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Non–Darcy Effects on Hydromagnetic Convective Heat and Mass Transfer Flow of a Chemically Reacting Fluid in a Vertical Channel with Heat Generation.

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Abstract

Non-Darcy effects on two-dimensional laminar simultaneous Heat and Mass Transfer flow of a viscous, incompressible, electrically conducting and chemically reacting fluid through a porous medium confined in a vertical channel. A similarity transformation is used to reduce the governing Partial Differential Equations into Ordinary ones. The behaviour of the velocity, temperature and concentration, Profiles as well as for skin friction, Nusselt number and Sherwood number are obtained and has been discussed for various parametric conditions.

Keywords: Heat transfer, Mass Transfer , Non_Darcy effect, Chemical reaction,

Introduction

Coupled Heat and Mass Transfer phenomenon in porous media is gaining attention due to its interesting applications. The flow phenomenon is relatively complex rather than that of the pure thermal convection process. Underground spreading of chemical wastes and other pollutants, grain storage, evaporation cooling and solidification are the few other application areas where the combined thermo-solutal natural convection in porous media are observed. Combined heat and mass transfer by free convection under boundary layer approximations has been studied by Bejan and Khair[1], Lai and Kulacki[2]. The free convection Heat and Mass Transfer in a porous enclosure has been studied recently by Angirasa et al[3].

For some industrial applications such as glass production and furnace design in space technology applications ,cosmic flight aerodynamics, rocket propulsion systems, plasma physics which operate at higher temperatures, radiation effects can be significant. In many chemical engineering processes ,there does occur the chemical reaction between a foreign mass and the fluid in which the plate is moving. These processes take place in numerous industrial applications viz, polymer production, manufacturing of ceramics or glassware and food processing. Das et al[4] have studied the effects of Mass Transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Chamkha et al[5] analysed the effects of radiation on free convection flow past a semi-infinite vertical plate with mass transfer.

Formulation of the problem

We consider a coupled Heat and Mass Transfer flow of a viscous electrically conducting fluid through a porous medium confined in a vertical channel bounded by porous flat walls in the presence of heat generating sources, transverse magnetic field effects and a first order chemical reaction. The flow is assumed to be steady, laminar and two-dimensional and the surface is maintained at constant temperature and concentration. All thermophysical properties are constant except the density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq approximation, Under these assumptions, the equations describing the physical situation are given by

$$\left(\frac{\partial v}{\partial y}\right) = 0 \tag{2.1}$$

$$v\left(\frac{\partial u}{\partial y}\right) = \nu\left(\frac{\partial^2 u}{\partial y^2}\right) + \beta g(T - T_e) + \beta^{\bullet} g(C - C_e) - \left(\frac{\sigma B_0^2 u}{\rho}\right) - \left(\frac{\nu}{k}\right)u - \left(\frac{\delta F}{\sqrt{k}}\right)u^2 \tag{2.2}$$

$$\rho_0 C_p v\left(\frac{\partial T}{\partial y}\right) = \lambda\left(\frac{\partial^2 T}{\partial y^2}\right) + Q(T - T_e) - \frac{\partial(q_R)}{\partial y} \tag{2.3}$$

$$v\left(\frac{\partial C}{\partial y}\right) = D\left(\frac{\partial^2 C}{\partial y^2}\right) - \gamma C \tag{2.4}$$

where y is the horizontal or transverse coordinate, u is the axial velocity,v is the transverse velocity,T is the fluid temperature, C is the species concentration ,T_e the ambient temperature, C_e is the ambient concentration and $\rho, g, \beta, \beta^{\bullet}, \mu, \sigma, B_0, Q, D$ and γ are the density, gravitational acceleration, coefficient of thermal expansion, coefficient of concentration expansion, dynamic viscosity, fluid electrical conductivity, magnetic induction, heat generation/absorption coefficient, mass diffusion coefficient and chemical reaction parameter respectively. The physical boundary conditions for the problem are

$$u(-L)=0, v(-L)=v_w, .T(-L)=T_1, C(-L)=C_1$$
$$u(+L)=0, v(+L)=v_w, .T(+L)=T_2, C(+L)=C_2 \tag{2.5}$$

where $v_w>0, T_1, T_2$ and C_1, C_2 are the suction velocity, surface temperatures and concentrations on $y=\pm L$ respectively.

Invoking Rosseland approximation for radiative heat flux

$$q_r = -\left(\frac{4\sigma^{\bullet}}{3\beta_R}\right)\left(\frac{\partial T'^4}{\partial y}\right)$$

Expanding T'^4 in Taylor series about T_e and neglecting higher order terms, we get

$$T'^4 \cong 4TT_e^3 - 3T_e^4$$

q_r represents the radiation heat flux in the y direction, $\sigma^{\bullet}$ the Stefan –Boltzman constant and β_R the mean absorption coefficient.

