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Constructions of Q-BI Fuzzy Ideals Over Sub Semi-Groups with Respect to (T,S) Norms

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Abstract

We consider the Q- Bifuzzification of the concept of several ideals in a semi-group G, and investigate some properties of such ideals.

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Introduction

After the introduction of fuzzy sets by L.A. Zadeh [11], several researchers explored on the generalization of the notion of fuzzy set, the concept of Intuitionistic fuzzy set was introduced by K.T. Atanassav [2] as a generalization of the notion of fuzzy set. In [5], N. Kuroki gave some properties of fuzzy ideals and fuzzy bi-ideals in a semi-groups then concept (1,2)- ideals in a semi-group was introduced by S. Lajos [8]. In this paper we consider the Q- Bifuzzification of the concept of several ideals in a semi-group G and investigate some properties of such ideals.

Preliminaries

Let 'G' be a semi-group. By a sub semi-groups of G we mean a non-empty subset A of G such that $A^2 \subseteq A$ and by a left (right) ideal of G we mean a non-empty subset A of G such that $GA \subseteq A$ $(AG \subseteq A)$. By two sided ideal or simply ideal, we mean a non – empty subset of G which is both left and right ideal of G. A sub semi-group 'A' of a semi-group G is called a bi-ideal of G if as $A \subseteq A$. A sub semi-group A of G is called a (1,2)- ideal of G if $AGA^2 \subseteq A$. A semi-group G is said to be (2, 2) – regular if x $\in$

x^2Gx^2 for $x \in G$. A semi-group ‘G’ is said to be regular if, for each $x \in G$, there exists $y \in G$ such that $x = xyx$. A semi-group ‘G’ is said to be completely regular if for each $x \in G$, there exists $y \in G$ such that $x = xyx$ and $xy = yx$. For a Semi-group ‘G’, note that G is completely regular iff G is a union of groups iff G is (2,2)- regular. A semi-group ‘G’ is said to be left (resp. right) ideal if every left (resp. right) ideal of G is a two sided ideal of G.

A Bi fuzzy set (briefly BFS) ‘A’ is a non-empty set X is an object having the form $A=\{(x, t_A(x), f_A(x) / x \in X\}$ where the functions $t_A : X \to [0,1]$ and $f_A : X \to [0,1]$ denote the truth degree of membership and false degree of membership respectively and as $t_A(x) + f_A(x) \leq 1$, for all $x \in X$.

In what follows, let G denote a semi-group unless otherwise specified.

Let ‘X’ be a non-empty set. A mapping $\mu : X \to [0,1]$ is called a fuzzy set in X. The complement of a fuzzy set μ in X, denoted by μ^c is the fuzzy set in X given by $\mu^c(x) = 1 - \mu(x)$ for all $x \in X$. In what follows, let Q and G denote a set and a semi-group, respectively unless otherwise specified. A mapping $\mu : G \times Q \to [0,1]$ is called a Q fuzzy set in X.

Definition 2.1: A Q-bi fuzzy set (QBFS) $A = (t_A , f_A)$ in G is called an Q- bi fuzzy sub semi-group of G
if

i. $t_A(xy,q) \geq T \{t_A(x,q), t_A(y,q)\}$
ii. $f_A(xy,q) \leq S \{f_A(x,q), f_A(y,q)\}$ for all $x, y \in G$.

Definition 2.2: A QBFS $A = (t_A , f_A)$ in G is called Q- bi fuzzy left ideal of G if $t_A(xy,q) \geq t_A(y,q)$ and $f_A(xy,q) \leq f_A(y,q)$, for $x,y \in G$. A Q- bifuzzy right ideal of G define in an analogous way. An BFS $A = (t_A , f_A)$ in G is called an Q- bifuzzy ideal of G if it is both an Q- bifuzzy left (right) ideal of G is an Q- bifuzzy subgroup of G.

Definition 2.3: A Q- bifuzzy sub semi-group $A = (t_A, f_A)$ of G is called Q- bifuzzy ideal of G if,

i. $t_A(xwy,q) \geq T \{t_A(x,q), t_A(y,q)\}$
ii. $f_A(xwy,q) \leq S \{f_A(x,q), f_A(y,q)\}$ for all $w,x, y \in G$.

