \upsilon_k(\overline{X}), \overline{X}\varepsilon R, \text{k} = 1,2,\ldots\ldots,\text{K} \end{aligned} \tag{3.5.3}$$

Subject to

$$\upsilon_k(\overline{X}) \geq 0,$$
$$\mu_k(\overline{X}) \geq \upsilon_k(\overline{X})$$
$$\mu_k(\overline{X}) + \upsilon_k(\overline{X}) < 1$$
$$\overline{X} \geq 0$$

The problem of equation (3.5.3) can be reduced following Angelov (1997) to the following form:

$$Max\ \alpha - \beta$$

Subject to

$$Z_k(\overline{X}) \leq U_k^{acc} - \alpha(U_k^{acc} - L_k^{acc})$$
$$Z_k(\overline{X}) \leq L_k^{rej} + \beta(U_k^{rej} - L_k^{rej})$$
$$\beta \geq 0$$
$$\alpha \geq \beta$$
$$\alpha + \beta < 1$$
$$\overline{X} \geq 0$$

Then the solution of the MOIM problem is summarized in the following steps:

Step 1. Pick the first objective function and solve it as a single objective IP subject to the constraint, continue the process K-times for K different objective functions. If all the solutions (i.e. $\overline{X}_1^* = \overline{X}_2^* = \ldots\ldots = \overline{X}_k^* (i = 1,2,\ldots\ldots,m;\ \text{j} = 1,2,\ldots\ldots,\text{n})$ same, then one of them is the optimal compromise solution and go to step 6. Otherwise go to step 2. However, this rarely happens due to the conflicting objective functions.

Then the intuitionistic fuzzy goals take the form

$$Z_k(\overline{X}) \mathrel{\tilde{\leq}} L_k(\overline{X})^*{}_k\ \ k = 1,2,\ldots\ldots\ldots,K.,$$

Step 2. To build membership function, goals and tolerances should be determined at first. Using the ideal solutions, obtained in step 1, we find the values of all the objective functions at each ideal solution and construct pay off matrix as follows:

$$\begin{bmatrix} Z_1(\overline{X}_1^{\ *}) & Z_2(\overline{X}_1^{\ *}) & .... & .... & .... & Z_k(\overline{X}_1^{\ *}) \\ Z_1(\overline{X}_2^{\ *}) & Z_2(\overline{X}_2^{\ *}) & .... & .... & .... & Z_k(\overline{X}_2^{\ *}) \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ .... & .... & .... & .... & .... & .... \\ Z_1(\overline{X}_k^{\ *}) & Z_2(\overline{X}_k^{\ *}) & .... & .... & .... & Z_k(\overline{X}_k^{\ *}) \end{bmatrix}$$

Step 3. From Step 2, we find the upper and lower bounds of each objective for the degree of acceptance and rejection corresponding to the set of solutions as follows:

$$U_k^{acc} = \max_{1 \le r \le k}(Z_k(\overline{X}_r^{\ *})) \quad \text{and} \quad L_k^{acc} = \min_{1 \le r \le k}(Z_k(\overline{X}_r^{\ *}))$$

For linear membership functions,

$$L_k^{rej} = L_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ where } 0 < t < 1$$

$$U_k^{rej} = U_k^{acc} + t(U_k^{acc} - L_k^{acc}) \text{ for } t = 0$$

Step 4. Construct the fuzzy programming problem of equation (3.5.3) and find its equivalent LP problem of equation (3.5.4).

Step 5. Solve equation (3.5.4) by using appropriate mathematical programming algorithm to get an optimal solution and evaluate the K objective functions at these optimal compromise solutions

Step 6. STOP.

Solution of Different Models

Stochastic Model: Demand follows Uniform distribution

We assume that demand for the period for the ith item is a random variable which follows uniform distribution and if the decision maker feels that demand values for item i below a_i or above b_i are highly unlikely and values between a_i and b_i are equally likely, then the probability density function $f_i(x)$ are given by:

$$f_i(x) = \begin{cases} \dfrac{1}{b_i - a_i} & \text{if } a_i \le x \le b_i \\ 0 & \text{otherwise} \end{cases} \qquad \text{for } i = 1, 2, \ldots, n.$$

So, $TC(Q_1, Q_2, \ldots\ldots, Q_n, r_1, r_2, \ldots\ldots, r_n) = \sum_{i=1}^{n} (H_i(r_i - \mu_i) + \dfrac{D_i K_i}{2Q_i(b_i - a_i)}(b_i - r_i)^2)$,

And $TC_i(Q_i, r_i) = H_i(r_i - \mu_i) + \dfrac{K_i D_i (b_i - r_i)^2}{2Q_i(b_i - a_i)}$

Where, $\mu_i = \dfrac{a_i + b_i}{2}$.

Solution of Multi-objective Stochastic Inventory Model with fuzzy constraint by Geometric Programming Technique

The model (2.2) can be formulated as an equivalent non-linear programming problem following Bellman and Zadeh(1970), Tiwari, Dharmar and Rao(1987) as:

$$MaxW(Q_1, Q_2, ..., Q_n, r_1, r_2, ..., r_n) =$$

$$\sum_{i=1}^{n}(w_i \mu_{TC_i}(Q_1, Q_2, ..., Q_n, r_1, r_2, ..., r_n) + w_B \mu_B(Q_1, Q_2, ...Q_n)) \qquad (4.2.1)$$

