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Reliability Analysis of a Bridge and Parallel Series Networks with Critical and Non- Critical Human Errors: A Block Diagram Approach.

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Abstract

This paper presents an approach for performing reliability analysis of bridge and parallel- series networks with critical and non-critical human errors. Reliability and mean time to failure formulas are developed for exponential and Rayleigh distributed failure times. Selective plots are shown for demonstrating the effect of human errors on system reliability and mean time to failure.

Keywords: Reliability, Availability, Human Errors, Mean time to failure, Bridge System

Introduction

Humans play a pivotal role in the design, development and operational phases of engineering systems. Reliability evaluation of systems without taking into consideration the human element does not provide a realistic picture. Hence, there is a definite need for incorporating the occurrence of human errors in system reliability evaluation.

A Human error is defined as a failure to perform a prescribed task (or the performance of a prohibited action) which could lead to disruption of scheduled operations or results in damage to property and equipment. Furthermore, depending upon the severity of human error consequences, human errors can be classified into two categories, namely, critical and non-critical. For our purpose the occurrence of a critical human error causes the entire system to fail where as the occurrence of a non-critical human error results in a single unit failure only.

This paper presents reliability analysis of bridge and parallel- series networks with critical and non-critical human errors [1-6].A newly developed approach [1,4-5,6]. is used to perform system reliability analysis. This approach is a modified version of the block diagram approach and is demonstrated in this which consists of a seven unit bridge network.

Assumptions

The following assumptions are associated with analysis given below:

1. A unit can fail either due to a hardware failure or due to a non-critical human error.
2. The occurrence of a critical human error can result in total system failure but the occurrence of a non-critical human error can cause the failure of a single unit only.
3. Each unit failure is independent of others.

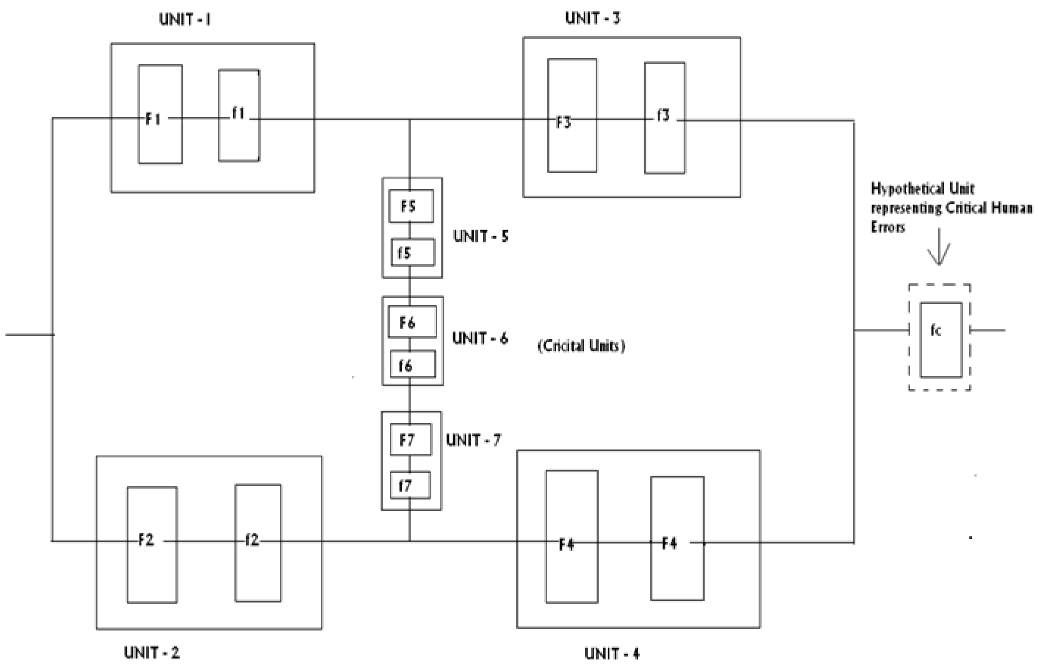

Figure 1.1: Block diagram for bridge network.