In order to write the governing equations and boundary conditions in the dimensionless form, the following non-dimensional quantities are introduced

$$y' = \frac{y}{L}, u' = \frac{u}{\left(\frac{\nu}{L}\right)}, \theta = \left(\frac{T-T_1}{T_2-T_1}\right), C' = \left(\frac{C-C_1}{C_2-C_1}\right)$$

the equations after dropping the dashes are

$$\left(\frac{d^2u}{dy^2}\right) + S\left(\frac{du}{dy}\right) + G\delta\left(\theta + N\,C\right) - \delta\left(M^2 + D^{-1}\right)u - \left(\delta^2\mathrm{A}\right)u^2 = 0 \tag{2.6}$$

$$\left(\frac{d^2\theta}{dy^2}\right) + SP\left(\frac{d\theta}{dy}\right) - \alpha\,P\,\theta = 0 \tag{2.7}$$

$$\left(\frac{d^2C}{dy^2}\right) + SSc\left(\frac{dC}{dy}\right) - k\,ScC = 0 \tag{2.8}$$

where

$P = \left(\frac{\mu C_p}{\lambda}\right)$ (Prandtl Number) $Sc = \left(\frac{\nu}{D}\right)$ (Schmidt Number)

$k = \left(\frac{\gamma L^2}{\nu}\right)$ (Chemical reaction parameter), $M^2 = \left(\frac{\sigma B_0^2 L^2}{\rho_0 \nu}\right)$ (Hartman Number)

$N = \left(\frac{\beta^{\bullet}\Delta C}{\beta\,\Delta T}\right)$ (Buoyancy ratio), $N_1 = \left(\frac{\lambda\beta_R}{4\sigma^{\bullet}T_e^3}\right)$ (Radiation parameter)

$\alpha = \left(\frac{Q\,L^2}{\lambda} \right)$ (Heat source parameter)

$\Lambda = FD^{-1/2}$ (Inertia parameter or Forchhimer Number)

The Non-dimensional boundary conditions are

$$u(\pm 1) = 0 \ , \theta(-1) = 0, C(-1) = 0$$
$$\theta(+1) = 1, C(+1) = 1$$
$$C_2 = 0$$

Stress, Nusselt number and Sherwood Number

The shear stress on the boundaries $y = \pm 1$ are given by

$$\tau = \frac{\tau^{\bullet}}{\left(\frac{\nu^2}{L^2} \right)} = \left(\frac{du}{dy} \right)_{y=\pm 1}$$
$$= \left(\frac{du_0}{dy} + \delta \frac{du_1}{dy} + \delta^2 \frac{du}{dy} \right)_{y=\pm 1}$$

The rate of Heat Transfer (Nusselt Number) on the boundaries $y = \pm 1$ are given by $(Nu)_{y=\pm 1} = \left(\frac{d\theta_0}{dy} + \delta \frac{d\theta_1}{dy} + \delta^2 \frac{d\theta_2}{dy} \right)_{y=\pm 1}$

The rate of Mass Transfer (Sherwood Number) on the boundaries $y = \pm 1$ are given by

$$(Sh)_{y=\pm 1} = \left(\frac{dC_0}{dy} + \delta \frac{dC_1}{dy} + \delta^2 \frac{dC_2}{dy} \right)_{y=\pm 1}$$

Discussion of the Numerical Results

The aim of this analysis is to investigate the convective Heat and Mass Transfer through a porous medium in a vertical channel in the presence of a chemical reaction with uniform suction. We consider three different cases k>0, k=0 and k<0,where k is the chemical reaction parameter .k>0 represents destructive chemical reaction, k=0 represents no chemical reaction and k<0 is for generative chemical reaction. The velocity u is represented in Figs.1 and 2. For different values of the governing parameters N_1, and k. u>0 is the actual flow and u<0 is the reversal flow. An increase in the radiation parameter N_1 leads to an enhancement in u in the flow region except in the vicinity of y=1(Fig.1).The behaviour of u with reference to and the chemical reaction parameter k. We notice that It is found that the velocity decreases during the generative reaction and enhances in the destructive reaction(Fig.2).

The Non-dimensional temperature θ is shown in Figs.4 and 5. The behaviour of θ with the radiation parameter N_1 is depicted in Fig.4.The temperature experiences a remarkable depreciation when the radiation parameter enhances and for higher values of N_1there is a marginal decrease in θ in the entire flow region. Fig.5 shows that θ decreases in the generative reaction and enhances in the destructive reaction.

