International Journal of Computational Science and Mathematics.
ISSN 0974-3189 Volume 2, Number 3 (2010), pp. 161--166

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Numerical Evaluation of European Option on a Non Dividend Paying Stock

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Abstract

In this paper an attempt has been made to numerically evaluate the value of European option employing Black-scholes formula. The important parameters like time, volatility, strike price, interest rate and spot price have been incorporated in the model. Appropriate boundary and initial conditions have been framed using financial conditions of the stock market. The numerical methods especially the implicit and explicit schemes have been employed to solve Black-Scholes PDE. The results obtained are used to study the relationships among various parameters.

Keywords: European option, non-dividend, strike price, volatility.

Introduction
Black and Scholes [9] proposed an explicit formula for evaluating European call options without dividends, which is still extensively used with underlying. The numerical solution of this equation has been of paramount interest due to the governing partial differential equation, which is very difficult to generate stable and accurate solutions [9,14]. A European call option gives its holder the right (but not the obligation) to purchase from the writer a prescribed asset for a prescribed price at a prescribed time in the future. The prescribed purchase price is known as the exercise price or strike price (E), and the prescribed time in the future is known as the expiry date (T). The direct opposite of a European call option is a European put option [13, 4, 5]. A European put option gives its holder the right (but not the obligation) to sell to the writer a prescribed asset for a prescribed price at a prescribed time in the future. Therefore, the value of the option at expiry, known as the pay-off function, is $V(S, T) = (S-E)^{+}$.

Numerical method of long standing for general parabolic free boundary problem is the time discrete. If differential equation is used to model the value of the option then the resulting problem no longer is formulated for a constant coefficient scalar (heat) equation but for a one dimensional non linear parabolic equation[12,10,2].

Finite difference theory has a long history and has been applied for more than 200 years to approximate the solutions of partial differential equations in the physical sciences and engineering. What is the relationship between FDM and financial mathematics? To answer this question we note that the behavior of a stock (or some other underlying) can be described by a stochastic differential equation [6 ,7]. Then, a contingent claim that depends on the underlying is modeled by a partial differential equation in combination with some initial and boundary conditions. Solving this problem means that we have found the value for the contingent claim [3].

Problem

In this paper we consider how it might be used to value an European option on a non-dividend paying stock. The Black-Scholes partial differential equation for European option on non-dividend paying stock is.

$$\frac{\partial V}{\partial t} + rS\frac{\partial V}{\partial S} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = rV \qquad (1)$$

Suppose that the life of the option is T. we divide this into N equally spaced intervals of length $\delta t = \frac{T}{N}$. A total of N+1 times are considered. i.e. $0,\ \delta t,\ 2\delta t, 3\delta t, ... T$ .Similarly $S_{\max}$ is a stock price of the option. We define $\delta S = \frac{S_{\max}}{M}$ and consider a total of M+ 1 equally spaced stock price of the options. $0, \delta S, 2\delta S, 3\delta S, ... S_{\max}$.

The (i, j) points on the grid is the point that corresponding to $i\delta t$ and stock price $j\delta S$. We will use the variable $V_{i,j}$ to denote the value of the option at the (i, j) point.

The following two methods are employed for solving the fundamental PDE numerically.

The implicit Finite Difference method (The backward Euler method)

The explicit Finite Difference method (The Euler method)

Implicit Finite Difference Method: -

For the interior point (i, j) on the grid $\frac{\partial V}{\partial S}$ can be approximated as

$$\frac{\partial V}{\partial S} = \frac{V_{i,j+1} - V_{i,j}}{\delta S} \qquad (2)$$

OR

$$\frac{\partial V}{\partial S} = \frac{V_{i,j} - V_{i,j-1}}{\delta S} \qquad (3)$$

Equation (2) is known as the forward difference approximation and equation (3) is known as the backward difference approximation. We use more symmetrical approximation by averaging the two.

$$\frac{\partial V}{\partial S} = \frac{V_{i\,j+1} - V_{i\,j-1}}{2\delta S} \tag{4}$$

For $\frac{\partial V}{\partial t}$, we will use a forward difference approximation, so that the value at time $i\delta t$ is related to the value at time (i+1) δt.

$$\frac{\partial V}{\partial t} = \frac{V_{i+1,j} - V_{i,j}}{\delta t} \tag{5}$$

The backward difference approximation for $\frac{\partial V}{\partial S}$ at the (i, j) point is given by equation (2), the backward difference at the (i,j+1) point is

$$\frac{V_{i,j+1} - V_{i,j}}{\delta S}$$

Hence a finite difference approximation for $\frac{\partial^2 V}{\partial S^2}$ at the (i, j) point is

$$\frac{\partial^2 V}{\partial S^2} = \left(\frac{V_{i,j+1} - V_{i,j}}{\delta S} - \frac{V_{i,j} - V_{i,j-1}}{\delta S}\right) \Big/ \delta S$$

$$\frac{\partial^2 V}{\partial S^2} = \frac{V_{i,j+1} + V_{i,j-1} - 2V_{i,j}}{\delta S^2} \tag{6}$$

Substituting equations (4), (5), and (6) into the differential equation (1) and S=j δS gives

$$\frac{V_{i+1,j} - V_{i,j}}{\delta t} + rj\delta S \frac{V_{i,j+1} - V_{i,j-1}}{2\delta S} + \frac{1}{2}\sigma^2 j^2 \delta S^2 \frac{V_{i,j+1} + V_{i,j-1} - 2V_{i,j}}{\delta S^2} = rV_{i,j}$$

For j=1,2,3…….., M-1 and i=0,1,2,3,…………,N-1,rearranging terms, we obtain

$$a_j V_{i,j-1} + b_j V_{i,j} + c_j V_{i,j+1} = V_{i+1,j} \tag{7}$$

Where

$$a_j = \frac{1}{2} rj\delta t - \frac{1}{2}\sigma^2 j^2 \delta t$$

$$b_j = 1 + \sigma^2 j^2 \delta t + r\delta t$$

$$c_j = -\frac{1}{2} rj\delta t - \frac{1}{2}\sigma^2 j^2 \delta t$$

A program has been developed in MATLAB 7.5 for this implicit finite difference scheme to evaluate the financial options.