Subject to

$$\mu_{TC_i}(Q_1, Q_2, ..., Q_n, r_1, ..., r_n) = 1 - \frac{TC_i(Q_1, Q_2, ..., Q_n, r_1, ..., r_n) - TC_i'}{\delta_i}$$

$$\mu_B(Q_1, Q_2, ...Q_n) = 1 - \frac{\sum_{i=1}^{n} p_i Q_i - B}{\delta_B}$$

$$0 \le \mu_{TC_i}(Q_1, Q_2, ..., Q_n, r_1, ..., r_n) \le 1, \; 0 \le \mu_B(Q_1, Q_2, ...Q_n) \le 1$$

$$\delta_i = TC_{0i} - TC_i'$$

Here w_i and w_B are positive weights of $TC_i(Q_1, Q_2, ..., Q_n, r_1, r_2, ..., r_n)$ and budgetary constraints. They are also considered as normalized weights as:

$$\sum_{i=1}^{n} w_i + w_B = 1$$

(4.2.1) is equivalent to:

$$MinV(Q_1, Q_2, ..., Q_n, r_1, r_2, ..., r_n) =$$

$$\sum_{i=1}^{n}\left(\frac{w_i}{\delta_i} TC_i(Q_1, Q_2, ..., Q_n, r_1, r_2, ..., r_n) + \frac{w_B}{\delta_B} p_i Q_i\right)$$

When the Demand follows uniform distribution, using the section 4.1 and suppressing the term $H_i \mu_i$ as, it is a constant we get:

$$W(Q_1, Q_2, ...Q_n, r_1, r_2, ...r_n) = \left(\sum_{i=1}^{n}\left(w_i + \frac{w_i TC_i'}{\delta_i}\right) + w_B + \frac{w_B B}{\delta_B}\right) - V(Q_1, Q_2, ...Q_n, r_1, r_2, ...r_n)$$

As the first expression of the right hand side of the above expression is independent of the decision variables so omitting it and using section 3.1.3, we consider the problem given below to the standard form of signomial geometric programming problem as:

$$MinV(Q_1, Q_2, ..., Q_n, r_1, r_2, ..., r_n, M_1, M_2, ..., M_n) =$$

$$Min\sum_{i=1}^{n}(C_{1i} r_i + C_{2i} M_i^{\,2} Q_i^{-1} + C_{3i} Q_i)$$

Subject to

$C_{4i} M_i^{-1} - C_{5i} M_i^{-1} r_i \le 1$

$Q_i, r_i, M_i \ge 0$, $\forall i = 1,2,......., n.$

Where, $C_{1i} = \dfrac{w_i H_i}{\delta_i}, C_{2i} = \dfrac{D_i K_i}{2(b_i - a_i)}, C_{3i} = \dfrac{w_B p_i}{\delta_B}, C_{4i} = b_i, C_{5i} = 1.$

Clearly, (For i= 1, 2) it is a constrained signomial geometric programming problem with degree of difficulty 10 – 6 – 1= 3

The dual of the above signomial geometric programming problem for two items (i.e. for i=1, 2) can be written as:

$Max d(w) =$

$$\prod_{i=1}^{2}\left(\left(\frac{C_{1i}}{w_{1i}}\right)^{w_{1i}}\left(\frac{C_{2i}}{w_{2i}}\right)^{w_{2i}}\left(\frac{C_{3i}}{w_{3i}}\right)^{w_{3i}}\left(\frac{C_{4i}}{w_{4i}}\right)^{\sigma_{4i}w_{4i}}\left(\frac{C_{5i}}{w_{5i}}\right)^{\sigma_{5i}w_{5i}}\left(\sum_{j=1}^{2} w_{4j}\sigma_{4j}\right)^{\sigma_{4i}w_i}\left(\sum_{j=1}^{2} w_{5j}\sigma_{5j}\right)^{\sigma_{5i}w_i}\right)$$

Where, $\sigma_{1i} = 1$, $\sigma_{2i} = 1$, $\sigma_{3i} = 1$, $\sigma_{4i} = 1$, $\sigma_{5i} = -1$, $\sigma_1 = 1 = \sigma_2$ (for i=1, 2).

Using section 3.1 we have, for i = 1, 2:

$w_{1i} - w_{5i} = 0$, $w_{3i} - w_{2i} = 0$, $2w_{2i} - w_{4i} + w_{5i} = 0$

and $\sum_{i=1}^{2}(w_{1i} + w_{2i} + w_{3i}) = 1$

So, $w_{31} = w_{21}$

$w_{41} = 2w_{21} + w_{11}$

$w_{51} = w_{11}$

$w_{22} = w_{32} = \dfrac{1 - w_{11} - w_{12} - 2w_{21}}{2}$

$w_{42} = 1 - w_{11} - 2w_{21}$

Thus $d(w)$ can be expressed in terms of w_{11}, w_{12}, w_{21} and to determine $Max d(w)$ we also get the following equations:

$$\frac{\partial d}{\partial w_{11}} = 0 = \frac{\partial d}{\partial w_{12}} = \frac{\partial d}{\partial w_{21}}$$

Now

$$\log d = \sum_{i=1}^{2}\Big(w_{1i}(\log\frac{C_{1i}}{w_{1i}}) + w_{2i}(\log\frac{C_{2i}}{w_{2i}}) + w_{3i}(\log\frac{C_{3i}}{w_{3i}}) + \sigma_{4i}w_{4i}(\log\frac{C_{4i}}{w_{4i}} + \log\sum_{j=1}^{2}\sigma_{4j}w_{4j})$$

$$+\sigma_{5i}w_{5i}(\log\frac{C_{5i}}{w_{5i}} + \log\sum_{j=1}^{2}\sigma_{5j}w_{5j})\Big)$$

$$\frac{1}{d}\frac{\partial d}{\partial w_{11}} = (\log\frac{C_{11}}{w_{11}} + \frac{1}{2}(\log\frac{C_{22}}{w_{22}} - \log\frac{C_{32}}{w_{32}}) + (\log\frac{C_{41}}{w_{41}} - \log\frac{C_{42}}{w_{42}}) - \sigma_{51}(1 + \log w_{51})$$

