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Interaction of Peristalsis with Heat Transfer of Visco Elastic Rivlin Erickson Fluid through a Porous Medium under the Magnetic Field

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Abstract

The interaction of peristalsis with Heat transfer of Viscoelastic Rivlin Erickson fluid in a two-dimensional channel of the flexible wall through a porous medium under the magnetic field has been studied. Assuming that the wave length of the peristaltic wave is large in comparison to the mean half width of the channel, a perturbation method of solution is obtained in terms of wall slope parameter and closed form expressions have been derived for stream function, temperature and heat transfer coefficient. Furthermore, the effects of elasticity parameters and magnetic parameter on average temperature, average heat transfer have been discussed. It has been observed that the average temperature gradually enhances with increase in Hartman number M and the average heat transfer depreciates with increase in elastic parameters E_1 and E_2.

Keywords: Peristaltic transport, Viscoelastic fluid, Heat transfer, Porous medium.

Introduction

The Magneto hydrodynamic (MHD) flow of a fluid in a channel with peristalsis is of interest in connection with certain flow problems of the movement of conductive physiological fluids e.g. the blood and blood pump machines, and with the need for theoretical research on the operation of a peristaltic MHD Compressor. Blood is regarded as a suspension of small cells in plasma. Moreover, it is known that in arteries, blood flows in two layers, a plasma layer near the wall and a core layer consisting of suspension of cells in the plasma. Since the red blood cells, which contain iron, are magnetic in nature, the core may be treated as magnetic field.

The effect of moving magnetic field on blood flow was studied by Sud et al.[19] and they observed that the effect of suitable moving magnetic field accelerates the speed of blood. Prasada Rao et al [8] studied Peristaltic transport of a viscous conduction fluid under a uniform magnetic field. Deshikachar and Rama Chandra Rao [3] investigated the effect of a magnetic field on the blood oxygenation process. Mekheimer, Kh.S., [11] discussed the peristaltic flow of blood under effect of a magnetic field in a non-uniform channels. Radhakrishnamacharya and Radhakrishna Murthy [14] have studied Heat Transfer to peristaltic transport in a uniform channel in the presence of magnetic field. Maruthi Prasad et al., [10] discussed peristaltic transport of a Hershel-Bulkley fluid in a channel in the presence of magnetic field of low intensity. Rathod and Hosurker Shrikanth [16] studied the flow of RivlinErickson incompressible fluid through an inclined channel with two parallel flat walls under the influence of magnetic field and the MHD flow of RivlinEricksen fluid between two infinite parallel inclined plates. Mittra and Prasad [12] studied peristaltic transport in a two-dimensional channel considering the elasticity of the walls under the approximation of small amplitude ratio with dynamic boundary conditions.

The study of magnetic field with porous medium is very important both from theoretical as well as practical point of view, because most of natural phenomena of the fluid flow are connected with porous medium. For e.g., filtration of fluids, underground water, oil reservoir and fluid through pipes. Hayat,T, et.al.[5] studied effect of Heat transfer on the peristaltic flow of an electrically conducting fluid in a porous space. D.V. Krishna and Mallikarjuna Goud [9] studied the effect of a magnetic field on the peristaltic flow through a porous medium in a non uniform channel and its application to blood flow. Sobh [18] studied the slip flow of peristaltic transport of a magneto-Newtonian fluid through a porous medium with heat transfer. Kothandapani, M. Srinivas, S.,[7] studied On the influence of wall properties in the MHD peristaltic transport with heat transfer and porous medium. It has been observed that under the influence magnetic field, flow separation occurred only at high Reynolds number compared to the non-magnetic case.

Keeping the above facts in view, in this chapter we discuss the interaction of peristalsis with heat transfer and wall properties of Viscoelastic Rivlin Erickson fluid through a porous medium in the presence of Magnetic Field. Assuming that the wave length of the peristaltic wave is large in comparison to the mean-half width of the channel, a perturbation solution has been obtained in terms of the wall slope parameter and closed form expressions have been derived for stream function, velocity, temperature, heat transfer coefficient. The effects of elasticity parameters and magnetic parameter on temperature, heat transfer coefficient, average temperature, average heat transfer, pressure (rise) drop and frictional force have been discussed.

Formulation of the Problem

We consider a peristaltic flow of an incompressible Viscoelastic Rivlin Erickson fluid through porous medium under uniform transverse magnetic field in a two dimensional channel of uniform thickness. The channel is symmetric with respect to its axis and walls of are assumed to be flexible and are taken as a stretched membrane on which traveling sinusoidal waves of moderate amplitude are imposed.

The geometry of the flexible walls are represented by

$$y=\eta(x,t)=d+a\ \mathrm{Sin}\frac{2\pi}{\lambda}(x-ct) \tag{2.1}$$

Where, 'd' is the mean half width of the channel ' a ' is the amplitude of the peristaltic wave, ' c ' is the wave velocity, ' λ ' is the wave length and 'c' is the phase speed of the wave.

