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Heat and Mass Transfer in a Visco-Elastic MHD Flow Past a Vertical Plate Under Oscillatory Suction Velocity

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Abstract

The unsteady hydromagnetic flow of an electrically conducting visco-elastic fluid past an infinite vertical porous plate in a porous medium of time dependent permeability under oscillatory suction velocity normal to the plate has been investigated. The flow with heat and mass transfer is characterized by the second-order fluid model. It is considered that the uniform magnetic field acts normal to the flow and the permeability of the porous medium fluctuates with time. The perturbation technique has been used to solve the problem. The profiles of velocity and skin friction have been presented graphically for different values of parameters involved in the solution to observe the effects of the visco-elastic parameter.

Keywords: Heat Transfer; Mass Transfer; Visco-elastic; Porous Plate; Suction Velocity.

Introduction

The flow past an infinite porous plate and the phenomenon of heat and mass transfer have been of great interest due to its applications in industries. Flows which arise in fluids due to the unsteady motion of a boundary, boundary temperature, density differences caused by the diffusion of thermal energy etc. have many applications in geophysics, chemical engineering, turbo-machinery and aerospace technology. Some important contributions where the transfer of heat and mass take place simultaneously as a result of buoyancy reduced motions have been given by several authors.

The investigations of the problems of free convective flow of a viscous fluid through a porous medium with heat and mass transfer have been studied by authors 1-11. The effects of permeability which varies with time, of free convective flow past a vertical porous wall have been investigated by Shreekanth et al[12]. Singh et al [13] have discussed hydromagnetic free convective and mass transfer flow of such fluid considering permeability variation with direction. Acharya et al[14] have extended the study in steady flow with constant suction in the presence of magnetic field. Singh et al[15] have studied the effects of permeability variation and oscillatory suction velocity in presence of time dependent viscosity along with the uniform magnetic field.

The objective of the present paper is to study the unsteady hydromagnetic flow of a visco-elastic fluid past an infinite vertical plate in a porous medium of time dependent permeability under oscillatory suction velocity normal to the plate with heat and mass transfer.

The constitutive equation for second-order fluid [Coleman & Noll (1960)] is

$$\sigma_{ij} = -p\delta_{ij} + \mu_1 A_{(1)ij} + \mu_2 A_{(2)ij} + \mu_3 A_{(1)ik} A_{(1)kj} \tag{1.1}$$

where σ_{ij} are the stress tensors, p the hydrostatic pressure, $A_{(i)}$ are kinematic Rivlin-Erickson tensors; μ_1, μ_2, μ_3 are material constants describing viscosity, elasticity and cross-viscosity where $\mu_2 < 0$ from thermodynamic consideration [Coleman and Markovitz (1964)]. Equation (1.1) is valid for low rates of shear.

Mathematical Analysis

We introduce a co-ordinate system where $\bar{x}$ -axis is taken along the infinite vertical plate in the direction of flow and $\bar{y}$ -axis normal to it. The permeability of the porous medium is considered to be of the form $K(t) = K_0(1+\varepsilon e^{i\bar{n}\bar{t}})$ and the suction velocity is assumed to be $v(t) = -v_0(1+\varepsilon e^{i\bar{n}\bar{t}})$, where $\varepsilon << 1$ being the amplitude of the permeability variation, is a positive constant, $v_0 > 0$ is a constant and negative sign indicates that the suction is towards the plate, and K_0 the mean permeability of the medium. All the fluid properties are assumed to be constant except that the influence of the density variation with temperature. Let $\bar{u}$ be the component of velocity in the $\bar{x}$ -direction.

The boundary conditions relevant to the problem are

$$\begin{aligned} &\bar{y} = 0 : \bar{u} = 0, \bar{T} = \bar{T}_w, \bar{C} = \bar{C}_w \\ &\bar{y} \to \infty : \bar{u} \to 0, \bar{T} = \bar{T}_\infty, \bar{C} = \bar{C}_\infty \end{aligned} \tag{2.1}$$

where $\bar{T}_w, \bar{T}_\infty$ and $\bar{C}_w, \bar{C}_\infty$ are respectively the temperature and the molar concentration of the fluid at the plate and far away from the plate.