$$+\sigma_{41}\log(\sum_{j=1}^{2}\sigma_{4j}w_{4j}) - \sigma_{42}\log(\sum_{j=1}^{2}\sigma_{4j}w_{4j}) + \sigma_{41}w_{41}\left(\frac{\sigma_{41}-\sigma_{42}}{\sum_{j=1}^{2}\sigma_{4j}w_{4j}}\right) + \sigma_{42}w_{42}\left(\frac{\sigma_{41}-\sigma_{42}}{\sum_{j=1}^{2}\sigma_{4j}w_{4j}}\right)$$

$$+\sigma_{51}(\log\sum_{j=1}^{2}\sigma_{5j}w_{5j})+\sigma_{51}w_{51}(\frac{\sigma_{51}}{\sum_{j=1}^{2}\sigma_{5j}w_{5j}})=0$$

$$\frac{1}{d}\frac{\partial d}{\partial w_{12}}=(\log\frac{C_{12}}{w_{12}}-\frac{1}{2}(\log\frac{C_{22}}{w_{22}}-\log\frac{C_{32}}{w_{32}})-\sigma_{52}(1+\log w_{52}-\frac{w_{52}\sigma_{52}}{\sum_{j=1}^{2}\sigma_{5j}w_{5j}}-\log\sum_{j=1}^{2}\sigma_{5j}w_{5j})=0$$

$$\frac{1}{d}\frac{\partial d}{\partial w_{21}}=(\log\frac{C_{21}}{w_{21}}-2\log\frac{C_{42}}{w_{42}}-\log\frac{C_{32}}{w_{32}}+\log\frac{C_{41}}{w_{41}}+\log\frac{C_{31}}{w_{31}})-\sigma_{51}(1+\log w_{51})+$$

$$\sigma_{41}\log(\sum_{j=1}^{2}\sigma_{4j}w_{4j})-2\sigma_{42}\log(\sum_{j=1}^{2}\sigma_{4j}w_{4j})+2\sigma_{41}w_{41}(\frac{\sigma_{41}-\sigma_{42}}{\sum_{j=1}^{2}\sigma_{4j}w_{4j}})+2\sigma_{42}w_{42}(\frac{\sigma_{41}-\sigma_{42}}{\sum_{j=1}^{2}\sigma_{4j}w_{4j}})=0$$

Using the above ten equations we can easily determine the optimal dual variables and according to primal-dual relation

$$V^{*}(Q_1^{*},Q_2^{*},r_1^{*}r_2^{*},M_1^{*},M_2^{*})=d(w^{*})$$

Where,

$$d(w^{*})=\prod_{i=1}^{2}((\frac{C_{1i}}{w_{1i}^{*}})^{w_{1i}^{*}}(\frac{C_{2i}}{w_{2i}^{*}})^{w_{2i}^{*}}(\frac{C_{3i}}{w_{3i}^{*}})^{w_{3i}^{*}}(\frac{C_{4i}}{w_{4i}^{*}})^{\sigma_{4i}w_{4i}^{*}}(\frac{C_{5i}}{w_{5i}^{*}})^{\sigma_{5i}w_{5i}^{*}}(\sum_{j=1}^{2}w_{4j}^{*}\sigma_{4j})^{\sigma_{4i}w_i^{*}}(\sum_{j=1}^{2}w_{5j}^{*}\sigma_{5j})^{\sigma_{5i}w_i^{*}})$$

The optimal values of the decision variables are obtained from the relations (For i =1, 2)

$$C_{1i}r_i^{*}=d^{*}w_{1i}^{*}$$

$$C_{3i}Q_i^{*}=d^{*}w_{3i}^{*}$$

$$C_{2i}(M_i^{*})^{2}(Q_i^{*})^{-1}=d^{*}w_{2i}^{*}$$

$$(M_i^{*})^{-1}r_i^{*}=\frac{w_{5i}^{*}}{\sum_{j=1}^{2}w_{5j}^{*}\sigma_{5j}}$$

Using these values of $Q_1^{*},Q_2^{*},r_1^{*}r_2^{*}$ we can obtain easily the optimal values of $TC_i(Q_1^{*},Q_2^{*},r_1^{*},r_2^{*})$ for i = 1,2.

Solution of Constrained Multi-Objective Inventory Model By Fuzzy Geometric Programming Technique [FGPT]

We consider the model described in (2.1) and applying the above method when the Demand follows uniform distribution, using the section 4.1 and suppressing the term $H_i\mu_i$ as, it is a constant and also using section 3.2 and section 3.1.3, we proceed and according to section 3.2.1 we solve:

$$MaxV(Q_1,Q_2,r_1,r_2)=w_1\mu_1\ (Q_1,r_1)+w_2\mu_2\ (Q_2,r_2)$$

Subject to

$$\sum_{i=1}^{2} p_i Q_i \le B$$

$$\mu_i(Q_i, r_i) = \begin{cases} 0 & \text{for } \mathrm{TC_i}(Q_1, r) \le \mathrm{L_i} \\ \dfrac{U_i - \mathrm{TC_i}(Q_i, r_i)}{U_i - L_i} & \text{for } \mathrm{L_i} \le \mathrm{TC_i}\ (Q_i, r_i) \le \mathrm{U_i} \\ 1 & \text{for } \mathrm{TC_i}(Q_i, r_i) \ge \mathrm{U_i} \end{cases}$$

$\forall \mathrm{i} = 1{,}2.$

$0 \le \mu_{TC_i}(Q_1, Q_2, r_1, r_2) \le 1$

$Q_1, Q_2, r_1, r_2 > 0$

$$V(Q_1, Q_2, r_1, r_2) = \left(\sum_{i=1}^{2} \frac{w_i U_i}{(U_i - L_i)} - W(Q_1, Q_2, r_1, r_2)\right)$$

