International Journal of Computational Science and Mathematics.
ISSN 0974-3189 Volume 3, Number 3 (2011), pp. 303-315

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Flow of an Unsteady Conducting Dusty Fluid between a Non-torsional Oscillating Plate and a Long Wavy Wall

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Abstract

The geometry of flow of a dusty viscous conducting fluid between a non-torsional oscillating parallel plate and a long wavy wall in an anholonomic coordinate system has been studied. The velocity distribution of fluid and dust for different pressure gradients is obtained analytically. The effect of strength of magnetic field on velocity profiles at fixed time has been discussed with the help of graphs. Finally the skin fraction at the boundaries is calculated.

Keywords: Frenet frame field system; non-torsional oscillating plate, long wavy wall, conducting dusty fluid; velocity of dust phase and fluid phase, magnetic field, laminar flow, skin friction.

AMS Subject Classification (2000): 76T10, 76T15;

Introduction

The study of the flow of dusty fluids has attracted many researchers to his applications in the fields of fluidization, combustion, use of dust in gas cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology, high-energy solid rocket propellant, fluid-droplet sprays, the electro static precipitation of dust, blood flow fluid droplets sprays and so on.

The importance of study of fluids having non-conducting fine dust particles was traced out by many mathematicians. In 1962 Saffman [16] has given the equations describing the motion of gas containing the small dust particles. On basis of these equations Rossow [15] studied the flow of a viscous, incompressible and electrically conducting fluid in presence of an external magnetic field due to the impulsive motion of an infinite flat plate.

Srinivasacharya, G.Radhakrishnamacharya and Ch.Srinivasulu [19] has discussed the effects of wall properties on peristaltic transport of a dusty fluid, Liu [11], Michael and Miller [13] have studied the flow produced by the motion of an infinite plane in a steady fluid occupying the semi-infinite space above it. A.Eric et al. [5] has studied the quantitative assessment of steady and pulsatile flow fields in a parallel plate flow chamber. Thierry Feraille et al. [17] discussed the channel flow induced by wall injection of fluid and particles.

During the second part of the 20th century, some researchers like Kanwal [10], Truesdell [18], Indrasena [9], Purushotham [14], Bagewadi and Gireesha [1], [2] have applied differential geometry techniques to investigate the kinematical properties of fluid flows in the field of fluid mechanics. Further, recently the authors [6], [7], have studied two-dimensional dusty fluid flow in Frenet frame field system. The paper deals with the study of flow of an electrically conducting viscous incompressible fluid which suspended non-conducting small spherical dust particles between a non-torsional oscillating plate and a long wavy wall. The flow is due to the presence of a uniform transverse magnetic field, non-torsional oscillations of the plate and time dependent pressure gradient. Initially it is assumed that both the conducting fluid and the non-conducting dust particles are to be at rest. Applying Laplace transform technique, the velocity fields for fluid and dust particles have been obtained. Also the skin friction at both the walls has been calculated. Finally the graphs are plotted for different values of Hartmann number and number density.

Equations of Motion

The modified Saffman's [16] equations for the conducting dusty gas and non-conducting dust particle are:

For fluid phase

(2.1) $$\nabla \cdot \vec{u} = 0$$
(Continuity)

(2.2) $$\frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \nabla)\vec{u} = -\rho^{-1}\nabla p + \nu \nabla^2 \vec{u} + \frac{kN}{\rho}(\vec{v} - \vec{u}) + \frac{1}{\rho}(\vec{J} \times \vec{B})$$
(Linear Momentum)

For dust phase

(2.3) $$\nabla \cdot \vec{v} = 0$$
(Continuity)

(2.4) $$\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = \frac{k}{m}(\vec{u} - \vec{v})$$
(Linear Momentum)

We have following nomenclature:

$\vec{u}$ - velocity of the fluid phase, $\vec{v}$ - velocity density of dust phase, ρ - density of the gas, p - pressure of the fluid, N - number of density of dust particles, ν - Kinematic viscosity, $K = 6\pi a\mu$ - Stoke's resistance (drag coefficient), a - spherical radius of dust particle, m - mass of the dust particle, μ - the co-efficient of viscosity of fluid particles, t - time and $\vec{J}$and $\vec{B}$ - given by Maxwell's equations and Ohm's law, namely,

(2.5) $$\nabla \times \vec{H} = 4\pi\vec{J}, \nabla \times \vec{B} = 0, \nabla \times \vec{E} = 0, \vec{J} = \sigma[\vec{J} + \vec{u} \times \vec{B}]$$

Here $\vec{H}$ - magnetic field, $\vec{J}$ - current density, $\vec{B}$ - magnetic Flux, $\vec{E}$ - electric field.

It is assumed that the effect of induced magnetic fields produced by the motion of the electrically conducting gas is negligible and no external electric field is applied. With those assumptions the magnetic field $\vec{J} \times \vec{B}$ of the body force in (2.2) reduces simply to $-\sigma B_0^2 \vec{u}$ where B_0 - the intensity of the imposed transverse magnetic field.

Frenet Frame Field System

Let $\vec{s}, \vec{n}, \vec{b}$ be triply orthogonal unit vectors tangent, principal normal, binormal respectively to the spatial curves of congruence's formed by fluid phase velocity and dusty phase velocity lines respectively as shown in the figure-1.