The equations governing the two-dimensional flow of Rivlin Erickson fluid are

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.2}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+\beta\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\frac{\nu}{k_1}u-\frac{\sigma\mu_e^2H_0^2}{\rho}u \tag{2.3}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)+\beta\frac{\partial^2}{\partial y^2}\left(\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)-\frac{\nu}{k_1}v \tag{2.4}$$

Equation of energy

$$\rho C_p\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\upsilon\left(\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}\right)^2-\beta\left(\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial t\partial y}+u\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}+v\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}\right) \tag{2.5}$$

Where u and v are the velocity components, 'p' is the fluid pressure, ' ρ ' is the density of the fluid, ' β ' is the coefficient of Visco-elasticity, ' ν ' is the coefficient of kinematic viscosity, k_1 is the permeability of the porous medium, μ_e is the magnetic permeability and H_0 is the magnetic field intensity and σ is the conductivity, 'k' is the coefficient of thermal conductivity, ' C_p ' is the specific heat at constant pressure, 'T' is the temperature.

The governing equation of motion of the flexible wall may be expressed as

$$L(\eta)=p-p_0 \tag{2.6}$$

Where 'L' is an operator, which is used to represent the motion of stretched membrane with damping forces such that

$$L \equiv -T^{*}\frac{\partial^2}{\partial x^2}+m\frac{\partial^2}{\partial t^2}+C\frac{\partial}{\partial t} \tag{2.7}$$

Here T^{*} is the elastic tension in the membrane, m is the mass per unit area and C is the coefficient of viscous damping forces, p_0 is the pressure on the outside surface of the wall due to tension in the muscles. For simplicity, we assume $P_0 = 0$.

The horizontal displacement assumed to be zero, gives

$$u=0 \quad \text{on} \quad y=\pm\,\eta=\pm\,[d+a\ \mathrm{Sin}\frac{2\pi}{\lambda}(x-ct)] \tag{2.8}$$

The dynamic boundary conditions at the flexible walls [12] are

$$\frac{\partial}{\partial x}L(\eta)=\rho\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)-\frac{\nu\rho}{K}u-\sigma\mu_e{}^2H_0{}^2u \tag{2.9}$$

$$\text{on} \quad y=\pm\eta$$

Where

$$\frac{\partial}{\partial x}L(\eta)=\frac{\partial p}{\partial x}=-T^{*}\frac{\partial^3\eta}{\partial x^3}+m\frac{\partial^3\eta}{\partial t^2\partial x}+C\frac{\partial^2\eta}{\partial t\,\partial x}$$

The conditions on temperature are

$$T=T_0 \ \text{on} \ \ y=-\eta\ , \quad T=T_1 \ \ \text{on} \ \ y=\eta \tag{2.10}$$

In view of the incompressibility of the fluid and two-dimensionality of the flow, we introduce the Stream function‘ ψ ‘such that

$$u=\frac{\partial\psi}{\partial y} \quad \text{and} \quad v=-\frac{\partial\psi}{\partial x} \tag{2.11}$$

and introducing non-dimensional quantities

$$x'=\frac{x}{\lambda},\ y'=\frac{y}{d},\ u'=\frac{u}{c},\ v'=\frac{v}{c\,\delta},\ \psi'=\frac{\psi}{cd},\ t'=\frac{ct}{\lambda},\ \eta'=\frac{\eta}{d},\ p'=\frac{p\,d^2}{\mu\,c\,\lambda},\ \theta=\frac{T-T_0}{T_1-T_0} \tag{2.12}$$

in equations of motion and the conditions (2.3) – (2.5), (2.8) – (2.10), and eliminating ‘p’, we finally get (after dropping primes)

$$\begin{aligned}
&\delta\left(\left(\frac{\partial}{\partial t}\left(\nabla^2\psi\right)\right)+\frac{\partial}{\partial y}\left(\frac{\partial\psi}{\partial x}\right)\left(\nabla^2\psi\right)+\frac{\partial\psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\nabla^2\psi\right)\right)-\frac{\partial}{\partial x}\left(\frac{\partial\psi}{\partial y}\right)\left(\nabla^2\psi\right)-\frac{\partial\psi}{\partial x}\left(\frac{\partial}{\partial y}\left(\nabla^2\psi\right)\right)\right)\\
&=S\delta\left(\frac{\partial}{\partial t}\left(\frac{\partial^4\psi}{\partial y^4}\right)+\frac{\partial\psi}{\partial y}\frac{\partial^5\psi}{\partial x\partial y^4}+\frac{\partial^4\psi}{\partial y^4}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial^4\psi}{\partial x\partial y^3}\frac{\partial^2\psi}{\partial y^2}-\frac{\partial\psi}{\partial x}\frac{\partial^5\psi}{\partial y^5}\right)\\
&+2S\delta^3\left(\frac{\partial}{\partial t}\left(\frac{\partial^4\psi}{\partial x^2\partial y^2}\right)+\frac{\partial^4\psi}{\partial y^4}\frac{\partial^2\psi}{\partial x^2}+\frac{\partial\psi}{\partial y}\frac{\partial^5\psi}{\partial x^2\partial y^3}-\frac{\partial^4\psi}{\partial x\partial y^3}\frac{\partial^2\psi}{\partial x^2}-\frac{\partial\psi}{\partial x}\frac{\partial^5\psi}{\partial x^2\partial y^3}\right)\\
&+\frac{1}{R}\left(\frac{\partial^2}{\partial y^2}\left(\nabla^2\psi\right)+\delta^2\left(\frac{\partial^2}{\partial x^2}\left(\nabla^2\psi\right)\right)\right)+D^{-1}\left(\nabla^2\psi\right)-\frac{M^2}{R}\frac{\partial^2\psi}{\partial y^2}
\end{aligned} \tag{2.13}$$