Let us introduce the following non-dimensional quantities

$$y = \frac{\bar{y}v_0}{\upsilon_1}, t = \frac{v_0{}^2\bar{t}}{4\upsilon_1}, u = \frac{\bar{u}}{v_0}, n = \frac{4\upsilon_1\bar{n}}{v_0^2}, T = \frac{\bar{T} - T_\infty}{T_w - T_\infty}, C = \frac{\bar{C} - C_\infty}{C_w - C_\infty} \tag{2.2}$$

Introducing the above non-dimensional variables in the governing equations for velocity, temperature and molar concentration; neglecting the induced magnetic field of strength B_0 and taking the usual Boussinesq's approximations, we obtain the following non-dimensional equations of the fluid motion:

$$\frac{1}{4}\frac{\partial u}{\partial t} - (1+\varepsilon e^{\text{int}})\frac{\partial u}{\partial y} = G_r T + G_m C + \frac{\partial^2 u}{\partial y^2} + \alpha[\frac{1}{4}\frac{\partial^2}{\partial y^2}(\frac{\partial u}{\partial t}) - (1+\varepsilon e^{\text{int}})\frac{\partial^3 u}{\partial y^3}] - \frac{u}{K_0(1+\varepsilon e^{\text{int}})} - M^2 u \tag{2.3}$$

$$\frac{1}{4}\frac{\partial T}{\partial t} - (1+\varepsilon e^{\text{int}})\frac{\partial T}{\partial y} = \frac{1}{P_r}\frac{\partial^2 T}{\partial y^2} \tag{2.4}$$

$$\frac{1}{4}\frac{\partial C}{\partial t} - (1+\varepsilon e^{\text{int}})\frac{\partial C}{\partial y} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} \tag{2.5}$$

subject to boundary conditions

$$\begin{aligned} &\mathrm{y} = 0 : \mathrm{u} = 0, \mathrm{T} = 1 + \varepsilon \mathrm{e}^{int}, \mathrm{C} = 1 + \varepsilon \mathrm{e}^{int} \\ &y \to \infty : u \to 0, T \to 0, C \to 0. \end{aligned} \tag{2.6}$$

where

$S_c = \dfrac{\upsilon_1}{D}$, Schmidt number

$P_r = \dfrac{\upsilon_1}{\kappa_0 / \rho C_p}$, Prandtl number

$G_r = \dfrac{\upsilon_1 g\beta(T_w - T_\infty)}{v_0{}^3}$, Grashof number for heat transfer

$G_m = \dfrac{\upsilon_1 g\bar{\beta}(C_w - C_\infty)}{v_0{}^3}$, Grashof number for mass transfer

$M = \sqrt{\dfrac{\sigma\upsilon_1}{\rho}}\dfrac{B_0}{v_0}$, Magnetic parameter

$\alpha = \dfrac{\upsilon_2 v_0}{\upsilon_1}$, visco-elastic parameter

and $\beta, \bar{\beta}$ are the co-efficient of volume expansion for heat and mass transfer respectively, *T* the fluid temperature, *C* the molar concentration, *D* the chemical molar diffusivity, C_p the specific heat at constant pressure, n the frequency of oscillation, t the time, ρ the density of the fluid, υ_1, υ_2 the kinematic co-efficient of viscosity and elasticity, σ the electric conductivity and g the acceleration due to gravity.

Method of Solution

To solve the equations (2.3) to (2.5) subject to boundary conditions (2.6), we assume the solutions for $\varepsilon << 1$ as follows:

$$\begin{aligned} u(y,t) &= u_0(y) + \varepsilon u_1(y)e^{\text{int}}, \\ T(y,t) &= T_0(y) + \varepsilon T_1(y)e^{\text{int}} \\ C(y,t) &= C_0(y) + \varepsilon C_1(y)e^{\text{int}} \end{aligned} \tag{3.1}$$

Substituting (3.1) into equations (2.3) to (2.5) and equating the harmonic and non-harmonic terms, we get

$$\alpha u_0''' - u_0'' - u_0' + a_1 u_0 = G_r T_0 + G_m C_0 \tag{3.2}$$

$$\alpha u_1''' - (1 + \alpha \frac{in}{4}) u_1'' - u_1' + a_2 u_1 = G_r T_1 + G_m C_1 - \alpha u_0''' + \alpha u_0' + \frac{u_0}{K_0} \tag{3.3}$$

$$T_0'' + P_r T_0' = 0 \tag{3.4}$$

$$T_1'' + P_r T_1' - \frac{in}{4} P_r T_1 = -P_r T_0' \tag{3.5}$$

$$C_0'' + S_c C_0' = 0 \tag{3.6}$$

$$C_1'' + S_c C_1' - \frac{in}{4} S_c C_1 = -S_c C_0' \tag{3.7}$$

where the primes denote differentiation with respect to y.