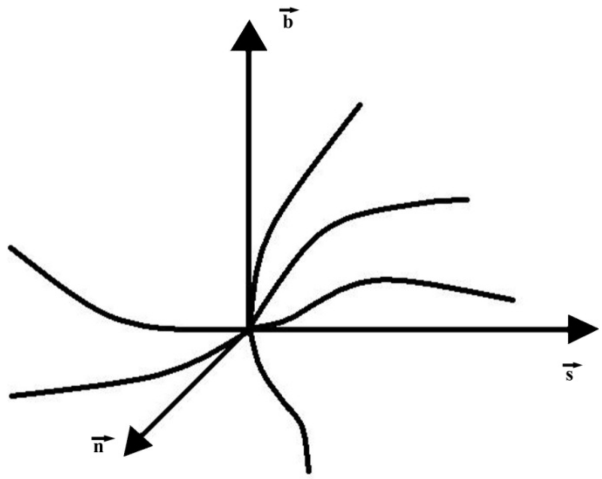

Figure 1: Frenet Frame Field System.

Geometrical relations are given by Frenet formulae [3]

$$\frac{\partial \vec{s}}{\partial s} = k_s \vec{n}, \frac{\partial \vec{n}}{\partial s} = \tau_s \vec{b}, \frac{\partial \vec{b}}{\partial s} = -\tau_s \vec{n},$$

$$\frac{\partial \vec{n}}{\partial n} = k'_n \vec{s}, \frac{\partial \vec{b}}{\partial n} = -\sigma_s' \vec{s}, \frac{\partial \vec{s}}{\partial n} = -\sigma_n' \vec{b} - k_n' \vec{n},$$

$$\frac{\partial \vec{b}}{\partial b} = k_b'' \vec{s}, \frac{\partial \vec{n}}{\partial b} = -\sigma_b'' \vec{s}, \frac{\partial \vec{s}}{\partial b} = \sigma_b'' \vec{n} - k_b'' \vec{b},$$

where $\frac{\partial}{\partial s}, \frac{\partial}{\partial n}$ and $\frac{\partial}{\partial b}$ are the intrinsic differential operators along fluid phase velocity (or dust phase velocity) lines, principal normal and binormal. The functions (k_s, k_n', k_b'') and $(\tau_s, \sigma'_n, \sigma_b'')$are the curvatures and torsions of the above curves and θ_{ns} and $\theta_{bs}$are normal deformations of these spatial curves along their principal normal and binormal respectively.

Formulation and Solution of the Problem

Consider a flow of an unsteady viscous incompressible, dusty fluid between a non-torsional oscillating plate and a long wavy wall as shown in the figure-2. Both the fluid and the dust particle clouds are supposed to be static at the beginning. The number density of the dust particles is taken as a constant throughout the flow and these are assumed to be spherical in shape, uniform in size and electrically non-conducting. The flow is due to magnetic field of uniform strength B_0, non-torsional oscillations of the plate and under the influence of time dependent pressure gradient. Under these assumptions the flow will be a parallel flow in which the streamlines are along the tangential direction and the velocities are varies with binormal direction and time *t*, since we extended the fluid to infinity in the principal normal direction.

For the above described flow the velocities of fluid and dust phase are of the form

$$\vec{u} = u_s\vec{s}, \vec{v} = v_s\vec{s}.$$

where $(u_s, u_n, u_b)$and $(v_s, v_n, v_b)$are velocity components of fluid and dust particles respectively.

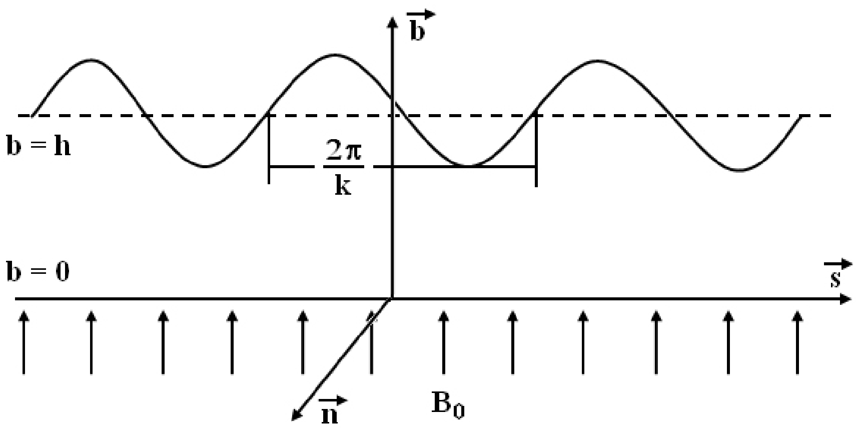

Figure 2: Geometry of the flow.

By virtue of system of equations (3.1) the intrinsic decomposition of equations (2.2) and (2.4) give the following forms

(4.1) $$\frac{\partial u_s}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial s} + \nu\left[\frac{\partial^2 u_s}{\partial b^2} - C_r u_s\right] + \frac{kN}{\rho}(v_s - u_s) - Eu_s,$$

(4.2) $$2u_s^2 k_s = -\frac{1}{\rho}\frac{\partial p}{\partial n} + \nu\left[2\sigma_b''\frac{\partial u_s}{\partial b} - 2k_b''\frac{\partial u_s}{\partial b}\right],$$

(4.3) $$0 = -\frac{1}{\rho}\frac{\partial p}{\partial b} + \nu[u_s k_s \tau_s],$$

(4.4) $$\frac{\partial v_s}{\partial t} = \frac{k}{m}(u_s - v_s),$$

(4.5) $$2v_s^2 k_s = 0,$$

where $E = \frac{\sigma B_0^2}{\rho}$ and $C_r = k_s^2 + \sigma_n^{1^2} + k_n^{1^2} + {\sigma^{11}}_b^2 + {k^{11}}_b^2$ is called curvature number [2].