Where

$$\nabla^2=\frac{\partial^2}{\partial y^2}+\delta^2\frac{\partial^2}{\partial x^2}$$

$$\begin{aligned}
R\delta\left(\frac{\partial\theta}{\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial\theta}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\theta}{\partial y}\right)&=\frac{1}{P_r}\left(\delta^2\frac{\partial^2\theta}{\partial x^2}+\frac{\partial^2\theta}{\partial y^2}\right)+E\left(\delta^2\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)^2\\
&-SRE\delta\left(\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial t\partial y^2}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial x\partial y^2}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\frac{\partial^3\psi}{\partial y^3}\right)
\end{aligned} \tag{2.14}$$

$$\frac{\partial\psi}{\partial y}=0 \quad \text{on} \quad y=\pm\eta=\pm[1+\varepsilon\ \mathrm{Sin}\,2\pi(x-t)] \tag{2.15}$$

$$\begin{aligned}
&\left(\frac{\partial^3\psi}{\partial y^3}+\delta^2\frac{\partial^3\psi}{\partial x^2\partial y}\right)-R\delta\left(\frac{\partial^2\psi}{\partial y\partial t}+\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial x\partial y}-\frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right)-\frac{R}{D_a}\frac{\partial\psi}{\partial y}-M^2\frac{\partial\psi}{\partial y}\\
&=\left(E_1\frac{\partial^3}{\partial x^3}+E_2\frac{\partial^3}{\partial x\partial t^2}+E_3\frac{\partial^2}{\partial x\partial t}\right)\eta \quad \text{on} \quad y=\pm\eta
\end{aligned} \tag{2.16}$$

$$\theta=0 \quad \text{on} \quad y=-\eta, \qquad \theta=1 \quad \text{on} \quad y=\eta \tag{2.17}$$

The non-dimensional parameters are

$\varepsilon=\frac{a}{d}$ and $\delta=\frac{d}{\lambda}$ are geometric parameters

$R=\frac{c\,d}{\nu}$ is the Reynolds number

$S=\frac{\beta}{d^2}$ is the Visco-elastic parameter

$P_r = \dfrac{\rho\, c_p \nu}{k}$ is the Prandtl number

$D_a = \dfrac{c\, k}{\nu\, d}$ is the Darcy number

$E = \dfrac{c^2}{\rho\, c_p (T_1 - T_0)}$ is the Eckert number

$M = \sqrt{\dfrac{\sigma\, \mu_e{}^2 H_0{}^2 d^2}{\mu}}$ is the Hartman number

$E_1 = -\dfrac{T\, d^3}{\lambda^3 \rho\, \nu\, c}$, $E_2 = \dfrac{m\, c\, d^3}{\lambda^3 \rho\, \nu}$, $E_3 = \dfrac{C\, d^3}{\lambda^2 \rho\, \nu}$ are the elasticity parameters

Method of Solution

We seek perturbation solution for the stream function (ψ), pressure gradient (p) and temperature coefficient (θ) in terms of small parameter δ as follows:

$$\psi = \psi_0 + \delta\, \psi_1 + \ldots\ldots \tag{3.1}$$

$$\theta = \theta_0 + \delta\, \theta_1 + \ldots\ldots \tag{3.2}$$

Substituting (3.1-3.2) in equations (2.13-2.17) and collecting the coefficients of various powers of δ

The zeroth order equations are

$$\frac{\partial^4 \psi_0}{\partial y^4} - \left(\frac{R}{D_a} + M^2\right)\frac{\partial^2 \psi_0}{\partial y^2} = 0 \tag{3.3}$$

$$\frac{1}{P_r}\left(\frac{\partial^2 \theta_0}{\partial y^2}\right) + E\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)^2 = 0 \tag{3.4}$$

The corresponding boundary conditions are

$$\frac{\partial \psi_0}{\partial y} = 0 \quad \text{on} \quad y = \pm\eta \tag{3. 5}$$

$$\frac{\partial^3 \psi_0}{\partial y^3} - \left(\frac{R}{D_a} + M^2\right)\frac{\partial \psi_0}{\partial y} = \left(E_1 \frac{\partial^3}{\partial x^3} + E_2 \frac{\partial^3}{\partial x\, \partial t^2} + E_3 \frac{\partial^2}{\partial x\, \partial t}\right)\eta \quad \text{on} \quad y = \pm\eta \tag{3.6}$$

$$\theta_0 = 0 \quad \text{on} \quad y = -\eta, \qquad \theta_0 = 1 \quad \text{on} \quad y = \eta \tag{3.7}$$