The shear stress τ at the walls $y = \pm 1$ are evaluated for various different parametric conditions are presented in Tables.1-4.It is found that the shear stress at y=1 is positive for G>0 and negative for G<0 while at y=-1 it is negative for G>0 and positive for G<0.The magnitude of stress enhances with increase in $|G|$.

The variation of τ with the chemical reaction parameter k shows that the magnitude of stress at y =1 enhances with increase in both destructive reaction and generative reaction. The behaviour of τ with k at y = -1 shows that $|\tau|$ depreciates during the destructive chemical reaction and enhances with $k \geq 2$ at $|G| = 10^3$ while for $G \geq 3x10^3$, $|\tau|$ enhances with k>0 and depreciates with k for $|G| \geq 3x10^3$. $|\tau|$ experiences a depreciation in generative chemical reaction for all values of $|G|$, and also the radiation parameter N_1 enhances $|\tau|$ at y= +1and -1. Tables.1and2.An increase in the chemical reaction parameter k shows that $|Nu|$ at y=1 experiences an enhancement with all values of k while at y= -1, $|Nu|$ depreciates during the destructive chemical reaction and enhances during the generative chemical reaction . The radiation parameter N_1 enhances $|Nu|$ at y=1 and depreciates it at y= -1 for all G. Tables.3 and 4.

Table 1: Shear Stress (τ) at y =1 P=0.71,k=2.0,M=2.

G/ τ	I	II	III	IV	V	VI
10^3	4.2404	3.3031	6.2755	9.1284	4.3432	4.4147
$3x10^3$	12.7219	9.9093	18.8265	27.3852	13.0296	13.2374
-10^3	-4.2404	-3.3031	-6.2754	-9.1284	-4.3432	-4.4126
$-3x10^3$	-12.7219	-9.9093	-18.8265	-27.3852	-13.0296	-13.2374

Table 2: Shear Stress (τ) at y = -1 P=0.71,k=2.0,N1=0.5,M=2.

G/ τ	I	II	III	IV	V	VI
10^3	-0.6072	-0.4357	2.6974	3.7612	-0.4827	-0.4019
$3x10^3$	-2.7054	-2.2941	2.6301	2.6306	-2.2798	-1.9969
-10^3	0.3125	0.1067	-4.5180	-6.6455	0.2054	0.1381
$-3x10^3$	0.0536	-0.6669	-19.0161	-28.5892	-0.2156	-0.3772
	I	II	III	IV	V	VI
K	2	1.2	0	-0.2	2	2
N_1	0.5	0.5	0.5	0.5	10	100

Table 3: Nusselt Number(Nu) at y =1 P=0.71,k=2.0,N1=0.5,M=2.

G/ Nu	I	II	III	IV	V	VI
10^3	0.49841	0.48861	0.35192	0.28483	0.52262	0.54794
$3x10^3$	0.52252	0.51922	0.35521	0.34391	0.54142	0.56354
-10^3	0.47782	0.47962	0.69831	0.82885	0.51540	0.54266
$-3x10^3$	0.48081	0.49242	1.39432	1.97606	0.51982	0.54770

Table 4: Nusselt Number(Nu) at y = -1 P=0.71,k=2.0,N1=0.5,M=2.

G/ Nu	I	II	III	IV	V	VI
10^3	0.01251	0.01683	-0.22998	-0.40596	0.02047	0.01917
$3x10^3$	-0.0749	-0.0921	-0.83253	-1.36042	-0.06271	-0.0601
-10^3	0.12001	0.1257	0.372512	0.54852	0.11164	0.10547
$-3x10^3$	0.21751	0.2345	0.974982	1.50294	0.20282	0.19177
	I	II	III	IV	V	VI
K	2.0	1.2	0	-0.2	-0.2	-1.2
N_1	0.5	0.5	0.5	0.5	10	100