From equation (4.5) we see that $v_s^2 k_s = 0$ which implies either $v_s = 0$ or $k_s = 0$. The choice $v_s = 0$ is impossible, since if it happens then $u_s = 0$, which shows that the flow doesn't exist. Hence $k_s = 0$, it suggests that the curvature of the streamline along tangential direction is zero. Thus no radial flow exists.

Let us consider the following non-dimensional quantities,

$$u_s^* = \frac{u_s h}{U}, v_s^* = \frac{v_s h}{\nu}, b^* = \frac{b}{h}, t^* = \frac{tU}{h^2}, p^* = \frac{ph^2}{\rho U^2}, s^* = \frac{s}{h},$$

where U is the characteristic velocity and h is the characteristic length.

Using the above non-dimensional quantities we get the non-dimensionalised form of the equations are as follows

(4.6) $$\frac{\partial u_s}{\partial t} = -\frac{\partial p}{\partial s} + \frac{\frac{h}{Re}\partial^2 u_s}{\partial b^2} - \frac{h^3 C_r}{Re}u_s + \frac{kNh^2}{\rho U}(v_s - u_s) - Mu_s,$$

(4.7) $$\frac{\partial v_s}{\partial t} = \frac{kh^2}{mU}(u_s - v_s),$$

where $Re = Uh/\nu$ is the Reynold's number, and $M = \frac{Eh^2}{U}$ is the Hartmann number, $\alpha = \alpha^*/h$ is the non-dimensional amplitude parameter and $\beta = \beta^* h$ is the non-dimensional frequency parameter.

(4.8) $$\begin{cases} \text{Initial condition: at } t = 0,\ u_s = 0, v_s = 0 \\ \text{Boundary condition for } t > 0, u_s = f(t), \text{ at } b = 0 \\ \text{and } u_s = -\alpha\sin(t+\beta), \text{ at } b = 1 - \epsilon\cos t \\ \text{where } \epsilon \text{ is a constant.} \end{cases}$$

We define Laplace transformations of u_s and v_s as

(4.9) $$U_s = \int_0^\infty e^{-xt}\ u_s dt \text{ and } V_s = \int_0^\infty e^{-xt}\ v_s dt,$$

By applying Laplace transform to the equations (4.6) and (4.7) one can obtains

(4.10) $$xU_s = P(x) + \frac{h}{Re}\frac{d^2 U_s}{db^2} = \frac{h^3 C_r}{Re}\ U_s + \frac{h^2 l}{U\tau}(V_s - U_s) - MU_s,$$

(4.11) $$xV_s = \frac{h^2}{U\tau}(U_s - V_s),$$

where $l = \frac{mN}{\rho}$ and $\tau = \frac{m}{k}$. Equation (4.11) implies

(4.12) $$V_s = \frac{h^2}{(h^2+xU\tau)}U_s,$$

Eliminating V_s from (4.10) using (4.12) we obtain the following equation

(4.13) $$\frac{d^2U_s}{db^2} - Q^2U_s = -\frac{Re}{h}P(x),$$

where $Q^2 = h^2C_r + \frac{MRe}{h} + \frac{xRe}{h}(1 + \frac{lh^2}{(xU\tau+h^2)})$.

Case 1: Impulsive Motion

Suppose $-\frac{\partial p}{\partial s} = p_0\delta(t)$, is imposed on the flow and $f(t) = u_0\delta(t)$, where p_0 and u_0 are constant and $\delta(t)$ is the Dirac delta function. Now, solving the equation (4.13) with the boundary conditions (4.8), one can obtain the fluid and dust phase velocities as follows,

$$u_s = \frac{u_0\sinh[(b_2-b)X_1]}{\sinh(b_2X_1)} - \frac{2hu_0\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1} r_1 \sin\left(\frac{r_1\pi[(b_2-b)X_1]}{b_2}\right)$$
$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)^2}{x_3[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)^2}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right]$$
$$-\alpha\left[\frac{k_2\sin t-k_1\cos t}{(C^2+D^2)}\right] + \frac{2h\alpha\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1} r_1 \sin\left(\frac{r_1\pi b}{b_2}\right)$$
$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)^2(\cos\beta+x_3\sin\beta)}{(x_3^2+1)[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)^2(\cos\beta+x_4\sin\beta)}{(x_4^2+1)[(h^2+x_4U\tau)^2+lh^4]}\right]$$
$$-\frac{Rep_0}{hX_1^2}\left[\frac{\sinh(bX_1)+\sinh[(b_2-b)X_1]-\sinh(b_2X_1)}{\sinh(b_2X_1)}\right]$$
$$-\frac{2p_0}{\pi}\sum_{r_1=0}^{\infty}\left[\frac{(-1)^{r_1}-1}{r_1}\right]\sin\left(\frac{r_1\pi b}{b_2}\right)\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)^2}{x_3[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)^2}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$v_s = \frac{\delta(t)u_0\sinh[(b_2-b)X_1]}{\sinh(b_2X_1)} - \frac{2h^3u_0\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1} r_1 \sin\left(\frac{r_1\pi[(b_2-b)X_1]}{b_2}\right)$$
$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)}{x_3[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right]$$
$$-\alpha h^2\left[\frac{L_2\sin t-L_1\cos t}{(C^2+D^2)(h^4+\tau^2U^2)}\right] + \frac{2h^2\alpha\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1} r_1 \sin\left(\frac{r_1\pi b}{b_2}\right)$$
$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)\ (\cos\beta+x_3\sin\beta)}{(x_3^2+1)[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)\ (\cos\beta+x_4\sin\beta)}{(x_4^2+1)[(h^2+x_4U\tau)^2+lh^4]}\right]$$
$$-\frac{Rep_0}{hX_1^2}\left[\frac{\sinh(bX_1)+\sinh[(b_2-b)X_1]-\sinh(b_2X_1)}{\sinh(b_2X_1)}\right]$$