The first order equations are

$$\frac{\partial}{\partial t}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)+\frac{\partial}{\partial y}\left(\frac{\partial \psi_0}{\partial x}\right)\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)+\frac{\partial \psi}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right)-\frac{\partial}{\partial x}\left(\left(\frac{\partial \psi_0}{\partial y}\right)\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right)$$

$$-\frac{\partial \psi_0}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right)=\frac{1}{R}\left(\frac{\partial^4 \psi_1}{\partial y^4}\right)-\left(\frac{1}{D_a}+\frac{M^2}{R}\right)\frac{\partial^2 \psi_1}{\partial y^2}+ \tag{3.8}$$

$$+S\left(\frac{\partial}{\partial t}\left(\frac{\partial^4 \psi_0}{\partial y^4}\right)+\frac{\partial \psi_0}{\partial y}\frac{\partial^5 \psi_0}{\partial x \partial y^4}+\frac{\partial^4 \psi_0}{\partial y^4}\frac{\partial^2 \psi_0}{\partial x \partial y}-\frac{\partial^4 \psi_0}{\partial x \partial y^3}\frac{\partial^2 \psi_0}{\partial y^2}-\frac{\partial \psi_0}{\partial x}\frac{\partial^5 \psi_0}{\partial y^5}\right)$$

$$R\left(\frac{\partial \theta_0}{\partial t}+\frac{\partial \psi_0}{\partial y}\frac{\partial \theta_0}{\partial x}-\frac{\partial \psi_0}{\partial x}\frac{\partial \theta_0}{\partial y}\right)=\frac{1}{P_r}\left(\frac{\partial^2 \theta_1}{\partial y^2}\right)+2E\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\left(\frac{\partial^2 \psi_1}{\partial y^2}\right)$$

$$-SRE\left(\frac{\partial^2 \psi_0}{\partial y^2}\frac{\partial^3 \psi_0}{\partial t \partial y^2}+\frac{\partial \psi_0}{\partial y}\frac{\partial^2 \psi_0}{\partial y^2}\frac{\partial^3 \psi_0}{\partial x \partial y^2}-\frac{\partial \psi_0}{\partial x}\frac{\partial^2 \psi_0}{\partial y^2}\frac{\partial^3 \psi_0}{\partial y^3}\right) \tag{3.9}$$

The corresponding boundary conditions are

$$\frac{\partial \psi_1}{\partial y}=0 \quad \text{on} \quad y=\pm\eta \tag{3.10}$$

$$\frac{\partial^3 \psi_1}{\partial y^3}-\left(\frac{R}{D_a}+M^2\right)\frac{\partial \psi_1}{\partial y}-R\left(\frac{\partial}{\partial t}\left(\frac{\partial \psi_0}{\partial y}\right)+\frac{\partial \psi_0}{\partial y}\left(\frac{\partial}{\partial x}\left(\frac{\partial \psi_0}{\partial y}\right)\right)-\left(\frac{\partial \psi_0}{\partial x}\right)\left(\frac{\partial^2 \psi_0}{\partial y^2}\right)\right)=0 \tag{3.11}$$

$$\text{on} \quad y=\pm\eta$$

$$\theta_1=0 \quad \text{on} \quad y=-\eta, \qquad \theta_1=1 \quad \text{on} \quad y=\eta \tag{3.12}$$

Zeroth -order problem

On solving the equations (3.3-3.4), subject to the conditions (3.5-3.7), we get

$$\psi_0 = A_1 y + A_2 \operatorname{Sinh} \alpha y \tag{3.13}$$

$$\theta_0 = \frac{A_9}{4\alpha^2}(2\alpha^2 y^2 - \operatorname{Cosh} 2\alpha y) + A_5 y + A_6 \tag{3.14}$$

First -order problem

On solving the equations (3.8 - 3.9), subject to the conditions (3.10-3.12), we get

$$\psi_1 = A_3 y + A_4 \operatorname{Sinh}\alpha y + \left(\frac{B_1}{2\alpha^3}-\frac{5B_2}{4\alpha^4}\right) y \operatorname{Cosh}\alpha y + \frac{B_2}{4\alpha^3} y^2 \operatorname{Sinh}\alpha y + \frac{B_3}{24\alpha^4}\operatorname{Sinh}2\alpha y \tag{3.15}$$

$$\theta_1 = \frac{g_3}{12}y^4 + \frac{g_2}{6}y^3 + \frac{1}{4}(2g_1 - g_{13})y^2 + \frac{1}{\alpha^4}(\alpha^2 g_5 - 2\alpha g_8 + 6g_{11})\,\mathrm{Sinh}\,\alpha y +$$
$$\frac{1}{4\alpha^4}(4\alpha^2 g_4 - 8\alpha g_9 + 24 g_{10} - \alpha^2 g_{16} - 4\alpha^2 g_{14})\,\mathrm{Cosh}\,\alpha y + \frac{g_7}{4\alpha^2}\mathrm{Sinh}\,2\alpha y +$$
$$\frac{1}{8\alpha^3}(2\alpha g_6 - 2g_{12} + \alpha g_{13} - g_{15})\mathrm{Cosh}\,2\alpha y + \frac{1}{\alpha^3}(\alpha g_9 - 4g_{10})\,y\,\mathrm{Sinh}\,\alpha y + \tag{3.16}$$
$$\frac{1}{\alpha^3}(\alpha g_8 - 4g_{11})\,y\,\mathrm{Cosh}\,\alpha y + \frac{g_{10}}{\alpha^2}y^2\,\mathrm{Cosh}\,\alpha y + \frac{g_{11}}{\alpha^2}y^2\,\mathrm{Sinh}\,\alpha y +$$
$$\frac{1}{8\alpha^2}(2g_{12} + g_{15})y\,\mathrm{Sinh}\,2\alpha y + \frac{1}{72\alpha^2}(9g_{14} + 2g_{16})\,\mathrm{Cosh}\,3\alpha y + A_7 y + A_8$$