Analysis

Bridge Network

This paper represents a seven unit bridge network with critical and non-critical human errors as shown in Figure 1.1. In this figure, each real unit is represented by a rectangle. The failure probability of each unit is divided into two components, namely, hardware failure probability and non-critical human error probability. These failure probabilities are represented by block connected in series as shown in each rectangle in figure 1.1. A hypothetical unit representing critical human errors is connected in series with the bridge network. The total system can fail due to the failure of this hypothetical unit.

Notation

The following symbols are associated with this:

F_j - Hardware failure probability of jth unit, for j=1,2,3,4,56,7.

f_j - jth unit failure probability with respect to non-critical human errors, for j=1,2,3,4,5,6,7.

f_c - critical human error occurrence probability associated with the system

R_{Hj} – hardware reliability of the jth unit.

R_{NCj} - reliability of the jth unit with respect to non-critical human errors.

R_j - reliability of the jth unit with respect to hardware failures and non-critical human errors.

R_c - system reliability with respect to critical human errors.

$R_{H,NC}$ - system reliability with respect to hardware failures and non-critical human errors.

R_b - bridge system reliability with respect to hardware failure, critical and non-critical human errors.

s- Laplace transform variable

t- time

The time-independent reliability analysis are developed for the following two cases:

Case(1) : Non-identical units

The hardware reliability of jth unit is given by

$$R_{Hj}=1-F_j, \text{ for } j=1,2,3,4,5,6,7. \quad (1)$$

The reliability of jth unit with respect to non-critical human errors is

$$R_{NCj}=1-f_j, \text{ for } j=1,2,3,4,5,6,7. \quad (2)$$

The reliability of the jth unit with respect to hardware failures and non-critical human errors is

$$R_j=R_{Hj}R_{NCj}, \text{ for } j=1,2,3,4,5,6,7. \quad (3)$$

The bridge network's reliability with respect to hardware failures and non-critical human errors is:

$$R_{H,NC}=2R_1R_2R_3R_4R_5R_6R_7- R_2R_3R_4R_5R_6R_7- R_1R_3R_4R_5R_6R_7- R_1R_2R_4R_5R_6R_7-R_1R_2R_3R_5R_6R_7- R_1R_2R_3R_4+ R_2R_3R_5R_6R_7+ R_1R_4R_5R_6R_7+R_2R_4+R_1R_3 \quad (4)$$

The reliability of the bridge network with respect to critical human errors only is

$$R_c=1-f_c \quad (5)$$

Finally, using equations (4) and (5), we get

$$R_b=R_c * R_{H,NC} \quad (6)$$

Case (2) : Identical units

By setting $R_j=R$(i.e., $F_j=F$ and $f_j=f$), for j=1,2,3,4,5,6,7 in equation (6) yields

$$R_b=R_c(2R^7 -4R^6 +2 R^5 - R^4 +2 R^2), \quad (7)$$

Where $R=R_H R_{NC}$, $R_H=1-F$ and $R_{NC}=1-f$.

The plots of equation (7) are shown in figure 1.2 for the specified values of F, f and f_c.

These plots clearly show the impact of varying critical human error probability f_c and non-critical human error probability f on bridge system reliability. It is evident from these plots that the system reliability decreases with increasing values of f and f_c.