Thus the following signomial GPP can be constructed as

$$MinW(Q_1, Q_2, r_1, r_2, M_1, M_2) =$$

$$Min \sum_{i=1}^{2} (C_{1i} r_i + C_{2i} M_i^{\,2} Q_i^{\,-1})$$

Subject to

$$C_{3i} M_i^{\,-1} - C_{4i} M_i^{\,-1} r_i \le 1$$

$$\sum_{i=1}^{n} C_{5i} Q_i \le 1$$

$$Q_i, r_i, M_i \ge 0,\ \forall i = 1{,}2. \tag{4.3.1}$$

Where, $C_{1i} = \dfrac{H_i}{(U_i - L_i)}, C_{2i} = \dfrac{D_i K_i}{2(b_i - a_i)(U_i - L_i)}, C_{5i} = \dfrac{p_i}{B}, C_{4i} = 1, C_{3i} = b_i, w_1 = w_2.$

Clearly, it is a constrained signomial geometric programming problem with degree of difficulty 10 – 6 – 1= 3

The dual of the above signomial geometric programming problem can be written as:

$$Maxd(w) = \prod_{i=1}^{2} \left(\left(\frac{C_{1i}}{w_{1i}}\right)^{w_{1i}} \left(\frac{C_{2i}}{w_{2i}}\right)^{w_{2i}} \left(\frac{C_{3i}}{w_{3i}}\right)^{\sigma_{3i} w_{3i}} \left(\frac{C_{4i}}{w_{4i}}\right)^{\sigma_{4i} w_{4i}} \left(\frac{C_{5i}}{w_{5i}}\right)^{\sigma_{5i} w_{5i}} \left(\sum_{j=1}^{2} w_{4j} \sigma_{4j}\right)^{\sigma_{4i} w_i} \left(\sum_{j=1}^{2} w_{5j} \sigma_{5j}\right)^{\sigma_{5i} w_i} \left(\sum_{j=1}^{2} w_{3j} \sigma_{3j}\right)^{\sigma_{3i} w_{3i}} \right)$$

Where, $\sigma_{3i} = 1$, $\sigma_{4i} = -1, \sigma_{5i} = 1$ (for i=1, 2).

Using sections 3.1.2A and 3.1.2B we have, for i = 1, 2:

$w_{2i} = w_{5i}$

$w_{1i} = w_{4i}$

$w_{3i} = 2w_{2i} + w_{4i}$

$$\sum_{i=1}^{2} w_{1i} + w_{2i} = 1$$

Again for $Maxd(w)$, we have:

$$\frac{\partial d}{\partial w_{11}} = 0 = \frac{\partial d}{\partial w_{12}} = \frac{\partial d}{\partial w_{21}}$$

$$\frac{1}{d}\frac{\partial d}{\partial w_{11}} = (\log\frac{C_{11}}{w_{11}} - \log\frac{C_{22}}{w_{22}} - 2\log\frac{C_{32}}{w_{32}} + \log\frac{C_{31}}{w_{31}} - \log\frac{C_{52}}{w_{52}} - 1) - \sigma_{41}(1+\log w_{41})$$

$$+\sigma_{31}\log(\sum_{j=1}^{2}\sigma_{3j}w_{3j}) - 2\sigma_{32}\log(\sum_{j=1}^{2}\sigma_{3j}w_{3j}) + \sigma_{31}w_{31}(\frac{\sigma_{41}-2\sigma_{42}}{\sum_{j=1}^{2}\sigma_{4j}w_{4j}}) + \sigma_{32}w_{32}(\frac{\sigma_{41}-2\sigma_{42}}{\sum_{j=1}^{2}\sigma_{4j}w_{4j}})$$

$$+\sigma_{41}(\log\sum_{j=1}^{2}\sigma_{4j}w_{4j}) + \sigma_{41}w_{41}(\frac{\sigma_{41}}{\sum_{j=1}^{2}\sigma_{4j}w_{4j}}) + +\sigma_{52}(\log\sum_{j=1}^{2}\sigma_{5j}w_{5j}) + \sigma_{51}w_{51}(\frac{\sigma_{51}}{\sum_{j=1}^{2}\sigma_{5j}w_{5j}}) = 0$$

$$\frac{1}{d}\frac{\partial d}{\partial w_{12}} = (\log\frac{C_{12}}{w_{12}} - \log\frac{C_{22}}{w_{22}} - \log\frac{C_{32}}{w_{32}} - \log\frac{C_{52}}{w_{52}} + 2) - \sigma_{42}(1+\log w_{42} - \frac{w_{42}\sigma_{42}}{\sum_{j=1}^{2}\sigma_{4j}w_{4j}} - \log\sum_{j=1}^{2}\sigma_{4j}w_{4j})$$

$$-2\sigma_{32}(\log\sum_{j=1}^{2}\sigma_{3j}w_{3j} + \frac{w_{32}}{\sum_{j=1}^{2}\sigma_{3j}w_{3j}}) - \frac{\sigma_{52}w_{52}}{\sum_{j=1}^{2}\sigma_{5j}w_{5j}} = 0.$$

$$\frac{1}{d}\frac{\partial d}{\partial w_{21}} = (\log\frac{C_{21}}{w_{21}} - 2\log\frac{C_{32}}{w_{32}} - \log\frac{C_{52}}{w_{52}} + \log\frac{C_{51}}{w_{51}} + \log\frac{C_{31}}{w_{31}} - 4) +$$