Explicit method

Here we use a backward difference approximation of the time derivative. This is the idea behind the explicit method, which approximates

$$\frac{\partial V}{\partial t} = \frac{V_{i,j} - V_{i-1,j}}{\delta t}$$

If this approximation is applied in conjunction with (4) and (6) in the PDE (1), then

$$\frac{V_{i,j} - V_{i,j}}{\delta t} + rj\delta S\frac{V_{i,j+1} - V_{i,j-1}}{2\delta S} + \frac{1}{2}\sigma^2 j^2 \delta S^2 \frac{V_{i,j+1} + V_{i,j-1} - 2V_{i,j}}{\delta S^2} = rV_{i,j}$$

For j=1,2,3…….., M-1 and i=0,1,2,3,…………,N-1, and rearranging term, we obtain

$$V_{i,j} = a'_j V_{i+1,j-1} + b'_j V_{i+1,j} + c'_j V_{i+!,j+1} \qquad (8)$$

Here

$$a'_j = \frac{1}{1+r\delta t}\left(-\frac{1}{2}rj\delta t + \frac{1}{2}\sigma^2 j^2 \delta t\right)$$

$$b'_j = \frac{1}{1+r\delta t}\left(1 - \sigma^2 j^2 \delta t\right)$$

$$c'_j = \frac{1}{1+r\delta t}\left(\frac{1}{2}rj\delta t + \frac{1}{2}\sigma^2 j^2 \delta t\right)$$

The program has been developed for explicit finite difference method in Matlab7.5 to evaluate the financial options.

Results and Discussion

The value of various parameters like time, volatility, strike price, stock price and interest rate which have been used for numerical computations are given in table (1). The numerical results have been obtained for European call and put options by the two numerical methods namely Implicit and Explicit finite difference method. The figure (1) & (2) represents the results obtained by implicit method while figure 3 & 4 represents the results obtained by explicit method.

In Fig 1 and 3 we observe that when time is .2 yrs (73days), option price start increasing when stock price is 50 and option price goes an increasing when time is 1yr (365 days). we see that this relation among option price, stock price and time is linear.

Further the relationship between option price and stock price for put option in figure (2) and (4) are just opposite of the same relationship for call option in figure (1) and (3). There is no significant difference in the results obtained by the implicit and explicit method. Such models can be developed to study relationship among the financial parameters for prediction of stock markets.

Table 1: Values of parameters for European options [16,17,18,19].

Parameters	Symbol	Numerical value
Stock price	S	100
Strike /Exercise Price	E	50
Interest rate	R	10%
Option Maturity	T	12 months =1yr
Volatility	σ	.06

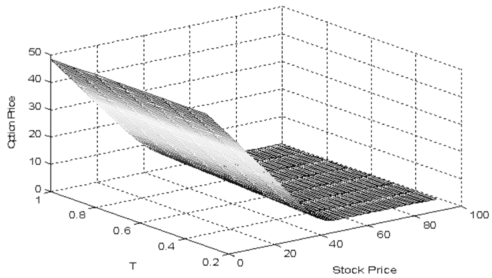

Figure 1: Graph among Option Price, stock Price and time for call Option by Implicit method.

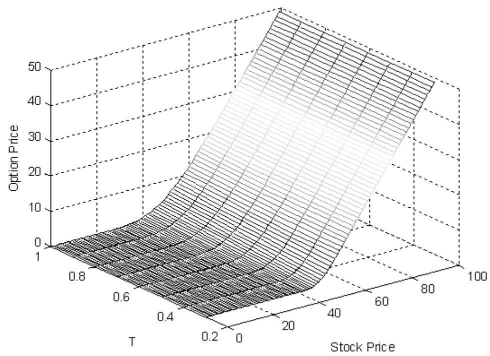

Figure 2: Graph among Option Price, stock Price and time for put option by implicit method.

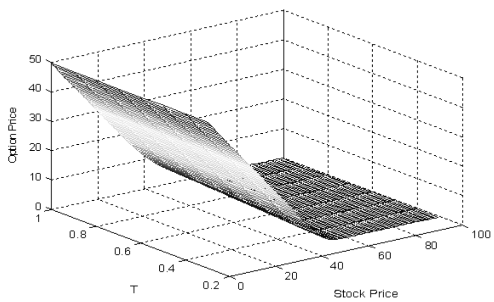

Figure 3: Graph among Option Price, stock Price and time for call option by Explicit method.

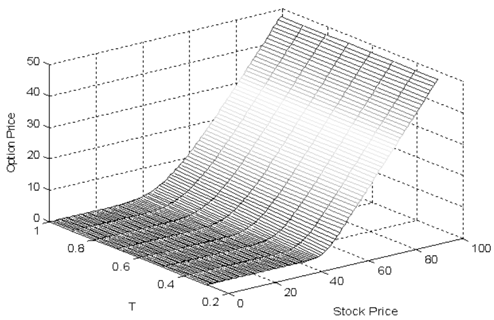

Figure 4: Graph among Option Price, stock Price and time for put option by explicit method.

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