$$-\frac{2p_o h^2}{\pi}\sum_{r_1=0}^{\infty}\left[\frac{(-1)^{r_1}-1}{r_1}\right]\sin\left(\frac{r_1\pi b}{b_2}\right)\left[\frac{e^{x_3t}(h^2+x_3U\tau)}{x_3[(h^2+x_3U\tau)^2+lh^4]}+\frac{e^{x_4t}(h^2+x_4U\tau)}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right]$$

Shearing Stress (Skin Friction): The Shear stress at the boundaries $b=0$ and $b=1-\epsilon\cos t$ are given by

$$D_0=\frac{-u_0X_1\cosh[(b_2X_1)]}{\sinh(b_2X_1)}+\frac{2hu_0\pi^2 r_1}{Reb_2^3}\left[\frac{e^{x_3t}(h^2+x_3U\tau)^2}{x_3[(h^2+x_3U\tau)^2+lh^4]}+\frac{e^{x_4t}(h^2+x_4U\tau)^2}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$-\alpha\left[\frac{k_2^1\sin tk_{1\ 1}^1\cos t}{(C^2+D^2)}\right]+\frac{2h\alpha\pi^2}{Reb_2^3}\Big[\frac{e^{x_3t}(h^2+x_3U\tau)^2(\cos\beta+x_3\sin\beta)}{(x_3^2+1)[(h^2+x_3U\tau)^2+lh^4]}$$

$$+\frac{e^{x_4t}(h^2+x_4U\tau)^2(\cos\beta+x_4\sin\beta)}{(x_4^2+1)[(h^2+x_4U\tau)^2+lh^4]}\Big]-\frac{Rep_0}{hX_1}\left[\frac{1-\cos\text{h}(b_2X_1)}{\sinh(b_2X_1)}\right]$$

$$-\frac{2p_o}{\pi}\sum_{r_1=0}^{\infty}[1-(-1)^{r_1}]\left[\frac{e^{x_3t}(h^2+x_3U\tau)^2}{x_3[(h^2+x_3U\tau)^2+lh^4]}+\frac{e^{x_4t}(h^2+x_4U\tau)^2}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right].$$

$$D_1=\frac{-u_0X_1}{\sinh(b_2X_1)}+\frac{2hu_0\pi^2}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1}r_1\Big[\frac{e^{x_3t}(h^2+x_3U\tau)^2}{x_3[(h^2+x_3U\tau)^2+lh^4]}$$

$$+\frac{e^{x_4t}(h^2+x_4U\tau)^2}{x_4[(h^2+x_4U\tau)^2+lh^4]}\Big]-\alpha\left[\frac{k_2^{11}\sin tk_{1\ 1}^{11}\cos t}{(C^2+D^2)}\right]+\frac{2h\alpha\pi^2 r_1^2}{Reb_2^2}\Big[\frac{e^{x_3t}(h^2+x_3U\tau)^2(\cos\beta+x_3\sin\beta)}{(x_3^2+1)[(h^2+x_3U\tau)^2+lh^4]}$$

$$+\frac{e^{x_4t}(h^2+x_4U\tau)^2(\cos\beta+x_4\sin\beta)}{(x_4^2+1)[(h^2+x_4U\tau)^2+lh^4]}\Big]-\frac{Rep_0}{hX_1}\left[\frac{1-\cos\text{h}(b_2X_1)}{\sinh(b_2X_1)}\right]$$

$$-\frac{2p_o}{\pi}\sum_{r_1=0}^{\infty}[1-(-1)^{r_1}]\left[\frac{e^{x_3t}(h^2+x_3U\tau)^2}{x_3[(h^2+x_3U\tau)^2+lh^4]}+\frac{e^{x_4t}(h^2+x_4U\tau)^2}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right].$$

Case 2: Transition Motion: Suppose $-\frac{\partial p}{\partial s}=p_0H(t)e^{-\lambda_1 t}$, where p_0 and λ_1 are constants and $H(t)$ is the Heaviside unit step function. By Solving the equation (4.13) when subject to the boundary conditions (4.8), with $f(t)=u_0H(t)e^{-\lambda_1 t}$,