Characteristic of Q-fuzzy bi-ideals

Proposition 3.1: Every Q- bifuzzy ideal is an Q- bifuzzy (1,2)-ideal.

Proof: Let $A = (t_A, f_A)$ be an Q- bifuzzy ideal of G and let $w, x, y, z \in G$ and $q \in Q$ then

$t_A (xw(yz), q) = t_A((xwy)z, q)$
$\geq T \{ t_A(xwy, q), t_A(z, q)\}$
$\geq T \{ T \{ t_A(x,q), t_A(y,q) t_A(z, q) \}$
$= T \{t_A(x,q), t_A(y,q), t_A(z,q)\}$

and

$f_A (xw(yz), q) = f_A((xwy)z, q)$
$\leq S \{ f_A(xwy, q), f_A(z, q)\}$
$\leq S \{ S \{ f_A(x,q), f_A(y,q) f_A(z, q) \}$

$= \quad S\{ f_A(x,q), f_A(y,q), f_A(z,q)\}$

Hence $A = (t_A, f_A)$ be an Q- bifuzzy (1, 2)- ideal of G.

To consider the converse of proposition 3.1, we need to strengthen the condition of a sub semi-group G.

Proposition 3.2: If 'G' is a regular semi-group, then every Q- bifuzzy (1,2)-ideal of G is an Q- bi fuzzy ideal of G.

Proof: Assume that a sub semi-group G is regular and let $A = (t_A, f_A)$ be an Q- bifuzzy (1,2)-ideal of G. Let $w, x, y \in G$ and $q \in Q$. Since G is regular, we have $xw \in (xsx)s \subseteq xsx$ which implies that $xw = xGx$ for some $s \in G$ thus,

$t_A(xwy, q) = t_A((xsx)y, q) = t_A(xs(xy), q)$

$\geq \quad T\{ t_A(x, q), t_A(x, q), t_A(y, q)\}$

$= \quad T\{t_A(x,q), t_A(y,q)\}$

and

$f_A(xwy, q) = f_A((xsx)y, q) = f_A(xs(xy), q)$

$\leq \quad S\{ f_A(x, q), f_A(x, q), f_A(y, q)\}$

Therefore $A = (t_A, f_A)$ is an Q- bi fuzzy bi-ideal of G.

Proposition 3.3: Let 'A' be an Q- bifuzzy ideal of G. If G is a completely regular, then $A(a,q) = A(a^2,q)$ for all $a \in G$ and $q \in Q$.

Proof: Let $a \in G$ and $q \in Q$, then there exists $x \in G$ such that $a = a^2xa^{2.}$

Hence ,

$t_A(a,q) = t_A(a^2xa^2,q)$

$\geq \; T\{ t_A(a^2,q), t_A(a^2,q)\}$

$= \; t_A(a^2,q)$

$\geq \; T\{ t_A(a,q), t_A(a,q)\}$

$= \; t_A(a,q)$

and

$f_A(a,q) = f_A(a^2xa^2,q)$

$\leq \; S\{ f_A(a^2,q), f_A(a^2,q)\}$

$= \; f_A(a^2,q)$

$\leq \; S\{ f_A(a,q), f_A(a,q)\}$

$= \; f_A(a,q)$

It follows that $t_A(a,q) = t_A(a^2,q)$ and $f_A(a,q) = f_A(a^2, q)$ so that $A(a,q) = A(a^2,q)$.

Proposition 3.4: Let A be an Q- bifuzzy ideal of G. If 'G' is an intra-regular then $A(a,q) = A(a^2,q)$ for all $a \in G$ and $q \in Q$.

Proof: Let $a \in G$ then G is intra-regular there exists x and y in G such that $a = xa^2y$. Hence since A is Q- bifuzzy ideal.