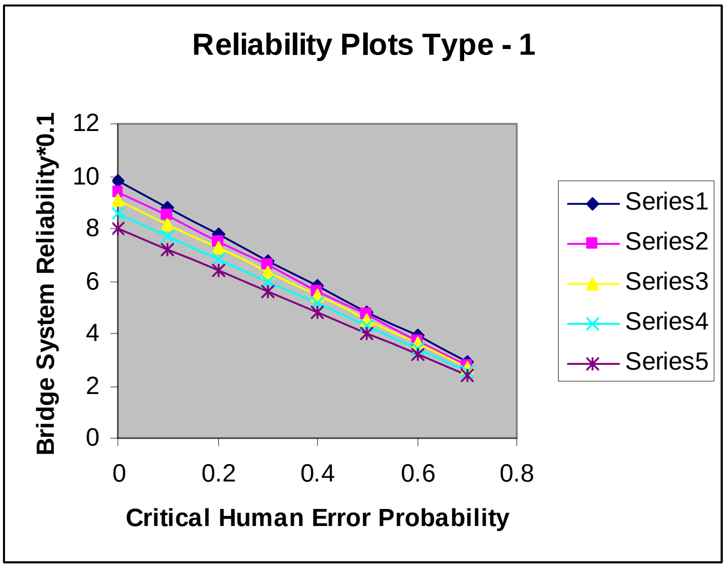

Figure 1.2: F=0.1.

Series	1	2	3	4	5
	f=0	f=0.05	f=0.1	f=0.15	f=0.2

Time dependent analysis for the following two cases are developed:

Case A: Exponentially distributed failure times

For exponentially distributed hardware failure, critical and non-critical human error times the time dependent equations for R_H, R_{NC},R and R_c are as follows:

$$R_H(t)=e^{-\lambda_H t} \tag{8}$$

Where λ_H is the constant hardware failure rate of a unit.

$$R_{NC}(t)=e^{-\lambda_{NC} t} \tag{9}$$

Where λ_{NC} is the constant non-critical human error rate associated with a unit.

$$R(t)=e^{-Xt},. \tag{10}$$

Where $X=\lambda_H+\lambda_{NC}$

$$R_c(t)=e^{-\lambda_c t} \tag{11}$$

Where λ_c is the constant critical human error rate associated with the system.

Using equations (7) to (11), we get the reliability of the seven identical unit networks as follows:

$$R_b(t)=2e^{-(7X+\lambda_c)t}-4e^{-(6X+\lambda_c)t}+2e^{-(5X+\lambda_c)t}-e^{-(4X+\lambda_c)t}+2e^{-(2X+\lambda_c)t} \tag{12}$$

The plots of equation (12) are shown in figure 1.3. for the assumed values of the model parameters.

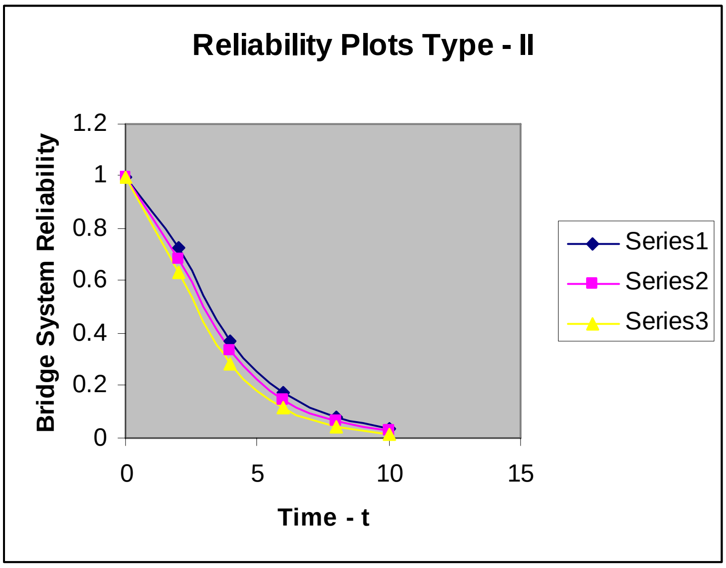

Figure 1.3: λ_H=0.12, λ_{NC}=0.12.

Series	1	2	3
	λ_C=0	λ_C=0.03	λ_C=0.07

These plots clearly demonstrate the effect of varying time t and bridge system reliability. It is evident from these plots that the system reliability decreases with increasing values of time.