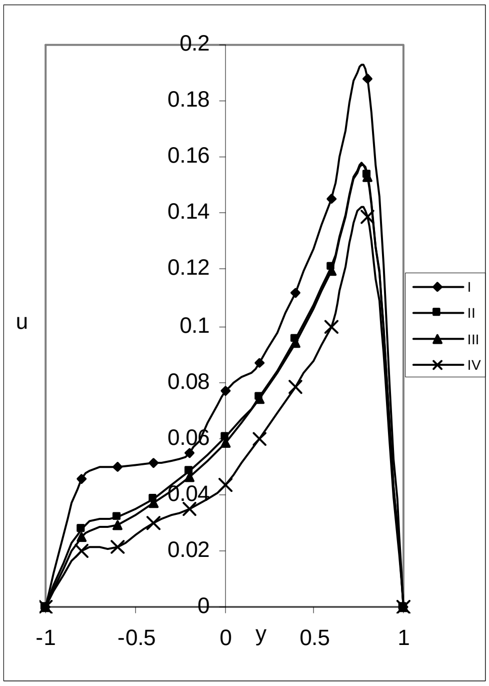

Figure 1: u with N_1 $G=10^3$, $D^{-1}=5x10^2$,M=2,N=1

	I	II	III	IV
N_1	0	5	10	100

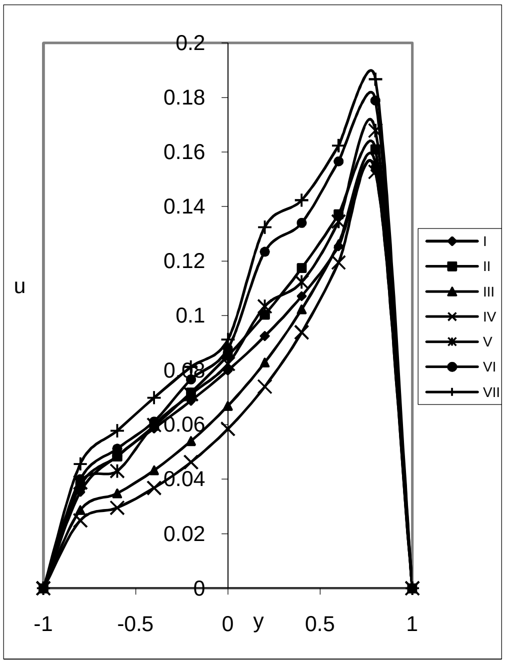

Figure 2: u with k G=10^3, D^{-1}=5x10^2,M=2,N=1

	I	II	III	IV	V	VI	VII
K	0	0.2	1.2	2.0	-0.2	-1.2	-2.0

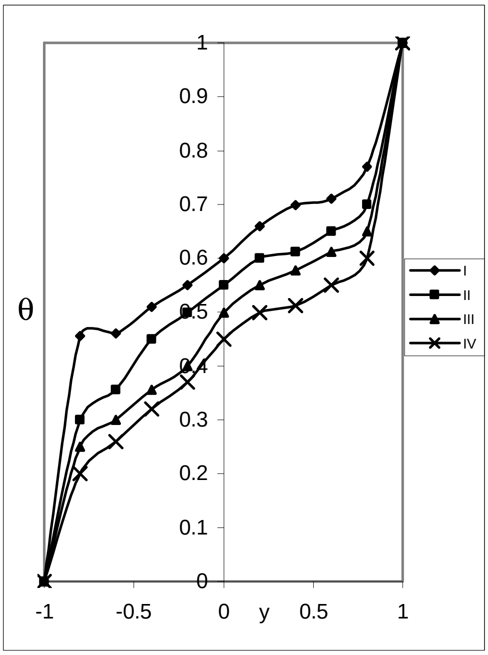

Figure 3: θ with N_1 G=10^3, D^{-1}=5x10^2,M=2,N=1

	I	II	III	IV
N_1	0	5	10	100

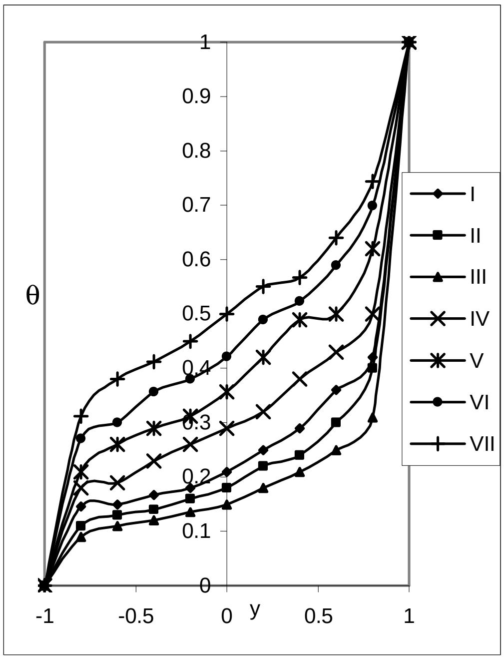

Figure 4: θ with k $G=10^3$, $D^{-1}=5x10^2$,M=2,N=1.

	I	II	III	IV	V	VI	VII
K	0	0.2	1.2	2.0	-0.2	-1.2	-2.0

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