The heat transfer coefficient in terms of wall slope parameter 'δ' is

$$z = \left(\frac{\partial\eta}{\partial x}\right)\left(\frac{\partial\theta_0}{\partial y}\right) + \delta\left[\left(\frac{\partial\theta_0}{\partial x}\right) + \left(\frac{\partial\eta}{\partial x}\right)\left(\frac{\partial\theta_1}{\partial y}\right)\right] + \ldots.. \tag{3.17}$$

The average temperature, $\bar{\theta}$ is given by

$$\bar{\theta} = \int_0^1 \theta \; dt \tag{3.18}$$

The average Heat transfer, $\bar{z}$ is given by

$$\bar{z} = \int_0^1 z \; dt \tag{3.19}$$

It can be noticed that when Inverse Darcy number $D_a^{-1} = 0$, Visco-elastic parameter S = 0 and Hartman number M = 0, this problem reduces to Radhakrishnamacharya and Srinivasulu [15].

The constants $A_1, A_2, \ldots A_9, B_1, B_2, B_3, g_1, g_2, \ldots. g_{16}$ are given in appendix.

Results and Discussion

In this analysis we analyzed effect of temperature variation on the peristaltic action of Viscoelastic RivlinErickson fluid in the presence Magnetic field through porous medium for different values of elastic parameters Reynolds number (R),the rigidity of the wall (E_1), the stiffness of the wall (E_2), damping nature of the wall (E_3) and Hartman number (M).

The average temperature $\bar{\theta}$ with different parameters is depicted in figs.(4.1-4.5). From figs.(4.1-4.3) ,it is observed that average temperature $\bar{\theta}$ depreciates in the region $0.1 \le y \le 0.9$ with increase in R, E_1 and E_2 and the enhancement is marginal at the boundary, while we observe that the depreciation in average temperature $\bar{\theta}$ with

E_3 is more significant at the centre of the channel y = 0 than at the boundary fig.(4.4). From fig.(4.5) we observe that the average temperature $\bar{\theta}$ gradually enhances with increase in Hartman number M. It is more significant at the centre of the channel than at the boundary and is insignificant at y =1.

The average heat transfer $\bar{z}$ is numerically evaluated and graphically depicted in figs.(4.6-4.10) for different parameters. The variation of average heat transfer $\bar{z}$ with Reynolds number R is shown in fig.(4.6). It is observed that the variation of $\bar{z}$ with R is almost linear in the region y = 0 to y = 0.6 and $\bar{z}$ graphically enhances in magnitude in the remaining region, the variation of $\bar{z}$ at y = 0 is comparably smaller than at y = 1. From figs.4.7 & 4.8 we find that $|\bar{z}|$ depreciates with increase in E_1 and E_2. The average heat transfer $\bar{z}$ is appreciably large at boundary of the channel y = 1, than at the centre of the channel y = 0, while the variation of average heat transfer $\bar{z}$ with E_3 shows that $\bar{z}$ enhances marginally at y = 0 fig.(4.9). From fig.(4.10) we find that $\bar{z}$ increases in the region $0 \le y \le 0.6$, for higher Hartman number M, smaller the $|\bar{z}|$ decreases.

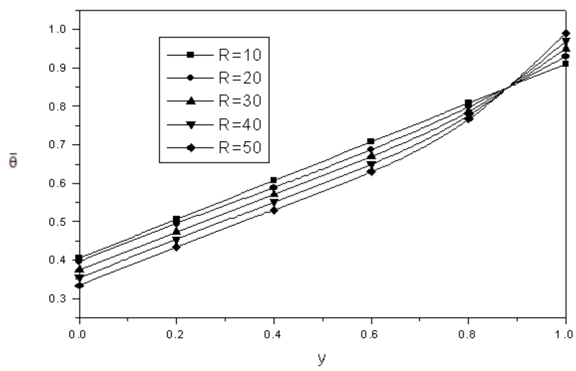

Fig.4.1-Effect of **R** on $\bar{\theta}$ (ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, α =10, S=0.5, E=0.5, P_r=0.7, M=10, D_a=10000)

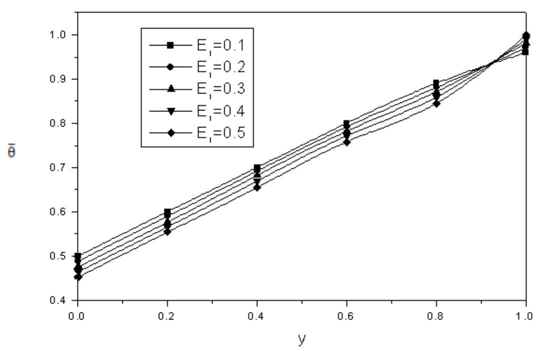

Fig.4.2-Effect of E_1 on $\bar{\theta}$ (ε=0.01, δ=0.01, E_2=0.2, E_3=0.3, α =10, R=10, S=0.5, E=0.5, P_r=0.7, M=10, D_a=10000)