$$2\sigma_{31}\log(\sum_{j=1}^{2}\sigma_{3j}w_{3j}) - 2\sigma_{32}\log(\sum_{j=1}^{2}\sigma_{3j}w_{3j}) + 2\sigma_{31}w_{31}(\frac{\sigma_{31}-\sigma_{32}}{\sum_{j=1}^{2}\sigma_{3j}w_{3j}}) + 2\sigma_{32}w_{32}(\frac{\sigma_{41}-\sigma_{42}}{\sum_{j=1}^{2}\sigma_{4j}w_{4j}})$$

$$\sigma_{51}\log(\sum_{j=1}^{2}\sigma_{5j}w_{5j}) - \sigma_{52}\log(\sum_{j=1}^{2}\sigma_{5j}w_{5j}) + \sigma_{51}w_{51}(\frac{\sigma_{51}-\sigma_{52}}{\sum_{j=1}^{2}\sigma_{5j}w_{5j}}) - \sigma_{52}w_{52}(\frac{\sigma_{51}-\sigma_{52}}{\sum_{j=1}^{2}\sigma_{5j}w_{5j}}) = 0$$

Using the above ten equations we can easily determine the optimal dual variables and according to primal-dual relation

$$W^*(Q_1^*, Q_2^*, r_1^* r_2^*, M_1^*, M_2^*) = d(w^*)$$

Where,

$$d(w^*) = \prod_{i=1}^{2}[(\frac{C_{1i}}{w_{1i}^*})^{w_{1i}^*}(\frac{C_{2i}}{w_{2i}^*})^{w_{2i}^*}(\frac{C_{3i}}{w_{3i}^*})^{\sigma_{3i}w_{3i}^*}(\frac{C_{4i}}{w_{4i}^*})^{\sigma_{4i}w_{4i}^*}(\frac{C_{5i}}{w_{5i}^*})^{\sigma_{5i}w_{5i}^*}(\sum_{j=1}^{2}w_{4j}^*\sigma_{4j})^{\sigma_{4i}w_i^*}(\sum_{j=1}^{2}w_{5j}^*\sigma_{5j})^{\sigma_{5i}w_i^*}\sum_{j=1}^{2}(w_{3j}^*\sigma_{3j})^{\sigma_{3i}w_i^*})$$

The optimal values of the decision variables are obtained from the relations (For i =1, 2)

$$C_{1i}r_i^* = d^* w_{1i}^*$$

$$C_{3i}(M_i^*)^{-1} = d^* w_{3i}^*$$

$$C_{2i}(M_i^*)^2(Q_i^*)^{-1} = d^* w_{2i}^*$$

$$(M_i^*)^{-1} r_i^* = \frac{w_{4i}^*}{\sum_{j=1}^{2} w_{4j}^* \sigma_{4j}}$$

$$\frac{p_i Q_i}{B} = \frac{w_{5i}^*}{\sum_{j=1}^{2} w_{5j}^* \sigma_{5j}}$$

Using these values of $Q_1^*, Q_2^*, r_1^* r_2^*$ we can obtain easily the optimal values of $TC_i(Q_1^*, Q_2^*, r_1^*, r_2^*)$ for i = 1,2.

Solution of Constrained Multi-Objective Inventory Model By Intuitionistic Fuzzy Geometric Programming Technique [IFGPT]

We consider the model described in (2.1) and applying the method of section 4.1. Now according to section (3.5) we have to solve the following problem:

$$Max V'(Q_1, Q_2, r_1, r_2) = \sum_{i=1}^{2} (\mu_i(Q_i, r_i) - \nu_i(Q_i, r_i))$$

Subject to

$$\sum_{i=1}^{2} p_i Q_i \le B$$

$$\mu_i(Q_i, r_i) = \begin{cases} 0 & \text{for } \mathrm{TC_i}(Q_1, r) \le \mathrm{L_i}^{acc} \\ \dfrac{U_i^{acc} - \mathrm{TC_i}(Q_i, r_i)}{U_i^{acc} - L_i^{acc}} & \text{for } \mathrm{L_i}^{acc} \le \mathrm{TC_i}(Q_i, r_i) \le \mathrm{U_i}^{\mathrm{acc}} \\ 1 & \text{for } \mathrm{TC_i}(Q_i, r_i) \ge \mathrm{U_i}^{acc} \end{cases}$$

$\forall \mathrm{i} = 1,2.$

$$\nu_i(Q_i, r_i) = \begin{cases} 0 & \text{for } \mathrm{TC_i}(Q_1, r) \ge \mathrm{U_i}^{rej} \\ \dfrac{\mathrm{TC_i}(Q_i, r_i) - L_i^{rej}}{U_i^{rej} - L_i^{rej}} & \text{for } \mathrm{L_i}^{rej} \le \mathrm{TC_i}(Q_i, r_i) \le \mathrm{U_i}^{\mathrm{rej}} \\ 1 & \text{for } \mathrm{TC_i}(Q_i, r_i) \le \mathrm{L_i}^{rej} \end{cases}$$

$\forall \mathrm{i} = 1,2.$

$0 \le \mu_{TC_i}(Q_i, r_i) \le 1$

$0 \le \nu_{TC_i}(Q_i, r_i) \le 1$

$Q_1, Q_2, r_1, r_2 > 0$

$$V'(Q_1, Q_2, r_1, r_2) = (\sum_{i=1}^{2} (\frac{U_i^{acc}}{(U_i^{acc} - L_i^{acc})} - \frac{U_i^{rej}}{(U_i^{rej} - L_i^{rej})}) - W'(Q_1, Q_2, r_1, r_2))$$

Thus using section 3.1.3 as earlier, the following signomial GPP can be constructed as

$$MinW'(Q_1, Q_2, r_1, r_2, M_1, M_2) = Min\sum_{i=1}^{2}(C_{1i}' r_i + C_{2i}' M_i^2 Q_i^{-1})$$

Subject to

$$C_{3i} M_i^{-1} - C_{4i} M_i^{-1} r_i \le 1$$

$$\sum_{i=1}^{n} C_{5i} Q_i \le 1$$

$$Q_i, r_i, M_i \ge 0, \ \forall i = 1,2. \tag{4.4.1}$$

Where, $C_{1i}' = \tilde{H}_i(\frac{1}{N_i} + \frac{1}{N_i'}), C_{2i}' = \frac{D_i \tilde{K}_i}{2(b_i - a_i)}(\frac{1}{N_i} + \frac{1}{N_i'}), C_{5i} = \frac{p_i}{B}, C_{4i} = 1, C_{3i} = b_i$.