The corresponding boundary conditions are

$$\begin{aligned} &\mathrm{y} = 0 : \mathrm{u}_0 = 0 = \mathrm{u}_1, \mathrm{T}_0 = 1 = \mathrm{T}_1, \mathrm{C}_0 = 1 = C_1 \\ &y \to \infty : u_0, u_1, T_0, T_1, C_0, C_1 \to 0 \end{aligned} \tag{3.8}$$

Substituting the solutions of equations (3.2) to (3.7) under the boundary conditions (3.8), we obtain

$$T(y,t) = e^{-P_r y} + \varepsilon[(1 - i\frac{4P_r}{n})e^{-m_1 y} + i\frac{4P_r}{n} e^{-P_r y}]e^{\text{int}} \tag{3.9}$$

$$C(y,t) = e^{-S_c y} + \varepsilon[(1 - i\frac{4S_c}{n})e^{-m_2 y} + i\frac{4S_c}{n} e^{-S_c y}]e^{\text{int}} \tag{3.10}$$

where

$$m_1 = \frac{1}{2}[P_r + \sqrt{P_r^2 + inP_r}], m_2 = \frac{1}{2}[S_c + \sqrt{S_c^2 + inS_c}]$$

and

$$
\begin{aligned}
u = {} & [(a_3+a_4)e^{-m_3 y} - a_3 e^{-P_r y} - a_4 e^{-S_c y} + \alpha\{(A_1 - A_2)e^{-m_3 y} + A_3 e^{-P_r y} + A_4 e^{-S_c y}\}] \\
& +\varepsilon(\cos nt + i\sin nt)[\{B_1 e^{-m_4 y} - G_r(B_2 e^{-m_1 y} + B_3 e^{-P_r y}) \\
& -G_m(B_4 e^{-m_2 y} + B_5 e^{-S_c y}) + B_6 e^{-m_3 y} - B_7 P_r e^{-P_r y} - B_8 S_c e^{-S_c y}\} \\
& +\alpha\{(C_1 + C_2)e^{-m_4 y} + C_3 e^{-m_1 y} + C_4 e^{-m_2 y} + C_5 e^{-m_3 y} + C_6 e^{-P_r y} \\
& +C_7 e^{S_c y}\}]
\end{aligned} \tag{3.11}
$$

where

$$a_1 = M^2 + \frac{1}{K_0}, a_2 = a_1 + \frac{in}{4}, 2m_3 = 1 + \sqrt{1+4a_1}, 2m_4 = 1+\sqrt{1+4a_2} \text{ and}$$

A_1, A_2,............. B_1, B_2,........ C_1, C_2,........are constants which are determined, but not presented here for the sake of brevity.

Separating real and imaginary parts and taking only the real part, we obtain the velocity, temperature and concentration fields in terms of fluctuating parts in the form

$$u(y,t) = u_0(y) + \varepsilon(M_r \cos nt - M_i \sin nt)$$
$$T(y,t) = T_0(y) + \varepsilon(T_r \cos nt - T_i \sin nt)$$
$$C(y,t) = C_0(y) + \varepsilon(C_r \cos nt - C_i \sin nt) \quad .$$

Hence expressions for transient velocity, temperature and concentration field for $nt = \pi$ are

$$u(y, \frac{\pi}{n}) = u_0(y) - \varepsilon M_r, T(y, \frac{\pi}{n}) = T_0(y) - \varepsilon T_r, C(y, \frac{\pi}{n}) = C_0(y) - \varepsilon C_r,$$

Skin Friction, Rate of Heat and Mass Transfer

The non-dimensional skin friction τ_w at the plate (*y=0*) is given by

$$\tau_\omega = \frac{d^2\sigma_{xy}}{\rho v^2} = [u_0'(0) + \alpha u_0''(0)] + \varepsilon e^{\text{int}}[u_1'(0) + \alpha u_1''(0)] \tag{4.1}$$

The dimensionless heat transfer co-efficient at the plate is given by

$$N_u = -\left(\frac{\partial T}{\partial y}\right)_{y=0} = -\left[T_0'(0) + \varepsilon e^{\text{int}} T_1'(0)\right] = P_r + \varepsilon|R|\cos(nt+\beta)$$

where R and β are the amplitude and the rate of heat transfer.

