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Abstract

This paper presents an approach for performing reliability analysis of bridge and parallel-series networks with critical and non-critical human errors. Reliability and mean time to failure formulas are developed for exponential and Rayleigh distributed failure times. Selective plots are shown for demonstrating the effect of human errors on system reliability and mean time to failure.

Keywords: Reliability, Availability, Human Errors, Mean time to failure, Bridge System

Introduction

Humans play a pivotal role in the design, development and operational phases of engineering systems. Reliability evaluation of systems without taking into consideration the human element does not provide a realistic picture. Hence, there is a definite need for incorporating the occurrence of human errors in system reliability evaluation.

A Human error is defined as a failure to perform a prescribed task (or the performance of a prohibited action) which could lead to disruption of scheduled operations or results in damage to property and equipment. Furthermore, depending upon the severity of human error consequences, human errors can be classified into two categories, namely, critical and non-critical. For our purpose the occurrence of a
critical human error causes the entire system to fail where as the occurrence of a non-critical human error results in a single unit failure only.

This paper presents reliability analysis of bridge and parallel-series networks with critical and non-critical human errors [1-6]. A newly developed approach [1,4-5,6] is used to perform system reliability analysis. This approach is a modified version of the block diagram approach and is demonstrated in this which consists of a seven unit bridge network.

**Assumptions**

The following assumptions are associated with analysis given below:

1. A unit can fail either due to a hardware failure or due to a non-critical human error.
2. The occurrence of a critical human error can result in total system failure but the occurrence of a non-critical human error can cause the failure of a single unit only.
3. Each unit failure is independent of others.

**Figure 1.1:** Block diagram for bridge network.

**Analysis**

**Bridge Network**

This paper represents a seven unit bridge network with critical and non-critical human errors as shown in Figure 1.1. In this figure, each real unit is represented by a
Reliability Analysis of a Bridge and Parallel Series Networks

rectangle. The failure probability of each unit is divided into two components, namely, hardware failure probability and non-critical human error probability. These failure probabilities are represented by block connected in series as shown in each rectangle in figure 1.1. A hypothetical unit representing critical human errors is connected in series with the bridge network. The total system can fail due to the failure of this hypothetical unit.

### Notation

The following symbols are associated with this:

- $F_j$: Hardware failure probability of $j$th unit, for $j=1,2,3,4,5,6,7$.
- $f_j$: $j$th unit failure probability with respect to non-critical human errors, for $j=1,2,3,4,5,6,7$.
- $f_c$: Critical human error occurrence probability associated with the system.
- $R_{Hj}$: Hardware reliability of the $j$th unit.
- $R_{NCj}$: Reliability of the $j$th unit with respect to non-critical human errors.
- $R_j$: Reliability of the $j$th unit with respect to hardware failures and non-critical human errors.
- $R_c$: System reliability with respect to critical human errors.
- $R_{H,NC}$: System reliability with respect to hardware failures and non-critical human errors.
- $R_b$: Bridge system reliability with respect to hardware failure, critical and non-critical human errors.
- $s$: Laplace transform variable.
- $t$: Time.

The time-independent reliability analysis are developed for the following two cases:

#### Case(1): Non-identical units

The hardware reliability of $j$th unit is given by

$$R_{Hj} = 1 - F_j \quad \text{for} \quad j=1,2,3,4,5,6,7. \quad (1)$$

The reliability of $j$th unit with respect to non-critical human errors is

$$R_{NCj} = 1 - f_j \quad \text{for} \quad j=1,2,3,4,5,6,7. \quad (2)$$

The reliability of the $j$th unit with respect to hardware failures and non-critical human errors is

$$R_j = R_{Hj} R_{NCj} \quad \text{for} \quad j=1,2,3,4,5,6,7. \quad (3)$$

The bridge network’s reliability with respect to hardware failures and non-critical human errors is:
The reliability of the bridge network with respect to critical human errors only is
\[ R_c = 1 - f_c \] (5)

Finally, using equations (4) and (5), we get
\[ R_b = R_c \cdot R_{H,NC} \] (6)

**Case (2) : Identical units**
By setting \( R_j = R \) (i.e., \( F_j = F \) and \( f_j = f \)), for \( j = 1, 2, 3, 4, 5, 6, 7 \) in equation (6) yields
\[ R_b = R_c (2R_7 - 4R_6 + 2R_5 - R_4 + 2R_2), \] (7)

Where \( R = R_{H,NC}, R_H = 1 - F \) and \( R_{NC} = 1 - f \).