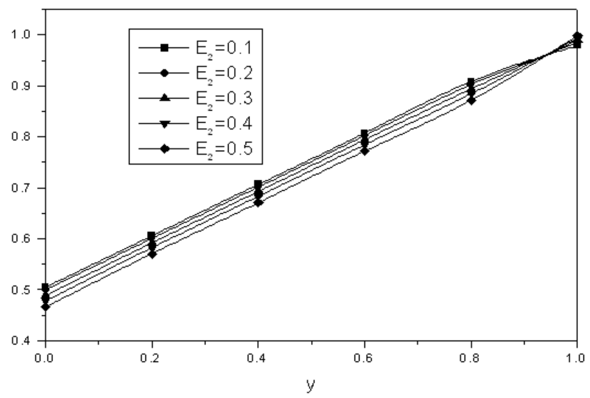

Fig.4.3-Effect of E_2 on $\overline{\theta}$ (ε=0.01, δ=0.01, E_1=0.1, E_3=0.3, α =10, R=10, S=0.5, E=0.5, P_r =0.7, M=10, D_a=10000)

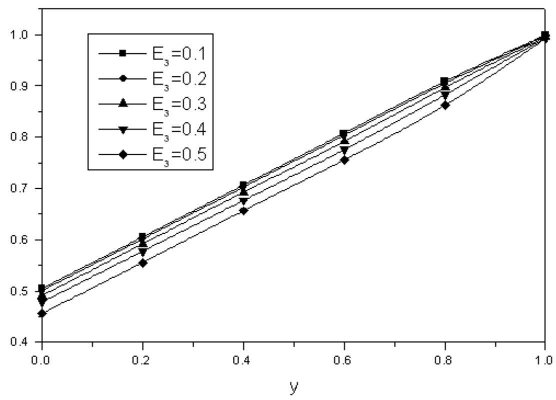

Fig.4.4-Effect of E_3 on $\overline{\theta}$ (ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, α =10, R=10, S=0.5, E=0.5, P_r =0.7, M=10, D_a=10000)

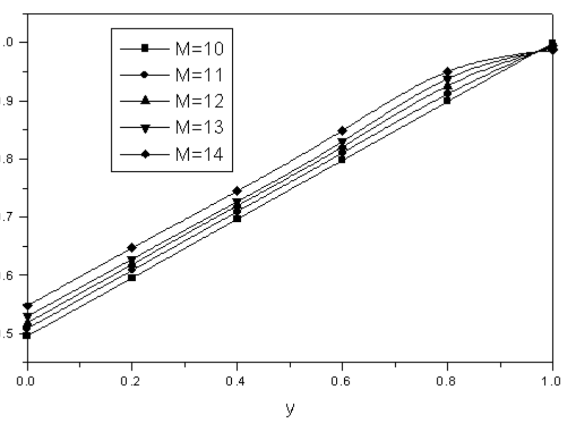

Fig.4.5-Effect of **M** on $\overline{\theta}$ (ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, α =10, R=10, P_r =0.7, S=0.5, E=0.5, D_a=10000)

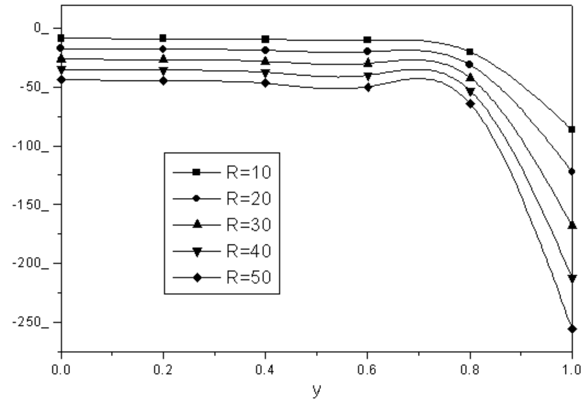

Fig.4.6-Effect of **R** on $\overline{z}$ (ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, α =10, S=0.5, E=0.5, P_r=0.7, M=10, D_a=10000)

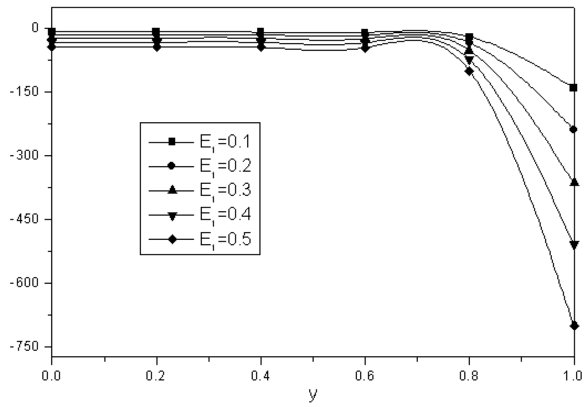

Fig.4.7-Effect of E_1 on $\overline{z}$ (ε=0.01, δ=0.01, E_2=0.2, E_3=0.3, R=10, α =10, S=0.5, E=0.5, P_r=0.7, M=10, D_a=10000)