For this case one can obtain the expressions for both fluid and dust velocities as

$$u_s=\frac{H(t)u_0e^{-\lambda_1 t}\sinh[(b_2-b)Y]}{\sinh(b_2Y)}-\frac{2hu_0\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1}r_1\sin\left(\frac{r_1\pi b_1}{b_2}\right)$$

$$\times\left[\frac{e^{x_3t}(h^2+x_3U\tau)^2}{(x_3+\lambda_1)[(h^2+x_3U\tau)^2+lh^4]}+\frac{e^{x_4t}(h^2+x_4U\tau)^2}{(x_4+\lambda_1)[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$-\alpha\left[\frac{k_2\sin t-k_1\cos t}{(C^2+D^2)}\right]+\frac{2h\alpha\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1}r_1\sin\left(\frac{r_1\pi b}{b_2}\right)$$

$$\times\left[\frac{e^{x_3t}(h^2+x_3U\tau)^2(\cos\beta+x_3\sin\beta)}{(x_3^2+1)[(h^2+x_3U\tau)^2+lh^4]}+\frac{e^{x_4t}(h^2+x_4U\tau)^2(\cos\beta+x_4\sin\beta)}{(x_4^2+1)[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$-\frac{Rep_0e^{-\lambda_1 t}}{hY}\left[\frac{\sinh(bY)+\sinh[(b_2-b)Y]-\sinh(b_2Y)}{\sinh(b_2Y)}\right]$$

$$-\frac{2p_o}{\pi}\sum_{r_1=0}^{\infty}\left[\frac{(-1)^{r_1}-1}{r_1}\right]\sin\left(\frac{r_1\pi b}{b_2}\right)\left[\frac{e^{x_3t}(h^2+x_3U\tau)^2}{x_3[(h^2+x_3U\tau)^2+lh^4]}+\frac{e^{x_4t}(h^2+x_4U\tau)^2}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right],$$

$$v_s = \frac{u_0\,\mathrm{h}^2\mathrm{e}^{-\lambda_1 \mathrm{t}}\sinh[(b_2-b)Y]}{(\mathrm{h}^2-\lambda_1\tau\mathrm{U})\sinh(b_2Y)} - \frac{2h^3u_0\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1}\, r_1 \sin\left(\frac{r_1\pi b_1}{b_2}\right)$$

$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)}{(x_3+\lambda_1)[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)}{(x_4+\lambda_1)[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$-\alpha h^2\left[\frac{L_2\sin t-L_1\cos t}{(C^2+D^2)(h^4+\tau^2U^2)}\right] + \frac{2h^2\alpha\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1}\, r_1 \sin\left(\frac{r_1\pi b}{b_2}\right)$$

$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)\quad(\cos\beta+x_3\sin\beta)}{\left(x_3^2+1\right)[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)\quad(\cos\beta+x_4\sin\beta)}{\left(x_4^2+1\right)[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$-\ \frac{Rep_0\,he^{-\lambda_1 t}}{(h^2-\lambda_1\tau U)}\left[\frac{\sinh(bY)+\sinh[(b_2-b)Y]-\sinh(b_2Y)}{\sinh(b_2Y)}\right]$$

$$-\frac{2p_oh^2}{\pi}\sum_{r_1=0}^{\infty}\left[\frac{(-1)^{r_1}-1}{r_1}\right]\sin\left(\frac{r_1\pi b}{b_2}\right)\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)}{x_3[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right].$$

Case 3: Periodic Motion for a Finite Time: In this case, we take $-\frac{\partial p}{\partial s} = p_0\sin(\alpha t)$ and $f(t) = u_0\sin(\alpha t)$, where p_0 and u_0 are constants. Now one obtains the velocity profiles for both fluid and dust phase as, we obtain the fluid and dust phase velocities as

$$u_s = \frac{u_0}{C_1^2+D_1^2}[k_3\sin(\alpha t) + k_4\cos(\alpha t)] - \frac{2hu_0\pi\alpha t}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1}\, r_1 \sin\left(\frac{r_1\pi b}{b_2}\right)$$

$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)^2}{(x_3+\alpha^2)[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)^2}{(x_4+\alpha^2)[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$-\alpha\left[\frac{k_2\sin t-k_1\cos t}{(C^2+D^2)}\right] + \frac{2h\alpha\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1}\, r_1 \sin\left(\frac{r_1\pi b}{b_2}\right)$$

$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)^2(\cos\beta+x_3\sin\beta)}{\left(x_3^2+1\right)[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)^2(\cos\beta+x_4\sin\beta)}{\left(x_4^2+1\right)[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$-\ \frac{Rep_0}{hY}\left[\frac{k_7\sin(\alpha t)+k_8\cos(\alpha t)}{\sinh(b_2Y)}\right]$$

$$-\frac{2p_o\alpha}{\pi}\sum_{r_1=0}^{\infty}\left[\frac{(-1)^{r_1}-1}{r_1}\right]\sin\left(\frac{r_1\pi b}{b_2}\right)\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)^2}{x_3[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)^2}{x_4[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$v_s = \frac{u_0h^2}{(C_1^2+D_1^2)(h^4+\alpha^2U^2\tau^2)}[k_9\sin(\alpha t) - k_{10}\cos(\alpha t)]$$

$$-\frac{2h^3\alpha u_0\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1}\, r_1 \sin\left(\frac{r_1\pi b}{b_2}\right)$$