The mean time to failure of the bridge system is given by

$$MTTF_b = \int_0^{\infty} R_b(t)dt$$

$$= \frac{2}{7X+\lambda_c} - \frac{4}{6X+\lambda_c} + \frac{2}{5X+\lambda_c} - \frac{1}{4X+\lambda_c} + \frac{2}{2X+\lambda_c} \quad (13)$$

The plots of the above equation are shown in figure 1.4

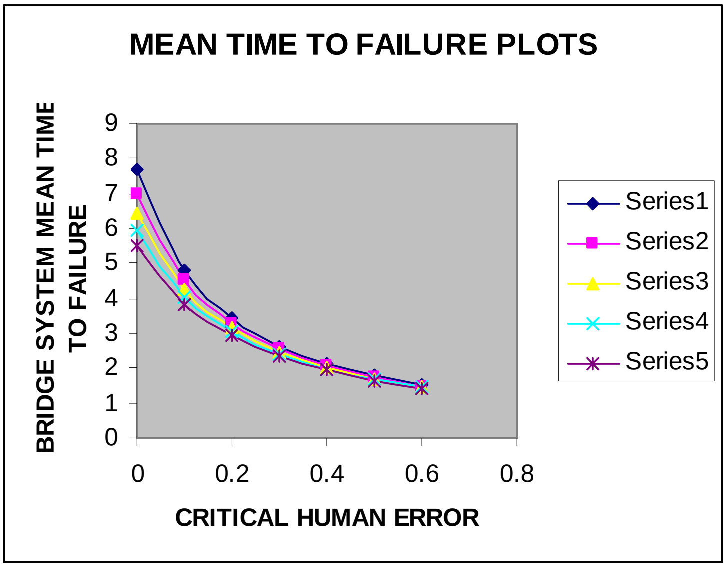

Figure 1.4: λ_H=0.1.

Series	1	2	3	4	5
	λ_{NC}=0	λ_{NC}=0.01	λ_{NC}=0.02	λ_{NC}=0.03	λ_{NC}=0.04

These plots clearly demonstrate the effect of varying critical and non-critical human error rates λ_c and λ_{NC} on the bridge system mean time to failure.

The variance of time to failure of the bridge system with human errors is

$$\sigma_b^2 = -2\lim_{s\to 0} R_b'(s) - (MTTF_b)^2$$

$$= \frac{4}{(7X+\lambda c)^2} -- \frac{8}{(6X+\lambda c)^2} + \frac{4}{(5X+\lambda c)^2} -- \frac{2}{(4X+\lambda c)^2} + \frac{4}{(2X+\lambda c)^2} -$$

$$\left[\frac{2}{(7X+\lambda c)} - \frac{4}{(6X+\lambda c)} + \frac{2}{(5X+\lambda c)} - \frac{1}{(4X+\lambda c)} + \frac{2}{(2X+\lambda c)}\right]^2 \quad (14)$$

Where R'_b(t) is the derivate of the laplace transform of R_b(t) with respect to s.

The bridge system failure density function is given by

$$f_b(t) = - R'_b(t)$$
$$= 2(7X+\lambda_c)e^{-(7X+\lambda c)t} - 4(6X+\lambda_c)e^{-(6X+\lambda c)t} + 2(5X+\lambda_c)e^{-(5X+\lambda c)t}$$
$$-(4X+\lambda_c)e^{-(4X+\lambda c)t} + 2(2X+\lambda_c)e^{-(2X+\lambda c)t} \quad (15)$$

Where

$$R'_b(t) = \frac{d}{dt} R_b(t)$$

The hazard rate function of the bridge system is

$$h_b(t) = \frac{f_b(t)}{R_b(t)} = \frac{2(7X+\lambda_c)e^{-7Xt} - 4(4X+\lambda_c)e^{-6Xt} + 2(5X+\lambda_c)e^{-5Xt} - (4X+\lambda_c)e^{-4Xt} + 2(2X+\lambda_c)e^{-2Xt}}{2e^{-7Xt} - 4e^{-6Xt} + 2e^{-5Xt} - e^{-4Xt} + 2e^{-2Xt}} \quad (16)$$

The plots of equation (16) are shown in figure 1.5.