$t_A(a,q) = t_A(xa^2y,q)$

$\geq \; t_A(xa^2,q)$

$\geq\ t_A(a^2,q)$
$\geq\ S\{\ t_A(a,q),\ t_A(a,q)\}$
$=\ t_A(a,q)$
and
$f_A(a,q)\ =\ f_A\ (xa^2y,q)$
$\leq\ f_A(xa^2,q)$
$\leq\ f_A(a^2,q)$
$\leq\ S\{\ f_A(a,q),\ f_A(a,q)\}$
$=\ f_A(a,q)$

Hence we have $t_A(a,q)\ =\ t_A\ (a^2,q)$ for all $x,y \in G$ and $q \in Q$.

Proposition 3.5: Let 'A' be an Q- bifuzzy ideal of G. If S is an intra-regular then $A(ab,q)\ =\ A(ba,q)$ for all a, b, $\in$ G and $q \in Q$.

Proof: Let a, b, $\in$ G and $q \in Q$ then by proposition (3.3), we have
$t_A(ab,q)\ =\ t_A\ (\ (ab)^2,q)$
$\geq\ t_A(\ a(ba)b,q)$
$\geq\ t_A(ba,q) = t_A((ba)^2,q)$
$\geq\ t_A((b(ab)a,q)$
$=\ t_A(ab,q)$
and
$f_A(ab,q)\ =\ f_A\ ((ab)^2,q)$
$\leq\ f_A(\ a(ba)b,q)$
$\leq\ f_A(ba,q) = f_A((ba)^2,q)$
$\leq\ f_A((b(ab)a,q))$
$=\ f_A(ab,q)$

So we have $t_A(ab,q)\ =\ t_A(ba,q)$ and $f_A(ab,q)\ =\ f_A(ba,q)$. Therefore $A(ab,q) = A(ba,q)$.

Proposition 3.6: A QBFS 'A' is Q- bifuzzy ideal of G if and only if the Q-fuzzy sets t_A and $\overline{f_A}$ are Q-fuzzy ideals of G.

Proof: Let 'A' be Q- bifuzzy ideal of G, then clearly t_A is a Q-fuzzy bi-ideal of G. Let x, a, y $\in$ G, $q \in Q$ then
$\overline{f_A}\ (xy,\ q)\ =\ 1 - f_A\ (xy,q)$
$\geq\ 1 - S\ \{f_A(x,q),\ f_A(y,q)\}$
$=\ T\ \{1 - f_A(x,q),\ 1 - f_A(y,q)\}$
$=\ T\ \{\ \overline{f_A}\ (x,q),\ \overline{f_A}\ (y,q)\}$ and
$\overline{f_A}\ (xay,q)\ =\ 1 - f_A\ (xay,q)$
$\geq\ 1 -\ S\{f_A(x,q),\ f_A(y,q)\}$
$=\ T\ \{1 - f_A(x,q),\ 1 - f_A(y,q)\}$
$=\ T\ \{\overline{f_A}\ (x,q),\ \overline{f_A}\ (y,q)\}$

Hence $\overline{f_A}$ is a Q-fuzzy ideal of G. Conversely, suppose that t_A and f_A are Q-fuzzy ideals of G. Let a, x, y $\in$ G.

$1 - f_A(xy,q) \;=\; \overline{f_A}\,(xy,q)$
$\geq\; T\,\{\overline{f_A}\,(x,q),\ \overline{f_A}\,(y,q)\}$
$=\; T\,\{1 - f_A(x,q),\ 1 - f_A(y,q)\}$
$=\; S\{f_A(x,q),\ f_A(y,q)\}$
$1 - f_A(xay, q) \;=\; \overline{f_A}\,(xay, q)$
$=\; T\,\{\overline{f_A}\,(x,q),\ \overline{f_A}\,(y,q)\}$
$=\; T\,\{1 - f_A(x,q),\ 1 - f_A(y,q)\}$
$=\; S\{f_A(x,q),\ f_A(y,q)\}$
which imply that $f_A\,(xy, q) \leq S\{f_A(x,q), f_A(y,q)\}$
and $f_A\,(xay, q) \leq S\{f_A(x,q), f_A(y,q)\}$
This completes the proof.

Proposition 3.7: An QBFS $A = (t_A, f_A)$ is an Q- bifuzzy ideal of G if and only if $\Box A = (t_A, \overline{t_A})$ and $\Diamond A = (\overline{f_A}, f_A)$ are Q- bifuzzy ideals of G.