Similarly, the mass transfer co-efficient S_h at the plate is given by

$$S_h = -\left(\frac{\partial C}{\partial y}\right)_{y=0} = -\left[C_0'(0) + \varepsilon e^{\text{int}} C_1'(0)\right] = S_c + \varepsilon|Q|\cos(nt+\gamma)$$

where Q and γ are the amplitude and the rate of mass transfer.

Results and Discussion

The aim of the problem is to bring out the effects of the visco-elastic parameter on the flow characteristics. The visco-elastic effect is exhibited through the non-dimensional parameter α. The corresponding results for Newtonian fluid can be deduced from the above results by setting $\alpha = 0$ and it is worth mentioning here that the results coincide with that of Singh et al.

Figure 1-10 reveal the transient velocity *u* and the skin friction τ_w against *y* for different values of magnetic parameter *M*, Grashof number for heat transfer G_r, Grashof number of mass transfer G_m and visco-elastic parameter α with consideration of Prandtl number $P_r = 5$, permeability parameter $K_0 = 10$, frequency parameter *n=5*, Schmidt number $S_c = 5$, perturbation parameter $\varepsilon = 0.005$ and $nt = \pi$. Two cases in general interest for Grashof number $G_r > 0$ corresponding to cooling of the plate and Grashof number $G_r < 0$ corresponding to heating of the plate are considered.

The figures reveal that due to cooling of the plate $(G_r > 0)$, the transient velocity *u* first increase and then decrease (Fig.1, Fig.2, Fig.3) in both Newtonian and non-Newtonian cases but the opposite pattern is observed (Fig.6, Fig.7, Fig.8) due to heating of the plate $(G_r < 0)$. Again, Fig.1 depicts that *u* increase with increasing values of the visco-elastic parameter $|\alpha|$ as compared to their corresponding values for Newtonian fluid, but decrease when the value of the magnetic parameter *M* increase with $|\alpha|$ (Fig.2, fig.3) due to cooling of the plate but the reverse behavior is observed due to heating of the plate (Fig.6, Fig.7, Fig.8).

Figures 4, 5 and Figures 9, 10 demonstrate the variations of the skin friction τ_w against *y*. From the figures, it is seen that the values of τ_w decrease for $G_r > 0$ and increase for $G_r < 0$ in both Newtonian and non-Newtonian cases. Again, the profiles reveal that τ_w increase with the increasing values of $|\alpha|$ in comparison with their corresponding values for Newtonian fluid and the magnetic parameter *M* due to cooling of the plate (Fig.4, Fig.5) but decrease due to heating of the plate (Fig.9, Fig.10) with the combination of other flow parameters.

It is noted that the dimensionless heat transfer co-efficient N_u and the mass transfer co-efficient S_h are not affected by the visco-elastic parameter.

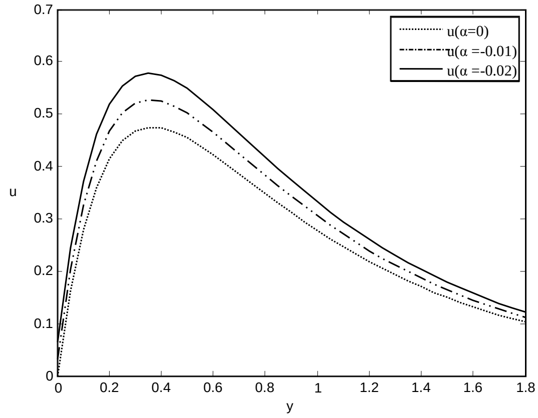

Figure 1: Transient velocity u against y for M=0.5, K^0=10, n=5.0, Gr=10, Gm=10, ε=0.005, Pr=5, Sc=5, nt=π.