$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)}{(x_3+\alpha)[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)}{(x_4+\alpha)[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$-\alpha\left[\frac{k_2\sin t-k_1\cos t}{(C^2+D^2)}\right] + \frac{2h^2\alpha\pi}{Reb_2^2}\sum_{r_1=0}^{\infty}(-1)^{r_1}\, r_1 \sin\left(\frac{r_1\pi b}{b_2}\right)$$

$$\times\left[\frac{e^{x_3t}\left(h^2+x_3U\tau\right)\quad(\cos\beta+x_3\sin\beta)}{\left(x_3^2+1\right)[(h^2+x_3U\tau)^2+lh^4]} + \frac{e^{x_4t}\left(h^2+x_4U\tau\right)\quad(\cos\beta+x_4\sin\beta)}{\left(x_4^2+1\right)[(h^2+x_4U\tau)^2+lh^4]}\right]$$

$$- \frac{Rep_0}{h}\left[\frac{k_{11}\sin(\alpha t) - k_{12}\cos(\alpha t)}{(C_1^2 + D_1^2)(y_3^2 + z_3^2)(h^4 + \alpha^2\tau^2U^2)}\right]$$

$$- \frac{2p_0\alpha}{\pi}\sum_{r_1=0}^{\infty}\left[\frac{(-1)^{r_1} - 1}{r_1}\right]\sin\left(\frac{r_1\pi b}{b_2}\right)\left[\frac{e^{x_3t}(h^2 + x_3U\tau)}{(x_3 + \alpha^2)[(h^2 + x_3U\tau)^2 + lh^4]} + \frac{e^{x_4t}(h^2 + x_4U\tau)}{(x_4 + \alpha^2)[(h^2 + x_4U\tau)^2 + lh^4]}\right].$$

where

$$M = \frac{Eh^2}{U},\ X_1 = \sqrt{h^2C_r + \frac{MRe}{h}}, b2 = 1 - \epsilon\cos t\,, a_1 = Reb_2^2\tau U,$$

$$a_2 = h^3C_r\tau U\, b_2^2 + MReb_2^2\tau U + Re\, b_2^2h^2 + Relh^2b_2^2 + r_1^2\pi^2h\tau U,$$

$$a_3 = h^5C_rb_2^2 + MReb_2^2h^2 + r_1^2\pi^2h^3, x_3 = \frac{-a_2 + \sqrt{a_2^2 - 4a_1a_3}}{2a_1},$$

$$x_4 = \frac{-a_2 - \sqrt{a_2^2 - 4a_1a_3}}{2a_1}, y_1 = h^2C_r + \frac{MRe}{h} + \frac{Relh\tau U}{(h^2 + \tau^2U^2)},$$

$$z_1 = \frac{Re}{h} + \frac{Relh\tau U}{(h^2 + \tau^2U^2)},\ y_2 = \sqrt{\frac{y_1 + \sqrt{y_1^2 + z_1^2}}{2}}, z\_2 = \sqrt{\frac{y_1 + \sqrt{y_1^2 + z_1^2}}{2}},$$

$$A = \sinh(y_2b)\cos(z_2b)\,, B = \cosh(y_2b)\sin(z_2b)\,, C = \sinh(y_2b_2)\cos(z_2b_2)$$

$$D = \cosh\,(y_2b_2)\sin(z_2b_2),\ k_1 = -\cos\beta(BC - AD) + \sin\beta(AC + BD),$$

$$k_2 = \cos\beta(AC + BD) - \sin\beta(BC - AD)\,, L_1 = k_1h^2 + k_2\tau U, L_2 = k_{12}h^2 - k_2\tau U,$$

$$Y = \sqrt{h^2C_r + \frac{MRe}{h} - \frac{\lambda Re}{h}\left(1 + \frac{lh^2}{(h^2 - \lambda\tau U)}\right)}, y_3 = \sqrt{h^2C_r + \frac{MRe}{h} + \frac{\alpha RelhU\tau}{(h^2 + \alpha^2\tau^2U^2)}},$$

$$z_3 = \frac{Re\alpha}{h} + \frac{\alpha Reh^3}{(h^2 + \alpha^2\tau^2U^2)}, y_4 = \sqrt{\frac{y_3^2 + \sqrt{y_3^2 + z_3^2}}{2}}, z_4 = \sqrt{\frac{-y_3^2 + \sqrt{y_3^2 + z_3^2}}{2}},$$

$$A_1 = \sinh(y_4b_1)\cos(z_4b_1)\,, B_1 = \cosh(y_4b_1)\sin(z_4b_1)\,, A_2 = \sinh(y_4b)\cos(z_4b),$$

$B_2 = \cosh(y_4b)\sin(z_4b)$, $C_1 = \sinh(y_4b_2)\cos(z_4b_2)\,, D_1 =$
$\cosh(y_4b\_2)\,\mathrm{sins}(z_4b_2)$,

$$k_5 = A_2 + A_1 - C_1, k_6 = B_2 + B_1 - D_1, k_7 = y_3(k_5C_1 + k_6D_1) + z_3(k_6C_1 - k_5D_1),$$

$k_8 = y_3(k_6C_1 - k_5D_1) - z_3(k_5C_1 + k_6D_1), k_9 = k_3h^2 - k_4\alpha\tau U, k_{10} = k_4h^2 +$
$k_3\alpha\tau U$

$$k_{11} = k_7h^2 - k_8\alpha\tau U, k_{12} = k_8h^2 + k_7\alpha\tau U,$$

$$k_1^1 = \cos(\beta)\,(Cz_2 - Dy_2) - \sin(\beta)\,(Cy_2 + Dz_2),$$

$$k_2^1 = \cos(\beta)\,(Cy_2 + Dz_2) - \sin(\beta)\,(Cz_2 + Dy_2,$$

$k_1^{11} =$
$\cos(\beta)\,[(Cz_2 - Dy_2)\cosh(y_2 z_2 b_2)\cos(z_2 b_2) + (Cy_2 - Dz_2)\sinh(y_2 b_2)\sin(z_2 b_2)]$