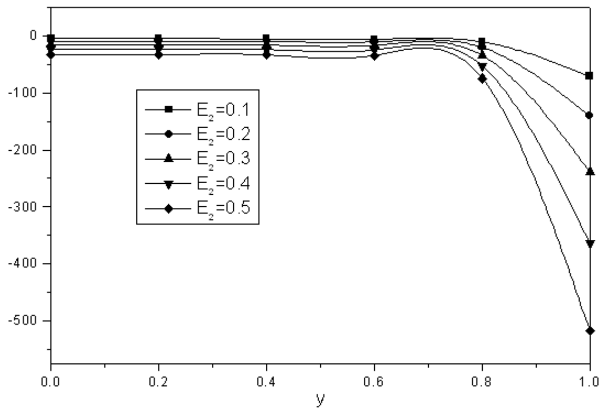

Fig.4.8-Effect of E_2 on $\overline{z}$ (ε=0.01, δ=0.01, E_1=0.1, E_3=0.3, R=10, α =10, S=0.5, E=0.5, P_r=0.7, M=10, D_a=10000)

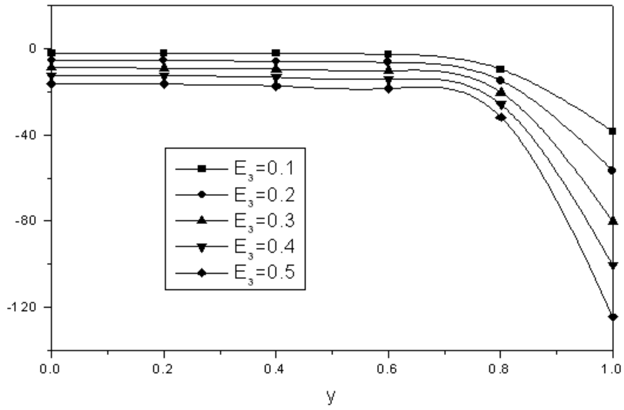

Fig.4.9-Effect of E_3 on $\bar{z}$ (ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, R=10, α =10, S=0.5, E=0.5, P_r=0.7, M=10, D_a=10000)

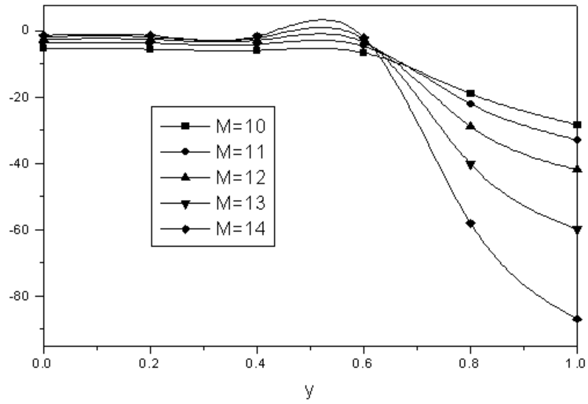

Fig.4.10-Effect of **M** on $\bar{z}$ (ε=0.01, δ=0.01, E_1=0.1, E_2=0.2, E_3=0.3, α =10, R=10, E=0.5,P_r=0.7, S=0.5, D_a=10000)

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Appendix

$$\alpha = \sqrt{\frac{R}{D_a} + M^2}$$

$$\eta = 1 + \varepsilon \, \mathrm{Sin}\, 2\pi (x - t) \quad A_1 = \frac{\varepsilon}{\alpha^2}[8\pi^3 \, \mathrm{Cos}\, 2\pi\,(x - t)(E_1 + E_2) - 4E_3\pi^2 \, \mathrm{Sin}\, 2\pi (x - t)]$$

$$A_2 = -\frac{\varepsilon}{\alpha^3} \mathrm{Sech}\, \alpha\eta [8\pi^3 \, \mathrm{Cos}\, 2\pi (x - t)(E_1 + E_2) - 4E_3\pi^2 \, \mathrm{Sin}\, 2\pi (x - t)]$$

$$A_3 = \frac{1}{\alpha^4}(\alpha B_1 - B_2 - \alpha^2(d_4 + d_{10} + d_9)) \, \mathrm{Cosh}\, \alpha\eta + \frac{1}{\alpha^3}(\alpha^3 d_{10} - B_2)\, \eta \, \mathrm{Sinh}\, \alpha\eta + \frac{B_3}{4\alpha^3} \mathrm{Cosh}\, 2\alpha\eta - \frac{1}{\alpha^2}(d_2 + d_8 + d_{11})$$

$$A_4 = \frac{1}{\alpha^3}(d_2 + d_8 + \alpha^2 d_{11}) \, \mathrm{Sech}\, \alpha\eta - \frac{1}{4\alpha^4}(4\alpha^3 d_{10} + 7B_2 + 2\alpha B_1)\, \eta \, \mathrm{Tanh}\, \alpha\eta - \frac{B_3}{3\alpha^4} \mathrm{Sech}\, \alpha\eta \, \mathrm{Cosh}\, 2\alpha\eta - \frac{B_2}{4\alpha^3}\eta^2 + \frac{1}{4\alpha^5}(9B_2 - 6\alpha B_1 + 4\alpha^3(d_4 + d_{10} + d_9))$$

