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An Integrated Inventory Model for Deteriorating Items with Time Dependent Demand and Allowable Shortages

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Abstract

This study develops a single vendor, multi-buyers production inventory policy for deteriorating items with a constant production and variable demand rate. The relevant cost function of this model is minimized analytically. Shortages in inventory are allowed. All these practical aspects are incorporated into the model with the purpose of making it more realistic.

Introduction

A new era has dawned in supply chain management with the advent of globalization. This has lead to increased competition and in order to achieve and sustain competitive advantage; Companies must be able to respond quickly to customer demand. In today's market environment, a buyer has to decide the numbers of deliveries during a fix interval of time to minimize his cost. But it is not necessary that this phenomenon will also be economical for the vendor. So if the no. of deliveries is decided in corporation with the vendor, the overall integrated cost can be minimized. Physical decay of the stocked items over time is taken in to account and shortages in inventory are allowed. Deteriorating inventory, in general, is defined as decay, damaged, spoilage, evaporation, obsolescence, pilferage, Loss of marginal value of a commodity that result in decreasing usefulness from the original one. The two parameter weibull distribution is used because the items are assumed to have a varying rate of deterioration. When there is a shortage, some of the customer will seek alternatives, while others will wait for the next replenishment. This behavior is known by the term partial backlogging.

A Consideration of partial back-ordering inventory for non-perishable product has been undertaken by Rosenberg (1979). Ghare & Scharader (1963) was the first who developed an economic order quantity model by assuming a constant rate of deterioration. Covert and Philip (1973) developed an EOQ model for items whose deterioration patterns follow the Weibull distribution. Shine (1990) developed an inventory model for exponentially deteriorating items with quantity discounts and partial back-ordering under a prescribed scheduling period. Banarjee (1986) developed a joint economic lot size model for a single-buyer, single-vendor system. Goyal (1988) generalized Banarjee's model by relaxing the assumption of the lot-for-lot policy of the vendor. Hill (1989) derived a central-warehouse, multi-retailer model with shortage by using simulation. Lu and Posner (1994) introduced two heuristic procedures for the one warehouse multi-retailer system. Ha and Kim (1997) made the analysis of integration between a buyer and a supplier by setting up the mathematical model in which inventory cost of vendor is derived. Yang and Wee (2002) derived a single vendor and multiple-buyers production inventory policy for a deteriorating item.

In this paper we develop a economic production quantity model for single vendor and multiple buyers in which demand and deterioration is taken as time dependant. The numerical illustration of this problem is also discussed and shows that the integrated approach is better than the independent.

Assumptions

1. The production rate is finite and is greater than the sum of all the buyers demand.
2. Demand rate is assumed to be time dependent.
3. A single item inventory is assumed.
4. The model is developed for single vendor and multiple buyers.
5. Holding cost applies to good units only.
6. Shortages are allowed for buyers.
7. Weibull distribution rate of deterioration is considered.

Notations

N	number of buyers
p	production rate per year
T	time length of each cycle
T_1	The length of production time in each production cycle T
T_2	The length of non production time in each production cycle T
$I_{v1}(t)$	Inventory level for vendor when t is between 0 and T
$I_{v2}(t)$	Inventory level for vendor when t is between 0 and T
$I_{bi1}(t)$	Inventory level for buyer i when t is between 0 and v
$I_{bi2}(t)$	Inventory level for buyer i when shortages occurs
η	The waiting time for next replenishment.
v	the time at which inventory level become zero.

n_i	Delivery times per period T for buyer i where i=1, 2, 3, 4.........................N.
I_{mv}	The maximum inventory level of vendor
I_{mbi}	The maximum inventory level of buyer i
p_v	The unit production cost for vendor.
p_b	The unit price for buyer
c_{sv}	The set up cost of each production cycle for vendor.
c_{ob}	The ordering cost per order for buyer
h_v	The holding cost per dollar per year for vendor
h_b	The holding cost per dollar per year for buyer
c_{sh}	The shortage cost per unit for buyer
c_r	The lost sale cost per unit for buyer.
α_i	Initial demand rate for the buyer i

Mathematical Formulation

The inventory system is represented by the following differential equations.

$$\frac{dI_{v1}(t)}{dt} + \alpha\beta t^{\beta-1} I_{v1}(t) = p - \sum_{i=1}^{N}(a_i + b_i t) \quad 0 \le t \le T_1 \tag{1}$$