$N_i = (U_i^{acc} - L_i^{acc}),\ N_i' = (U_i^{rej} - L_i^{rej})$.

Clearly, it is a constrained signomial geometric programming problem with degree of difficulty 10 – 6 – 1= 3

The dual of the above signomial geometric programming problem can be written as:

$$Maxd(w) = \prod_{i=1}^{2}((\frac{C_{1i}'}{w_{1i}})^{w_{1i}} (\frac{C_{2i}'}{w_{2i}})^{w_{2i}} (\frac{C_{3i}}{w_{3i}})^{\sigma_{3i} w_{3i}} (\frac{C_{4i}}{w_{4i}})^{\sigma_{4i} w_{4i}} (\frac{C_{5i}}{w_{5i}})^{\sigma_{5i} w_{5i}} \sum_{j=1}^{2}(w_{4j}\sigma_{4j})^{\sigma_{4i} w_i} \sum_{j=1}^{2}(w_{5j}\sigma_{5j})^{\sigma_{5i} w_i} \sum_{j=1}^{2}(w_{3j}\sigma_{3j})^{\sigma_{3i} w_i})$$

Where, $\sigma_{3i} = 1$, $\sigma_{4i} = -1, \sigma_{5i} = 1$ (for i=1, 2).

Using the similar method as described in section 4.1 we solve the above problem and we can easily determine the optimal dual variables and according to primal-dual relation:

$$W^*(Q_1^{*'}, Q_2^{*'}, r_1^{*'} r_2^{*'}, M_1^{*'}, M_2^{*'}) = d(w^{*'})$$

Where,

$$d(w^{*'}) = \prod_{i=1}^{2}((\frac{C_{1i}'}{w_{1i}^{*'}})^{w_{1i}^{*}} (\frac{C_{2i}'}{w_{2i}^{*'}})^{w_{2i}^{*}} (\frac{C_{3i}}{w_{3i}^{*'}})^{\sigma_{3i} w_{3i}^{*}} (\frac{C_{4i}}{w_{4i}^{*'}})^{\sigma_{4i} w_{4i}^{*}} (\frac{C_{5i}}{w_{5i}^{*'}})^{\sigma_{5i} w_{5i}^{*}} \sum_{j=1}^{2}(w_{4j}^{*'}\sigma_{4j})^{\sigma_{4i} w_i^{*}} \sum_{j=1}^{2}(w_{5j}^{*'}\sigma_{5j})^{\sigma_{5i} w_i^{*}} \sum_{j=1}^{2}(w_{3j}^{*'}\sigma_{3j})^{\sigma_{3i} w_i^{*'}})$$

The optimal values of the decision variables are obtained from the relations

(For I =1, 2)

$$C_{1i}' r_i^{*'} = d^{*'} w_{1i}^{*'}$$

$$C_{3i}(M_i^{*'})^{-1} = d^{*'} w_{3i}^{*'}$$

$$C_{2i}'(M_i^{*'})^2 (Q_i^{*'})^{-1} = d^{*'} w_{2i}^{*'}$$

$$(M_i^{*'})^{-1} r_i^{*'} = \frac{w_{4i}^{*'}}{\sum_{j=1}^{2} w_{4j}^{*'} \sigma_{4j}}$$

$$\frac{p_i Q_i}{B} = \frac{w_{5i}^{*'}}{\sum_{j=1}^{2} w_{5j}^{*'} \sigma_{5j}}$$

Using these values of $Q_1^{*'}, Q_2^{*'}, r_1^{*'} r_2^{*'}$ we can obtain easily the optimal values of $TC_i(Q_1^{*'}, Q_2^{*'}, r_1^{*'}, r_2^{*'})$ for I = 1,2.

Numericals

To solve the model (2.1) and (2.2) we consider the following data:

H_1=\$9; D_1=2400; a_1=10; b_1=40; K_1=\$11; $p_1 = \$4$; H_2=\$10; D_2=2000; a_2=20; b_2=50; K_2=12; $p_2 = \$3$; B=\$12000, $\delta_1 = 50; \delta_2 = 60; \delta_B = 200.$

$U_1^{acc} = 41.22$, $U_1^{rej} = 41.22$, $U_2^{acc} = 1787.30$, $U_2^{rej} = 1787.30$, $L_1^{acc} = 36.43,$ $L_1^{rej} = 38$, $L_2^{acc} = 1687.94$, $L_2^{rej} = 1712$.

Table A : Boundary level of decision variables:

i	Q_i		r_i	
	Lower limit (l_{Q_i})	Upper limit (u_{Q_i})	Lower limit (l_{r_i})	Upper limit (u_{r_i})
1	400	600	20	50
2	300	500	20	50

Table B: Target expenditure of total annual cost ($TC_{0i}, i = 1,2,....n$)

i	l_i (min value of TC_i without tolerance)	L_i (min value of TC_i with tolerance)	$TC_{0i} = \min(l_i, L_i)$
1	50.76	61.97	50.76
2	1802.56	1843.11	1802.56

Using the above data and section 4.2 and 4.1 the results of TABLE 1 and TABLE 2 are obtained. Similarly using section 3.2, 4.3 and section 3.4, 3.5 and 4.4, the results of TABLE 3 and TABLE 4 are achieved.