The plots of equation (7) are shown in figure 1.2 for the specified values of \( F, f \) and \( f_c \).

These plots clearly show the impact of varying critical human error probability \( f_c \) and non-critical human error probability \( f \) on bridge system reliability. It is evident from these plots that the system reliability decreases with increasing values of \( f \) and \( f_c \).

![Reliability Plots Type - 1](image)

**Figure 1.2:** \( F = 0.1 \).

<table>
<thead>
<tr>
<th>Series</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f )</td>
<td>0</td>
<td>0.05</td>
<td>0.1</td>
<td>0.15</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Time dependent analysis for the following two cases are developed:
Case A: Exponentially distributed failure times
For exponentially distributed hardware failure, critical and non-critical human error times the time dependent equations for $R_H$, $R_{NC}$, $R$ and $R_c$ are as follows:

$$R_H(t) = e^{-\lambda_H t}$$  \hspace{1cm} (8)

Where $\lambda_H$ is the constant hardware failure rate of a unit.

$$R_{NC}(t) = e^{-\lambda_{NC} t}$$  \hspace{1cm} (9)

Where $\lambda_{NC}$ is the constant non-critical human error rate associated with a unit.

$$R(t) = e^{-X t}$$  \hspace{1cm} (10)

Where $X = \lambda_H + \lambda_{NC}$

$$R_c(t) = e^{-\lambda_C t}$$  \hspace{1cm} (11)

Where $\lambda_c$ is the constant critical human error rate associated with the system.

Using equations (7) to (11), we get the reliability of the seven identical unit networks as follows:

$$R_b(t) = 2 e^{-\lambda_H t} - 4 e^{-\lambda_{NC} t} + 2 e^{-\lambda_c t} - e^{-\lambda_{NC} t} + 2 e^{-2\lambda_{NC} t}$$  \hspace{1cm} (12)

The plots of equation (12) are shown in figure 1.3. for the assumed values of the model parameters.

![Reliability Plots Type - II](image)

**Figure 1.3:** $\lambda_H=0.12$, $\lambda_{NC}=0.12$.

<table>
<thead>
<tr>
<th>Series</th>
<th>$\lambda_c=0$</th>
<th>$\lambda_c=0.03$</th>
<th>$\lambda_c=0.07$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td></td>
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<tr>
<td>2</td>
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<td>3</td>
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</tbody>
</table>
These plots clearly demonstrate the effect of varying time $t$ and bridge system reliability. It is evident from these plots that the system reliability decreases with increasing values of time.

The mean time to failure of the bridge system is given by

$$\text{MTTF}_b = \int_0^\infty R_b(t) \, dt$$

$$= \frac{2}{7X + \lambda_c} - \frac{4}{6X + \lambda_c} + \frac{2}{5X + \lambda_c} - \frac{1}{4X + \lambda_c} + \frac{2}{2X + \lambda_c}$$  \hspace{1cm} (13)$$

The plots of the above equation are shown in figure 1.4

![MEAN TIME TO FAILURE PLOTS](image)

**Figure 1.4:** $\lambda_H=0.1.$

<table>
<thead>
<tr>
<th>Series</th>
<th>$\lambda_{NC}=0$</th>
<th>$\lambda_{NC}=0.01$</th>
<th>$\lambda_{NC}=0.02$</th>
<th>$\lambda_{NC}=0.03$</th>
<th>$\lambda_{NC}=0.04$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
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<td>3</td>
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<td>4</td>
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<td>5</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

These plots clearly demonstrate the effect of varying critical and non-critical human error rates $\lambda_c$ and $\lambda_{NC}$ on the bridge system mean time to failure.