$-\sin(\beta)\,[(Cy_2 + Dz_2)\cosh(y_2 z_2 b_2)\cos(z_2 b_2) -$
$(Cz_2 - Dy_2)\sinh(y_2 b_2)\sin(z_2 b_2)]$

$k_2^{11} =$
$\cos(\beta)\,[(Cy_2 + Dz_2)\cosh(y_2 z_2 b_2)\cos(z_2 b_2) - (Cz_2 - Dy_2)\sinh(y_2 b_2)\sin(z_2 b_2)]$

$-\sin(\beta)\,[(Cz_2 - Dy_2)\cosh(y_2 z_2 b_2)\cos(z_2 b_2) +$
$(Cy_2 + Dz_2)\sinh(y_2 b_2)\sin(z_2 b_2)]$

Conclusion

One can observe the parabolic in nature of velocity profiles for the fluid and dust particles plotted as in graphs 3 to 8. It is observed that velocity of fluid particles is parallel to velocity of dust particles. It is evident from the graphs 3-5 that, as we increase the strength of the magnetic field, it has an appreciable effect on the velocities of fluid and dust particles. i.e., the magnetic effect has retarding influence. If we increase the number density of the dust particles it effect on the flow, i.e it decreases the velocities of both fluid and dust phase. Further one can can observe that if the dust is very fine i.e., mass of the dust particles is negligibly small then the relaxation time of dust particle decreases and ultimately as $\tau \to 0$ the velocities of fluid and dust particles will be the same.

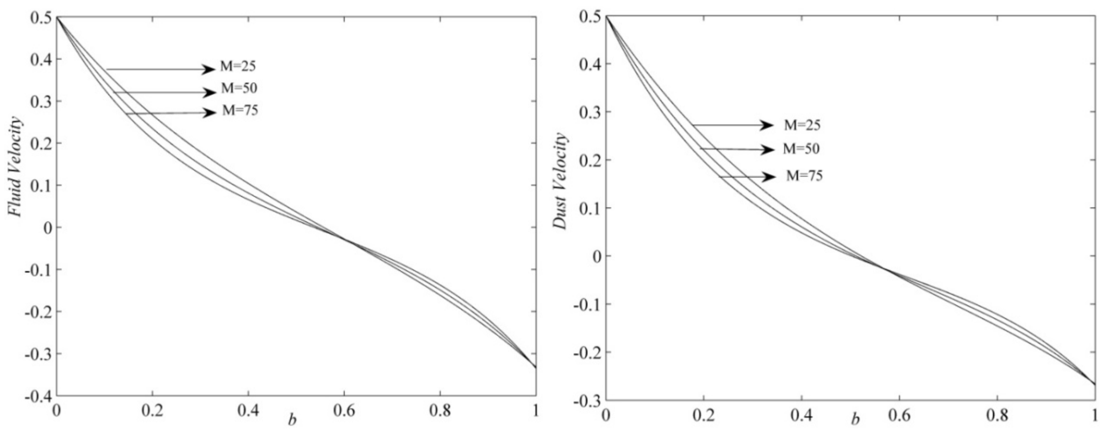

Figure 3: Variation of fluid and phase velocity with b (case-1).

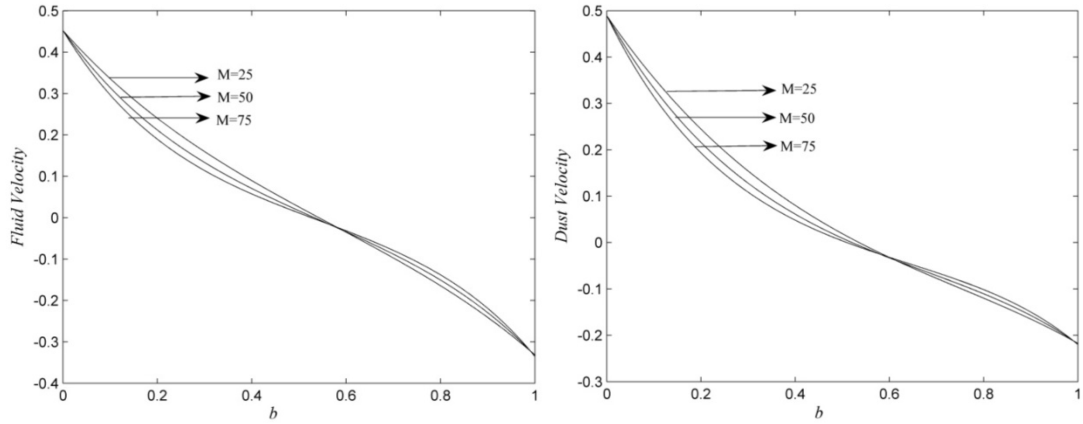

Figure 4: Variation of fluid and phase velocity with b (case-2).