Table 1 : Solution of the model (2.2) by FNLPT

TC_1^*(\$)	TC_2^*(\$)	Q_1	Q_2	r_1	r_2	BUDGET(\$)	TYPE	WEIGHTS (w_1,w_2,w_B)
43.76	1723.98	471	372	37	25	9096	I	(1/3, 1/3, 1/3)
34.43	1737.32	488	414	29	24	9147	II	(3/5, 1/5, 1/5)
44.07	1738.44	517	417	34	28	9118	III	(1/5, 3/5, 1/5)
43.98	1714.87	528	384	32	27	8089	IV	(1/5, 1/5, 3/5)

Table 2 Solution of the model (2.2) by FGPT

TC_1^*($)	TC_2^*($)	Q_1	Q_2	r_1	r_2	BUDGET($)	TYPE	WEIGHTS (w_1,w_2,w_B)
41.33	1718.43	489	376	35	27	9088	I	(1/3, 1/3, 1/3)
33.98	1735.06	487	409	30	23	9137	II	(3/5, 1/5, 1/5)
42.67	1732.65	519	422	32	30	9114	III	(1/5, 3/5, 1/5)
41.02	1711.07	518	393	31	25	8076	IV	(1/5, 1/5, 3/5)

Table 3 : Solution of the model (2.1) by FPT and FGPT

METHOD	TC_1^*($)	TC_2^*($)	Q_1	Q_2	r_1	r_2	Aspiration level
FPT	41.43	1741.87	418	378	37	25	μ_1=0.812 μ_2=0.798
FGPT	39.44	1739.23	432	391	31	21	μ_1=0.889 μ_2=0.801

Table 4 : Solution of the model (2.1) by IFPT and IFGPT

METHOD	TC_1^*($)	TC_2^*($)	Q_1	Q_2	r_1	r_2	Aspiration Level	
							μ	ν
IFPT	39.33	1737.21	437	371	36	26	μ_1=0.872 μ_2=0.863	ν_1=0.073 ν_2=0.098
IFGPT	38.41	1728.45	428	402	32	23	μ_1=0.887 μ_2=0.899	ν_1=0.071 ν_2=0.083

Following observations can be made from the above results:

1. If we consider TABLE 1 and TABLE 2 separately then it is observed that when w_1=0.6 i.e. more importance is given to TC_1, lowest value is obtained in case of TYPE II, in comparison to other two TYPES. Similarly for TC_2 lowest value is obtained in case of TYPE III corresponding to the weight 0.6.Similarly for the budget B, minimum value occurs for TYPE IV, corresponding to the weight 0.7.
2. But, if we compare TABLE 1 and TABLE 2 then, the values of TC_1 and TC_2 as well as budget B are more minimized in case of Fuzzy Geometric Programming Technique [FGPT] than usual Fuzzy Non-linear Programming Technique [FNLPT], in all the four types of data.
3. From TABLE 3 we conclude that, Fuzzy Geometric Programming Technique [FGPT] obtained more minimized values of TC_1 and TC_2 , in comparison to Fuzzy Programming Technique [FPT].
4. From TABLE 4 we conclude that, Intuitionistic Fuzzy Geometric Programming Technique [IFGPT] obtained more minimized values of TC_1

and TC_2 , in comparison to Intuitionistic Fuzzy Programming Technique [IFPT].

5. Two important conditions of Intuitionistic fuzzy Optimization [IFO] viz. $\mu_i > \nu_i$ and $\mu_i + \nu_i < 1[i = 1,2]$ are also satisfied from the results of TABLE 4.
6. Intuitionistic Fuzzy Geometric Programming Technique [IFGPT] determines the lowest values of TC_i (i=1, 2) and from TABLE 3 and TABLE 4 we conclude that $IFGPT_{TC_i} < IFPT_{TC_i} < FGPT_{TC_i} < FPT_{TC_i}$ (i=1, 2)

Conclusion

We solve this multi-objective inventory problem with uniform lead-time demand by fuzzy geometric programming technique. We consider this model with deterministic constraint and then with a fuzzy constraint. Objective of this paper is to prove that intuitionistic fuzzy geometric programming always obtains the better value of the objective function than the usual fuzzy geometric programming.

References

[1] Abou-El-Ata, M.O., Fergany H.A. and E1- Wakeel M.F. (2003), “ Probabilistic Multi-item Inventory Model With Varying Order Cost Under Two Restrictions: A Geometric Programming Approach” Intl. J. Productions Economics, 83: 223-231.

[2] Banerjee S. and Roy T. K. (2008), “Single and Multi-objective Stochastic Inventory Problems in Fuzzy Environment”, The Journal of Fuzzy Mathematics, Vol. 16, No. 4, 875-897.

[3] Banerjee S. and Roy T. K. (2009), “Application of the Intuitionistic Fuzzy Optimization in the Constrained Multi-Objective Stochastic Inventory Model” J. Tech, September 2009, Vol. XXXXI, pp. 83-98.

[4] Banerjee S. and Roy T. K. (2010), “Solution of Single and Multi-Objective Stochastic Inventory Models with Fuzzy Cost Components by Intuitionistic Fuzzy Optimization Technique” Advances in Operations Research (Accepted).

[5] Banerjee S. and Roy T. K. (2010), “Constrained and Unconstrained Stochastic Inventory Model with Fuzzy cost components and Fuzzy random variable”, Int. J. Appl. Math. Stat.; Vol. 19; No. D10; 72-89.