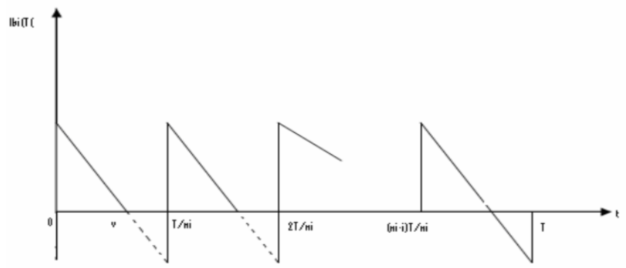
$$\frac{dI_{v2}(t)}{dt} + \alpha\beta t^{\beta-1} I_{v2}(t) = -\sum_{i=1}^{N}(a_i + b_i t) \quad 0 \le t \le T_2 \tag{2}$$

$$\frac{dI_{bi1}(t)}{dt} + \alpha\beta t^{\beta-1} I_{bi1}(t) = -\sum_{i=1}^{N}(a_i + b_i t) \quad 0 \le t \le v \tag{3}$$

$$\frac{dI_{bi2}(t)}{dt} = -(a_i + b_i t)\theta(\eta) \quad 0 \le t \le \frac{T}{n_i} \tag{4}$$

With boundary conditions

$$I_{v1}(0) = 0 \ , \ I_{v2}(T_2) = 0, \ \ I_{bi1}(v) = 0 \ \ I_{bi2}(\frac{T}{n_i}) = 0 \tag{5}$$



Inventory level for the buyer

The solutions of the above differential equations are

$$I_{vi}(t) = [pt - \sum_{i=1}^{n}(a_i t + b_i \frac{t^2}{2}) + \frac{\alpha p t^{\beta+1}}{\beta+1} - \sum_{i=1}^{n}\alpha(\frac{a_i t^{\beta+1}}{\beta+1} + \frac{b_i t^{\beta+2}}{\beta+2})e^{-\alpha t^{\beta}}]\ 0 \le t \le T_1 \tag{6}$$

$$I_{v2}(t) = [\sum_{i=1}^{N}\{a_i(T_2 - t) + \frac{b_i}{2}(T^2 - t^2)\} + \{\frac{\alpha a_i}{\beta+1}(T_2^{\ \beta+1} - t^{\beta+1}) + \frac{\alpha b_i}{\beta+2}(T_2^{\ \beta+2} - t^{\beta+2})\}]e^{-\alpha t^{\beta}}$$

$$0 \le t \le T_2 \tag{7}$$

$$I_{bi1}(t) = [\sum_{i=1}^{N}\{a_i(v - t) + \frac{b_i}{2}(v^2 - t^2)\} + \{\frac{\alpha a_i}{\beta+1}(v^{\beta+1} - t^{\beta+1}) + \frac{\alpha b_i}{\beta+2}(v^{\beta+2} - t^{\beta+2})\}]e^{-\alpha t^{\beta}}$$

$$0 \le t \le v \tag{8}$$

$$I_{bi2}(t) = \frac{n_i}{T}[\frac{a_i}{2}((\frac{T}{n_i})^2 - t^2) + \frac{b_i}{3}((\frac{T}{n_i})3 - t3)]\ \ 0 \le t \le \frac{T}{n_i} \tag{9}$$

From equation (7)

$$I_{mv} = I_{v2}(0)$$

$$I_{mv} = \sum_{i=1}^{N}(a_i T_2 + \frac{b_i}{2}T^2) + \sum_{i=1}^{N}(\frac{\alpha a_i}{\beta+1}T_2^{\ \beta+1} + \frac{\alpha b_i}{\beta+2}T_2^{\ \beta+2}) \tag{10}$$

From equation (8)

$$I_{mbi} = I_{bi1}(0)$$

$$I_{mbi} = a_i v + \frac{b_i}{2}v^2 + \frac{\alpha a_i}{\beta+1}v^{\beta+1} + \frac{\alpha b_i}{\beta+2}v^{\beta+2} \tag{11}$$

The yearly holding cost for all buyers

$$H_b = \frac{p_b h_b}{T}\sum_{i=1}^{N} n_i \int_0^v I_{bi1}(t)dt$$

$$H_b = \frac{p_b h_b}{T}\sum_{i=1}^{N} n_i[\frac{a_i}{2}v^2 + \frac{b_i}{3}v^3 + \frac{\alpha\beta a_i}{(\beta+1)(\beta+2)}v^{\beta+2} + \frac{\alpha\beta b_i}{(\beta+1)(\beta+3)}v^{\beta+3}] \tag{12}$$

The yearly holding cost for the vendor

$$H_v = \frac{p_v h_v}{T}[\int_0^{T_1} I_{v1}(t)dt + \int_0^{T_2} I_{v2}(t)dt]$$

$$H_v = \frac{p_v h_v}{T}[\{\frac{pT_1^2}{2} - \sum_{i=1}^{N}(\frac{a_i T_1^2}{2} + \frac{b_i T_1^3}{6}) + \frac{\alpha p T_1^{\beta+2}}{(\beta+1)(\beta+2)} - \sum_{i=1}^{N}\alpha(\frac{a_i T_1^{\beta+2}}{(\beta+1)(\beta+2)} + \frac{b_i T_1^{\beta+3}}{(\beta+2)(\beta+3)}) - \frac{\alpha p T_1^{\beta+2}}{(\beta+2)} + \sum_{i=1}^{N}(\frac{a_i \alpha T_1^{\beta+2}}{(\beta+2)} + \frac{\alpha b_i T_1^{\beta+3}}{2(\beta+3)}) - \frac{\alpha^2 p T_1^{2(\beta+1)}}{2(\beta+1)^2}\}$$