The variance of time to failure of the bridge system with human errors is

$$\sigma^2_b = -2 \lim_{s \to 0} R_b'(s) - (\text{MTTF}_b)^2$$

$$= \frac{4}{(7X + \lambda_c)} - \frac{8}{(6X + \lambda_c)} + \frac{4}{(5X + \lambda_c)} - \frac{2}{(4X + \lambda_c)} \left[ - \frac{2}{(2X + \lambda_c)} \right]^2$$  \hspace{1cm} (14)$$

Where $R_b'(t)$ is the derivate of the laplace transform of $R_b(t)$ with respect to $s.$
The bridge system failure density function is given by

\[ f_b(t) = R'_b(t) = 2(7X + \lambda c)e^{-7X + \lambda c}t - 4(6X + \lambda c)e^{-6X + \lambda c}t - 2(5X + \lambda c)e^{-5X + \lambda c}t + 2(4X + \lambda c)e^{-4X + \lambda c}t + 2(2X + \lambda c)e^{-(2X + \lambda c)t} \]

(15)

Where

\[ R'_b(t) = \frac{d}{dt} R_b(t) \]

The hazard rate function of the bridge system is

\[ h_b(t) = \frac{f_b(t)}{R_b(t)} = \frac{2(7X + \lambda c)e^{-7X} - 4(4X + \lambda c)e^{-4X} + 2(5X + \lambda c)e^{-5X} - 4(4X + \lambda c)e^{-4X} + 2(2X + \lambda c)e^{-2X}}{2e^{-3X} - 4e^{-6X} + 2e^{-5X} - 4e^{-4X} + 2e^{-2X}} \]

(16)

The plots of equation (16) are shown in figure 1.5.

![Hazard Rate Plots Type-1](image)

**Figure 1.5:** \( \lambda_H=0.12 \), \( \lambda_{NC}=0.08 \).

<table>
<thead>
<tr>
<th>Series</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda_c )</td>
<td>0</td>
<td>0.01</td>
<td>0.03</td>
<td>0.05</td>
<td>0.07</td>
</tr>
</tbody>
</table>

These plots clearly show the impact of varying time \( t \) and constant critical human error rate \( \lambda_c \). It is evident from these plots that the system hazard rate increases with the increasing values of \( t \) and \( \lambda_c \).

**Case B: Rayleigh distributed failure times**

For Rayleigh distributed failure times the time dependent equations for \( R_H, R_{NC}, R \) and...
\( R_c(t) = e^{-\beta_c t^2} \) (17) Where \( \beta_h = \frac{1}{\alpha_h} \); \( \alpha_h \) is the scale parameter associated with the Rayleigh distribution representing hardware failure times of a unit

\( R_{NC}(t) = e^{-\beta_{NC} t^2} \) (18) Where \( \beta_{NC} = \frac{1}{\alpha_{NC}} \); \( \alpha_{NC} \) is the scale parameter associated with the Rayleigh distribution representing failure times of a unit due to non-critical human errors.

\( R(t) = e^{-Y t^2} \) (19) Where \( Y = \beta_h + \beta_{NC} \)

\( R_c(t) = e^{-\beta_c t^2} \) (20)

Where \( \beta_c = \frac{1}{\alpha_c} \); \( \alpha_c \) is the scale parameter associated with the Rayleigh distribution representing bridge system failure times with respect to critical human errors.

Using equations (7), (17) to (20), the bridge system reliability with human error is

\[
R_b(t) = 2e^{-\left(\beta_h + Y\right) t^2} - 4e^{-\left(\beta_h + 6Y\right) t^2} + 2e^{-\left(\beta_h + 5Y\right) t^2} - e^{-\left(\beta_h + 4Y\right) t^2} + 2e^{-\left(\beta_c + 2Y\right) t^2}
\]

(21)

The numerical results pertaining to equation (21) are tabulated in the table 1.