[6] Bellman, R. E. and Zadeh, L. A.(1970), “Decision-making in a fuzzy environment”, Management Science, 17, B141-B164.

[7] Beightler, G.S. and Phillips, D.T. (1976), “Applied Geometric Programming”, Willey, New York.

[8] Beightler, G.S. and Phillips, D.T. and Wild, D.J. (1979), “Foundation of optimization”, Prentice-Hall, Englewood Cliffs, NJ.

[9] Biswal M.P. (1992), “Fuzzy Programming Technique to solve multi-objective geometric programming problem”, Fuzzy Sets and Systems, 51: 67-71.

[10] Cao, B.Y. (1993), “Fuzzy geometric programming (I)”, Fuzzy Sets and Systems, 53, 135-153.

[11] Cao, B.Y. (2002), “Fuzzy geometric programming”, Kluwer Academic Publishers, London.

[12] Das, K., Roy T.K. and Maiti, M. (2000), “Multi-item inventory model with quantity dependent inventory costs and demand-dependent unit cost under imprecise objective and restrictions: a geometric programming approach”, Production Planning and control, 11(8), 781-788.

[13] Duffin, R.J., Peterson, E.L. and Zener, C. (1966), “Geometric Programming Theory and Applications”, Willey, New York.

[14] Eckar, J. (1980), “Geometric programming methods, computational and applications”, SIAM Rev., 22, 338-362.

[15] Hariri, A.M.A and Abou-EI-Ata, M.O. (1997), “Multi-item EOQ inventory model with varying holding cost under two restrictions: a geometric programming approach”,Production Planning & Control, 8(5), 608-611.

[16] Islam Saidul and Roy T.K. (2004), “An economic production quantity model with flexibility and reliability consideration and demand dependent unit cost with negetive degree of difficulty: Geometric Programming Approach”, Tamsui Oxford Journal of Management Science, Taiwan, vol. 20(1), pp. 01-17.

[17] Jung, H. and Klein, C.M. (2001), “Optimal inventory Plolicies under decreasing cost functions via geometric programming”, European Journal of Operational Research, 132,628-642.

[18] K. Atanassov(1986): “ Intuitionistic fuzzy sets and system”, 20, 87-96.

[19] K. Atanassov , G. Gargov, (1989) “Interval valued intuitionistic fuzzy sets”, Fuzzy Sets and Systems, Vol. 31, No. 3, 343-349.

[20] K. Atanassov, V. Kreinovich, (1999) “Intuitionistic fuzzy interpretation of intetrval data”, Notes on Intuitionistic Fuzzy Sets, Vol. 5, No. 1, 1-8.

[21] K. Atanassov (1999) , “Intuitionistic Fuzzy Sets”, Springer Physica-Verlag, Berlin.

[22] K. Atanassov, “Open problems in intuitionistic fuzzy sets theory”, Proceedings of 6-th Joint Conf. on Information Sciences, Research Triangle Park (North Carolina, USA.

[23] Kaufmann, A. and Gupta, M., (1988), “Fuzzy Mathematical Models in Engineering and Management Science”, North Holland.

[24] Maji P.K., R. Biswas, A.R. Roy, (2001)“Intuitionistic fuzzy soft sets”, The Journal of Fuzzy Mathematics, Vol. 9, No. 3, 677-692.

[25] Mandal, N.K., Roy, T.K. and Maiti, M. (2005), “Multi-objective Fuzzy EOQ model with three constraints: A geometric programming approach”, Fuzzy Sets and Systems, 150(1), 87-106.

[26] Nikolova M., N. Nikolov, C. Cornelis, G.Deschrijver, (2002) “Survey of the research on intuitionistic fuzzy sets”, Advanced Studies in Contemporary Mathematics, Vol. 4, No. 2, 127-157.

[27] P.P.Angelov, (1995), “Intuitionistic fuzzy optimization”, Notes on Intutionistic Fuzzy Sets, 1, 27-33.

[28] P.P.Angelov, (1995), “Intuitionistic fuzzy optimization” , Notes on Intutionistic Fuzzy Sets, 1(2), 123-129.
[29] P.P.Angelov, (1997): “Optimization in an intuitionistic fuzzy environment” fuzzy sets and systems 86, 299-306.
[30] Pramanik, S. and Roy, T.K., (2005), “An Intuitionistic Fuzzy goal programming approach to vector optimization problem”, Notes on Intutionistic Fuzzy Sets, 11(1), 1-14.
[31] Rizvi S., H.J. Naqvi, D. Nadeem. (2002)“Rough intuitionistic fuzzy sets”, Proceedings of 6-th Joint Conf. on Information Sciences, Research Triangle Park (North Carolina, USA), March 8-13, 101-104.
[32] Tanaka, H., Okuda, T. and Asai, K. (1974), “On fuzzy mathematical programming”, Journal of Cybernetics, 3(4), 37-46.
[33] Tiwari, R.N., Dharmar, S. and Rao, J.R. (1987), “Fuuzzy goal programming: an additive model”, Fuzzy Sets and Systems, 24, 27-34.
[34] Verma R. K. (1990), “Fuzzy geometric programming with several objective functions”, Fuzzy Sets and Systems, 35: 115-120.
[35] Zadeh, L. A.(1965), “Fuzzy sets”, Information and Control 8, 338-356.
[36] Zener, C. (1971), “Engineering design by geometric programming”, Wiley.
[37] Zimmerman, H. J.(1976), “Description and optimization of fuzzy system”, International Journal of General Systems, 2, 209-215.
[38] Zimmermann, H.J. (1978), “Fuzzy linear programming with several objective functions”, Fuzzy Sets and Systems, 1, 46-55.