$$A_5 = \frac{1}{2\eta}$$

$$A_6 = \frac{1}{2} - \frac{A_9}{4\alpha^2}(\mathrm{Cosh}\, 2\alpha\eta - 4\alpha^2\eta^2)$$

$$A_7 = \frac{1}{2\eta} + \frac{g_2}{6}\eta^2 + \frac{1}{\eta\alpha^4}(\alpha^2 g_5 - 2\alpha g_8 + 6g_{11})\, \eta \, \mathrm{Sinh}\, \alpha\eta + \frac{g_7}{4\eta\alpha^2} \mathrm{Sinh}\, 2\alpha\eta + \frac{g_{11}}{\alpha^2}\, \eta \, \mathrm{Sinh}\, \alpha\eta$$

$$A_8 = \frac{1}{2} - \frac{g_3}{12}\eta^4 - \frac{1}{4}(2g_1 - g_{13})\eta^2 - \frac{1}{4\alpha^4}(4\alpha^2 g_4 - 8\alpha g_9 + 24 g_{10} - \alpha^2 g_{16} - 4\alpha^2 g_{14}) \, \mathrm{Cosh}\, \alpha\eta - \frac{1}{8\alpha^3}(2\alpha g_6 - 2g_{12} + \alpha g_{13} - g_{15}) \mathrm{Cosh}\, 2\alpha\eta - \frac{1}{\alpha^3}(\alpha g_9 - 4g_{10})\, \eta \, \mathrm{Sinh}\, \alpha\eta - \frac{1}{\alpha^3}(\alpha g_8 - 4g_{11})\, \eta \, \mathrm{Cosh}\, \alpha\eta - \frac{g_{10}}{\alpha^2}\eta^2 \, \mathrm{Cosh}\, \alpha\eta - \frac{1}{8\alpha^2}(2g_{12} + g_{15})\, \eta \, \mathrm{Sinh}\, 2\alpha\eta - \frac{1}{72\alpha^2}(9g_{14} + 2g_{16}) \, \mathrm{Cosh}\, 3\alpha\eta$$

$$A_9 = \frac{d_{20}\alpha^4 A_2^{\,2}}{2}$$

$$B_1 = \alpha^2(d_4 - d_9 - d_{10}) - S\, d_4\alpha^4$$

$B_2 = R\, d_{10}\alpha^3 (S\,\alpha^2 - 1) \qquad B_3 = R\, d_{11}\alpha^3 (2\,S\,\alpha^2 - 3)$

$d_1 = A_{1x} \qquad d_2 = A_{1t} \qquad d_3 = A_{2x}$

$d_4 = A_{2t} \qquad d_5 = A_{5t} \qquad d_6 = A_{6t}$

$d_7 = A_{9t} \qquad d_8 = A_1 A_{1x} \qquad d_9 = A_1 A_{2x}$

$d_{10} = A_2 A_{1x} \qquad d_{11} = A_2 A_{2x} \qquad d_{12} = A_5 A_{1x}$

$d_{13} = A_5 A_{2x} \qquad d_{14} = A_9 A_{1x} \qquad d_{15} = A_9 A_{2x}$

$d_{16} = A_2 A_{2t} \qquad d_{17} = A_2 d_{10} \qquad d_{18} = A_2 d_{11}$

$d_{19} = p_r R \qquad d_{20} = p_r E \qquad d_{21} = d_{19} d_{20}$

$g_1 = p_r\,(R\,d_6 + E\,A_3\,\alpha^2)\;, \qquad g_2 = d_{19}\,(d_5 - E\,d_{12})\;, \qquad g_3 = \frac{d_{19}}{2\,\alpha^2}(d_7 - 2\,d_{14})$

$$g_4 = \frac{d_{20}}{4\,\alpha^2}(4\,A_4\alpha^5 + 2\,\alpha\,B_1 - 5\,B_2)$$

$$g_5 = \frac{p_r}{2\,\alpha^3}(4\,E\,B_2 + 2\,E\,B_1\,\alpha - 2\,E\,A_4\alpha^5 - 2\,R\,d_{13}\,\alpha^3)$$

$g_6 = \frac{p_r}{12\,\alpha^2}(E\,B_3\,\alpha - 6\,R\,d_7) \qquad g_7 = -\frac{d_{20}\,B_3}{6\,\alpha^2} \qquad g_8 = \frac{d_{20}}{4\,\alpha^2}(B_2 - 2\,\alpha\,B_1)$

$g_9 = \frac{1}{4\,\alpha}(2\,d_{20}\,B_1\alpha - 3\,d_{20}\,B_2 - 8\,R\,d_{15}\alpha) \qquad g_{10} = \frac{d_{20}B_2}{4}$

$g_{11} = -\frac{g_{10}}{\alpha} \qquad g_{12} = \frac{d_{19}\,d_{14}}{\alpha}$

$g_{13} = d_{21}\,d_{16}\alpha^4 \qquad g_{14} = \frac{d_{19}\,d_{15}}{\alpha}$

$g_{15} = -\,d_{21}\,d_{17}\alpha^5 \qquad g_{16} = -\,d_{21}d_{18}\alpha^5$