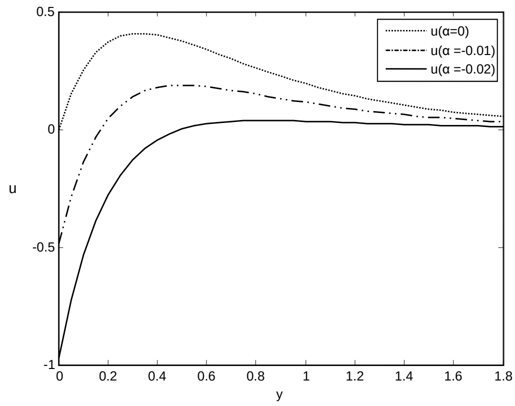

Figure 2: Transient velocity u against y for M=1.0, K^0=10, n=5.0, Gr=10, Gm=10, ε=0.005, Pr=5, Sc=5, nt=π.

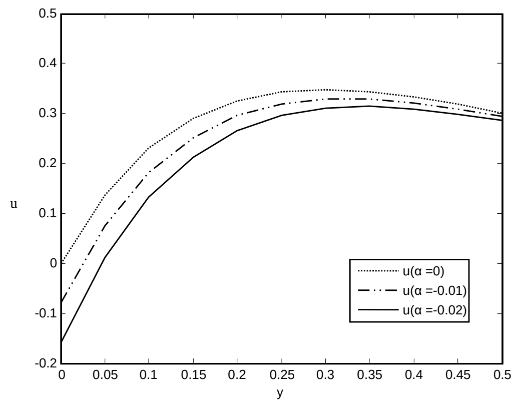

Figure 3: Transient velocity u against y for M=1.5, K_0=10, n=5.0, Gr=10, Gm=10, ε=0.005, Pr=5, Sc=5, nt=π.

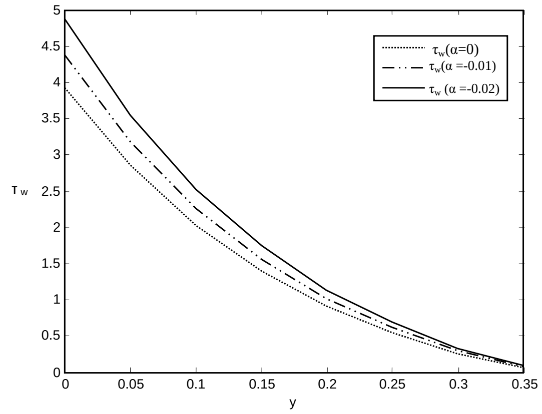

Figure 4: Skin Friction T_w against y for M=0.5, K_0=10, n=5.0, Gr=10, Gm=10, ε=0.005, Pr=5, Sc=5, nt=π.

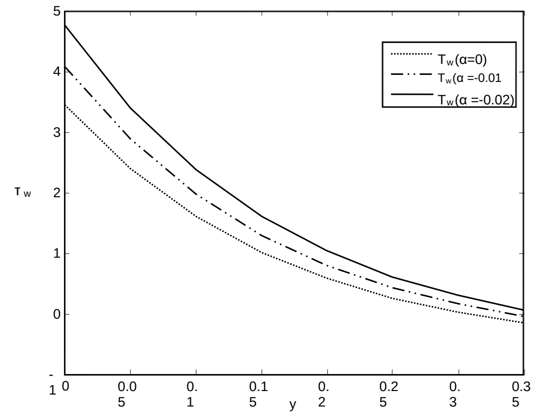

Figure 5: Skin Friction T_w against y for M=0.5, K_0=10, n=5.0, Gr=10, Gm=10, ε=0.005, Pr=5, Sc=5, nt=π.

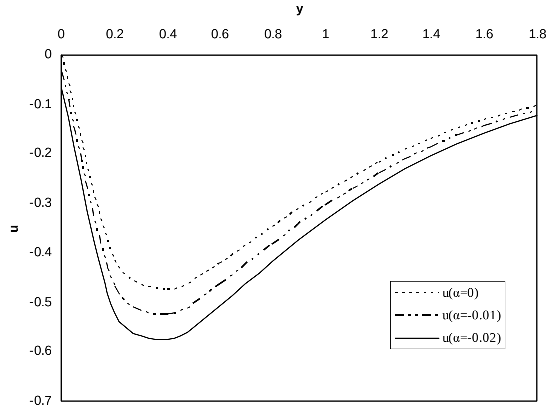

Figure 6: Transient velocity u against y for M=0.5, K_0=10, n=5.0, Gr=10, G^m=10, ε=0.005, P^r=5, Sc=5, nt=π.