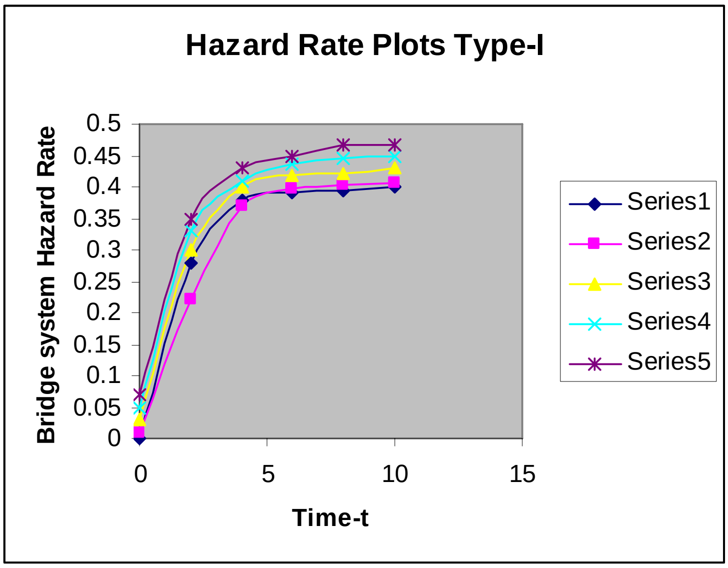

Figure 1.5: λ_H=0.12 λ_{NC}=0.08.

Series	1	2	3	4	5
	λ_c=0	λ_c=0.01	λ_c=0.03	λ_c=0.05	λ_c=0.07

These plots clearly show the impact of varying time t and constant critical human error rate λ_c. It is evident from these plots that the system hazard rate increases with the increasing values of t and λ_c.

Case B: Rayleigh distributed failure times

For Rayleigh distributed failure times the time dependent equations for R_H,R_{NC}, R and

R_c are: $R_H(t)= e^{-\beta_H t^2}$ (17) Where $\beta_H=\frac{1}{\alpha_H}$; α_H is the scale parameter associated with the Rayleigh distribution representing hardware failure times of a unit

$R_{NC}(t)= e^{-\beta_{NC}t^2}$ (18) Where $\beta_{NC}=\frac{1}{\alpha_{NC}}$; α_{NC} is the scale parameter associated with the Rayleigh distribution representing failure times of a unit due to non-critical human errors.

$R(t)= e^{-Yt^2}$ (19) Where $Y= \beta_H+\beta_{NC}$

$$R_c(t)=e^{-\beta_c t^2} \tag{20}$$

Where $\beta_c=\frac{1}{\alpha_c}$; α_c is the scale parameter associated with the Rayleigh distribution representing bridge system failure times with respect to critical human errors.

Using equations (7), (17) to(20), the bridge system reliability with human error is

$$R_b(t)=2e^{-(\beta_c+7Y)t^2}-4e^{-(\beta_c+6Y)t^2}+2e^{-(\beta_c+5Y)t^2}-e^{-(\beta_c+4Y)t^2}+2e^{-(\beta_c+2Y)t^2} \tag{21}$$

The numerical results pertaining to equation (21) are tabulated in the table1

The system mean time to failure is given by

$$MTTF_b= \int_0^{\infty} R_b(t)dt$$

$$=\left(\frac{\pi}{\beta_c+7Y}\right)^{1/2}-2\left(\frac{\pi}{\beta_c+6Y}\right)^{1/2}+\left(\frac{\pi}{\beta_c+5Y}\right)^{1/2}-\frac{1}{2}\left(\frac{\pi}{\beta_c+4Y}\right)^{1/2}+\left(\frac{\pi}{\beta_c+2Y}\right)^{1/2} \tag{22}$$

Table 1: Reliability values for Bridge system β_H=0.13, β_{NC}=0.09.