The system mean time to failure is given by

\[
MTTF_b = \int_0^\infty R_b(t) dt
\]

\[
= \left(\frac{\pi}{\beta_c + 7Y}\right)^{1/2} - 2\left(\frac{\pi}{\beta_c + 6Y}\right)^{1/2} + \left(\frac{\pi}{\beta_c + 5Y}\right)^{1/2} - \frac{1}{2}\left(\frac{\pi}{\beta_c + 4Y}\right)^{1/2} + \left(\frac{\pi}{\beta_c + 2Y}\right)^{1/2}
\]

(22)

**Table 1:** Reliability values for Bridge system \( \beta_h=0.13 \), \( \beta_{NC}=0.09 \).

<table>
<thead>
<tr>
<th>Time(t)</th>
<th>Bridge system reliability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>( \beta_c=0.0 )</td>
</tr>
<tr>
<td>0.0</td>
<td>1.000</td>
</tr>
<tr>
<td>0.3</td>
<td>0.999</td>
</tr>
<tr>
<td>0.6</td>
<td>0.986</td>
</tr>
<tr>
<td>0.9</td>
<td>0.932</td>
</tr>
<tr>
<td>1.2</td>
<td>0.819</td>
</tr>
<tr>
<td>1.5</td>
<td>0.631</td>
</tr>
<tr>
<td>1.8</td>
<td>0.438</td>
</tr>
<tr>
<td>2.1</td>
<td>0.273</td>
</tr>
<tr>
<td>2.4</td>
<td>0.154</td>
</tr>
<tr>
<td>2.7</td>
<td>0.079</td>
</tr>
<tr>
<td>3.0</td>
<td>0.038</td>
</tr>
</tbody>
</table>
Reliability Analysis of a Bridge and Parallel Series Networks

The failure density function of the bridge system is

\[ f_b(t) = \frac{d}{dt} R_b(t) = t \begin{bmatrix} 4(\beta_c + 7Y)e^{-(\beta_c + 7Y)^2} - 8(\beta_c + 6Y)e^{-(\beta_c + 6Y)^2} + 4(\beta_c + 5Y)e^{-(\beta_c + 5Y)^2} \\ -2(\beta_c + 4Y)e^{-(\beta_c + 4Y)^2} + 4(\beta_c + 2Y)e^{-(\beta_c + 2Y)^2} \end{bmatrix} \]  

(23)

The system hazard rate function is expressed as

\[ h_b(t) = \frac{f_b(t)}{R_b(t)} = \frac{A(t)}{B(t)} \]  

(24)

Where

\[ A(t) = 2t \begin{bmatrix} 2(\beta_c + 7Y)e^{-7Yt^2} - 4(\beta_c + 6Y)e^{-6Yt^2} + 2(\beta_c + 5Y)e^{-5Yt^2} \\ -(\beta_c + 4Y)e^{-4Yt^2} + 2(\beta_c + 2Y)e^{-2Yt^2} \end{bmatrix} \]

\[ B(t) = 2e^{-7Yt^2} - 4e^{-6Yt^2} + 2e^{-5Yt^2} - e^{-4Yt^2} + 2e^{-2Yt^2} \]

The plots of equation (24) are shown in figure 1.6.

![Hazard Rate Plots Type-II](image)

**Figure 1.6:** \( \beta_H = 0.12 \), \( \beta_{NC} = 0.08 \).

<table>
<thead>
<tr>
<th>Series</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_c = 0 )</td>
<td>( \beta_c = 0.02 )</td>
<td>( \beta_c = 0.04 )</td>
<td>( \beta_c = 0.06 )</td>
<td>( \beta_c = 0.08 )</td>
<td></td>
</tr>
</tbody>
</table>
These plots clearly show the impact of varying time $t$ and $\beta_c$. It is evident from these plots that the system Hazard rate increases with increasing values of $t$ and $\beta_c$.

References