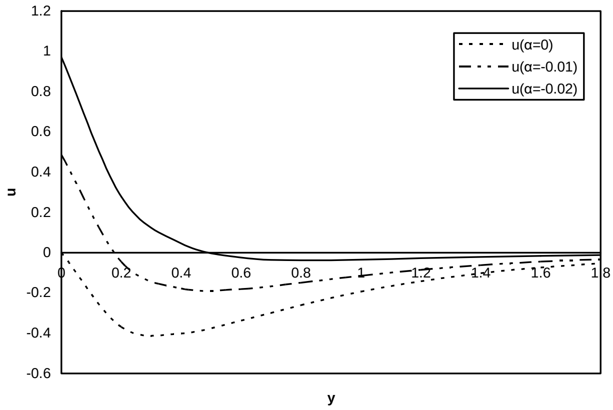

Figure 7: Transient velocity u against y for M=1.0, K_0=10, n=5.0, G^r=10, G^m=10, ε=0.005, Pr=5, Sc=5, nt=π.

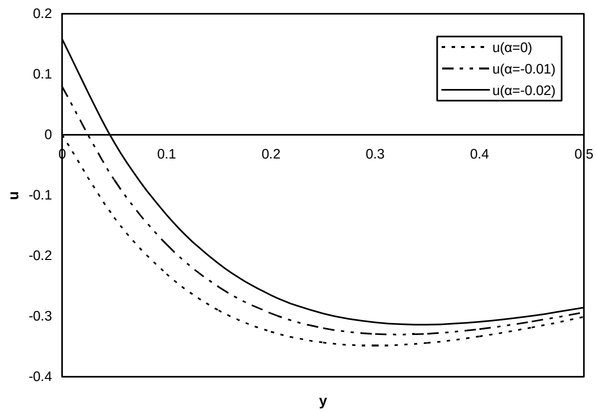

Figure 8: Transient velocity u against y for M=1.5, K_0=10, n=5.0, G^r=10, G^m=10, ε=0.005, Pr=5, Sc=5, nt=π.

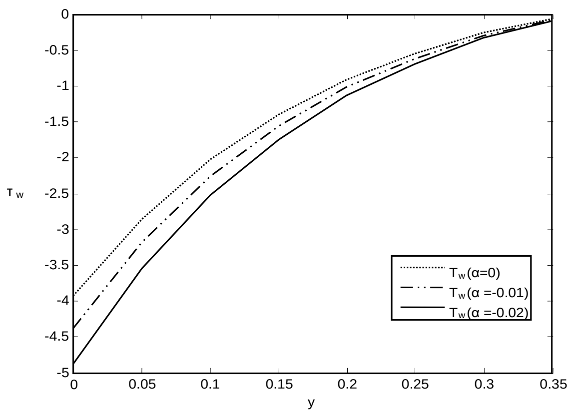

Figure 10: Skin Friction T_w against y for M=0.5, K_0=10, n=5.0, G^r=10, G^m=10, ε=0.005, Pr=5, Sc=5, nt=π.

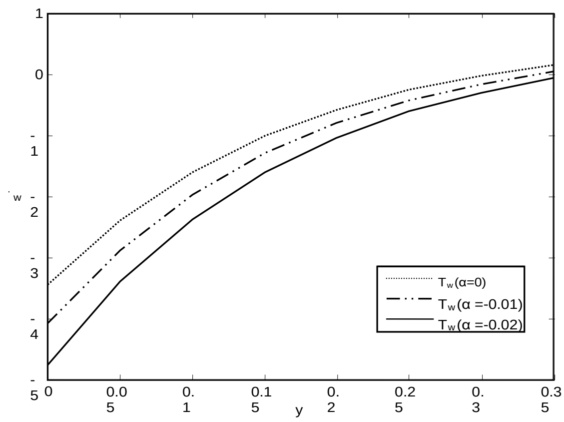

Figure 10: Skin Friction T_w against y for M=1.5, K_0=10, n=5.0, G^r=10, G^m=10, ε=0.005, Pr=5, Sc=5, nt=π.

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