	Bridge system reliability				
Time(t)	β_c=0.0	β_c=0.02	β_c=0.04	β_c=0.06	β_c=0.08
0.0	1.000	1.000	1.000	1.000	1.000
0.3	0.999	0.997	0.995	0.993	0.992
0.6	0.986	0.979	0.972	0.965	0.958
0.9	0.932	0.917	0.902	0.888	0.873
1.2	0.819	0.787	0.765	0.743	0.722
1.5	0.631	0.603	0.576	0.551	0.527
1.8	0.438	0.410	0.384	0.360	0.338
2.1	0.273	0.249	0.229	0.209	0.192
2.4	0.154	0.317	0.122	0.109	0.097
2.7	0.079	0.069	0.059	0.051	0.044
3.0	0.038	0.032	0.026	0.022	0.018

The failure density function of the bridge system is

$$f_b(t)=-\frac{d}{dt}R_b(t)$$

$$= t\begin{bmatrix} 4(\beta_c+7Y)e^{-(\beta_c+7Y)t^2}-8(\beta_c+6Y)e^{-(\beta_c+6Y)t^2}+4(\beta_c+5Y)e^{-(\beta_c+5Y)t^2} \\ -2(\beta_c+4Y)e^{-(\beta_c+4Y)t^2}+4(\beta_c+2Y)e^{-(\beta_c+2Y)t^2} \end{bmatrix} \tag{23}$$

The system hazard rate function is expressed as

$$h_b(t)=\frac{f_b(t)}{R_b(t)}=\frac{A(t)}{B(t)} \tag{24}$$

Where

$$A(t)=2t\begin{bmatrix} 2(\beta_c+7Y)e^{-7Yt^2}-4(\beta_c+6Y)e^{-6Yt^2}+2(\beta_c+5Y)e^{-5Yt^2} \\ -(\beta_c+4Y)e^{-4Yt^2}+2(\beta_c+2Y)e^{-2Yt^2} \end{bmatrix}$$

$$B(t)=2e^{-7Yt^2}-4e^{-6Yt^2}+2e^{-5Yt^2}-e^{-4Yt^2}+2e^{-2Yt^2}$$

The plots of equation (24) are shown in figure1.6.

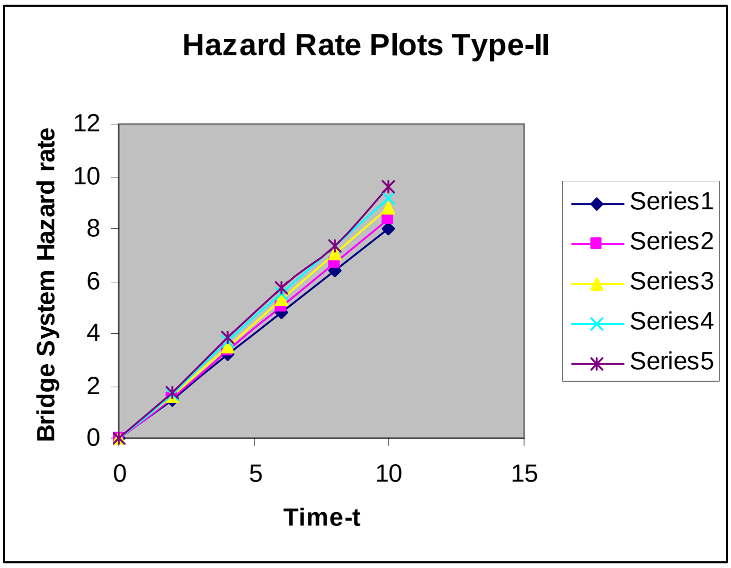

Figure 1.6: β_H=0.12 β_{NC}=0.08.

Series	1	2	3	4	5
	βc=0	βc=0.02	βc=0.04	βc=0.06	βc=0.08

These plots clearly show the impact of varying time t and β_c. It is evident from these plots that the system Hazard rate increases with increasing values of t and β_c.

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