$$+ \sum_{i=1}^{N}(\frac{a_i T_2^2}{2} + \frac{b_i T_2^3}{3}) + \sum_{i=1}^{N}(\frac{a_i \alpha T_2^{\beta+2}}{(\beta+2)} + \frac{\alpha b_i T_2^{\beta+3}}{(\beta+3)}) - \sum_{i=1}^{N}(\frac{a_i \alpha T_2^{\beta+2}}{(\beta+1)(\beta+2)} - \frac{\alpha b_i T_2^{\beta+3}}{(\beta+1)(\beta+3)})\}]$$

The annual deteriorated cost for all buyers

$$D.C_b = \sum_{i=1}^{N} (I_{mbi} - \int_0^v (a_i + b_i t)dt) \frac{n_i p_b}{T}$$

$$D.C_b = \sum_{i=1}^{N} (\frac{\alpha a_i}{\beta+1} v^{\beta+1} + \frac{\alpha b_i}{\beta+2} v^{\beta+2}) \frac{n_i p_b}{T} \tag{14}$$

The annual deteriorated cost for the vendor

$$D.C_v = \frac{p_v}{T}(pT_1 - \sum_{i=1}^{N} n_i I_{mbi})$$

$$DC_v = \frac{p_v}{T}\{pT_1 - \sum_{i=1}^{N} n_i (a_i v + \frac{b_i}{2} v^2 + \frac{\alpha a_i}{\beta+1} v^{\beta+1} + \frac{\alpha b_i}{\beta+2} v^{\beta+2})\} \tag{15}$$

The ordering cost per order for all buyers

$$O.C. = \sum_{i=1}^{N} \frac{n_i c_{ob}}{T} \tag{16}$$

The set up cost for vendor

$$S.C_v = \frac{c_{sv}}{T} \tag{17}$$

The yearly shortage cost for all buyers

$$Sh.C_b = \frac{c_s}{T} \sum_{i=1}^{N} \int_v^{\frac{T}{n_i}} I_{bi2}(t).dt$$

$$Sh.C_b = \frac{c_s}{T} \sum_{i=1}^{n} \frac{n_i^2}{T} [\frac{a_i}{3}(\frac{T}{n_i})^3 + \frac{b_i}{4}(\frac{T}{n_i})^4 - \frac{a_i}{2}((\frac{T}{n_i})^2 v - \frac{v^3}{3}) - \frac{b_i}{3}((\frac{T}{n_i})^3 v - \frac{v^4}{4})] \tag{18}$$

The yearly lost sale cost for all buyers

$$LSC_b = \frac{c_r}{T} \sum_{i=1}^{N} n_i \int_v^{\frac{T}{n_i}} (a_i + b_i t)(1 - \theta(\frac{T}{n_i} - t))dt$$

$$LSC_b = \frac{c_r}{T} \sum_{i=1}^{N} n_i \{\frac{a_i}{2}(\frac{T}{n_i}) + \frac{b_i}{6}((\frac{T}{n_i})^2 - a_i v + \frac{a_i n_i}{T} \frac{v^2}{2} - b_i \frac{v^2}{2} + \frac{b_i n_i}{T} \frac{v^3}{3}\} \tag{19}$$

Now the buyers total cost is given by

$$B.C. = H.C_b + D.C_b + O.C_b + Sh.C_b + LSC_b \tag{20}$$

And the vendors total cost is given by

$$V.C. = H.C_v + DC_v + S.C_v \tag{21}$$

The integrated total cost of the vendor and the buyers is given by
T.C. = B.C. + V.C. (22)

Numerical Example

n_1	n_2	B.C.	V.C.	T.C.
4	9	2292.69	78719.1	81011.8
5	7	1881.75	78970.8	80852.6
6	6	1729.53	79096.7	80826.3
6	5	1616.76	79222.6	80839.4
6	7	1881.75	78970.8	80852.6
8	4	1550.45	79348.5	80898.9
9	3	1548.03	79474.4	81022.4

Table for variation in α

α	n_1	n_2	T.C.
.000005	6	6	80825.7
.00001	6	6	80826.3
.000015	6	6	80826.8
.00002	6	6	80827.4
.000025	6	6	80827.9
.00003	6	6	80828.4
.000035	6	6	80829
.00004	6	6	80829.5

Table for variation in h_1

	h_1	T.C.
1	.0002	80638.8
2	.0004	80686.5
3	.0008	80781.9
4	.001	80826.3
5	.0014	80924.9
6	.0018	81020.3

Conclusion

In this study we developed a single vendor and multi-buyers production inventory model with variable deterioration rate and time dependent demand. A mathematical model incorporating the cost of both the vendors and the buyers is developed here. The result in this paper not only provides a valuable reference for decision makers in planning and controlling the inventory but also provide a useful model for many organizations.

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