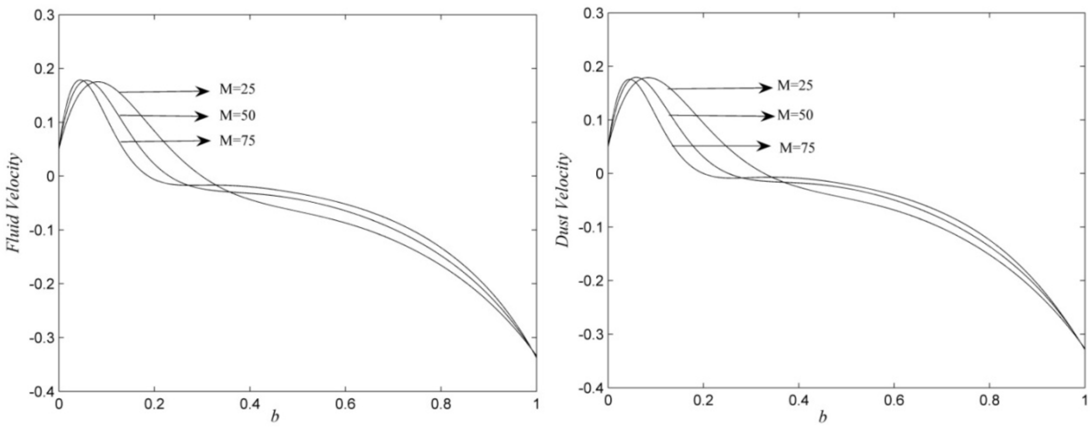

Figure 5: Variation of fluid and phase velocity with b (case-3).

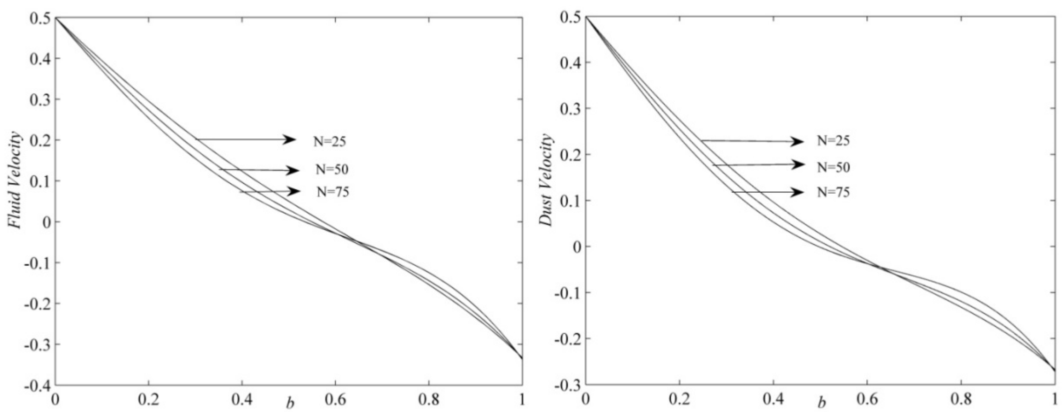

Figure 6: Variation of fluid and phase velocity with b (case-1).

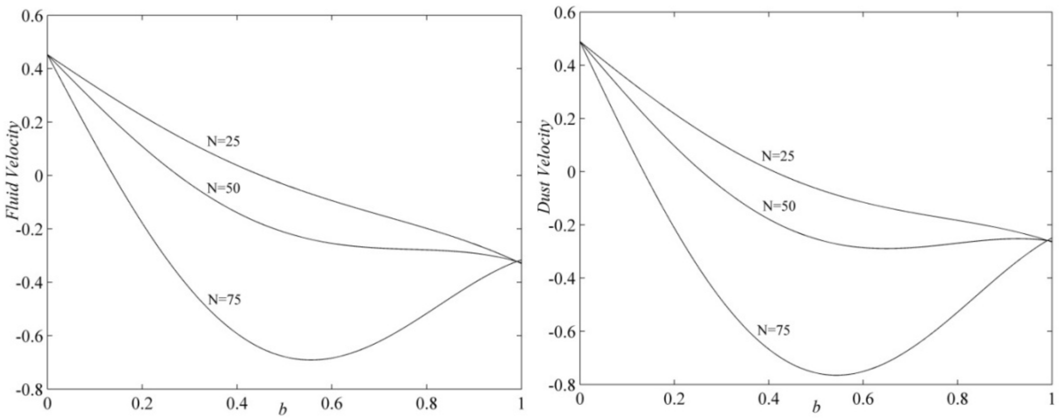

Figure 7: Variation of fluid and phase velocity with b (case-2).

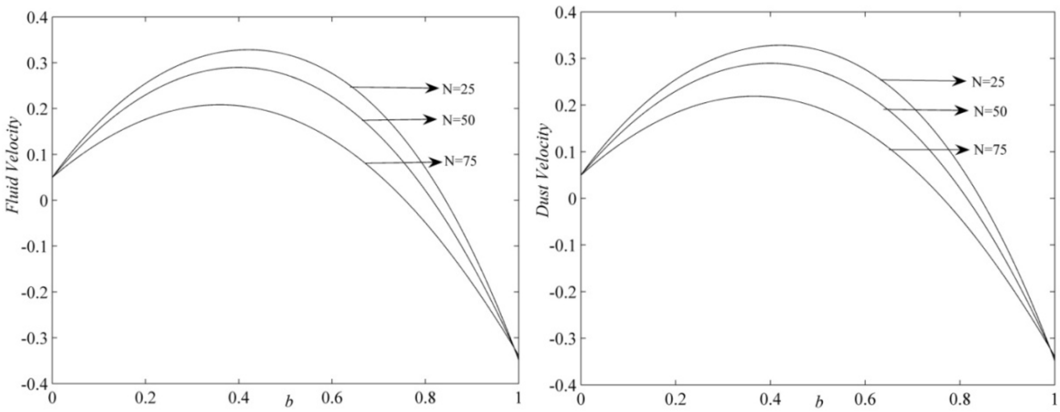

Figure 8: Variation of fluid and phase velocity with b (case-3).

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