Proof: It is sufficient to show that $\overline{t_A}$ satisfies the condition (i) in definition 2.1. and (ii) in definition of 2.3.

For any a, x, y ∈ G, we have
$\overline{t_A}\,(xy, q) \;=\; 1 - t_A\,(xy,q)$
$\leq\; 1 - T\,\{t_A(x,q),\ t_A(y,q)\}$
$=\; S\,\{1 - t_A(x,q),\ 1 - t_A(y,q)\}$
$=\; S\,\{\overline{t_A}\,(x,q),\ \overline{t_A}\,(y,q)\}$ and
$\overline{t_A}\,(xay, q) \;=\; 1 - t_A\,(xay, q)$
$\leq\; 1 - T\,\{t_A(x,q),\ t_A(y,q)\}$
$=\; S\,\{1 - t_A(x,q),\ 1 - t_A(y,q)\}$
$=\; S\,\{\overline{t_A}\,(x,q),\ \overline{t_A}\,(y,q)\}$
Therefore $\Box A$ is Q- bi fuzzy ideal of G.
Similarly, we can show $\Diamond A$ is Q- bi fuzzy ideal of G.

Proposition 3.8: Let $f : G \rightarrow T$ be a homomorphism of semi-groups. If $B = (t_B, f_B)$ is an Q- bifuzzy ideal of T, then the pre image $f^{-1}(B)$ of B under 'f' is an Q- bifuzzy ideal of G.

Proof: Assume that $B = (t_B, f_B)$ is an Q- bifuzzy bi-ideal of T and let x, y ∈ G then
$f^{-1}{}_{(tB)}(xy, q) \;=\; t_B(f(xy, q))$
$=\; t_B(f(x,q),\ f(y,q))$
$\geq\; T\,\{t_B(f(x,q),\ t_B(f(y,q)\}$
$=\; T\,\{f^{-1}{}_{(tB)}\,(x,q),\ f^{-1}{}_{(tB)}(y,q)\}$
Also
$f^{-1}{}_{(tB)}(xy, q) \;=\; f_B(f(xy, q))$
$=\; f_B\,(f(x,q),\ f(y,q))$
$\leq\; S\,\{f_B(f(x,q),\ f_B(f(y,q)\}$
$=\; S\,\{\ f^{-1}(f_B(x,q)),\ f^{-1}(f_B\,(y,q))\}$

Hence $f^{-1}(B) = (f^{-1}(t_B), f^{-1}(f_B))$ is Q- bifuzzy sub semi-group of G. For any

$x, a, y \in G$ we have

$f^{-1}(_{tB})\ (xay,q) = t_B(f(xay,q)$

$= t_B(f(x,a), f(a,q), f(y,q))$

$\geq T\ \{ t_B(f(x,q), t_B(y,q)\}$

$= T\ \{f^{-1}\ (t_B(x,q), f^{-1}(t_B(y,q))\}$

and

$f^{-1}(_{fB})\ (xay,q) = f_B(f(xay,q))$

$= f_B(f(x,a), f(a,q), f(y,q))$

$\leq S\ \{f_B(f(x,q), f_B(f(y,q)\}$

$= S\ \{f^{-1}\ (f_B(x,q), f^{-1}(f_B(y,q))\}$

Therefore $f^{-1}(B)$ is Q-bifuzzy ideal of G.

Proposition 3.9: If $\{A_i\}_{i\in A}$ is a family of Q- bifuzzy ideals of G then $\cap A_i$ is an Q- bifuzzy ideal of G, where

$\cap A_i = \{\wedge t_{Ai}, \vee f_{Ai}\}$ and

$\wedge\ t_{Ai}\ (x,q) = S\ \{ t_{Ai}(x,q) / i\in\wedge, x \in G\}$

$\vee f_{Ai}\ (x,q) = S\ \{f_{Ai}(x,q) / i\in\wedge, x \in G\}$

Proof: Let $x, y \in G$ then we have

$\wedge\ t_{Ai}\ (x,q) = \wedge\{T\ \{ t_{Ai}(x,q), t_{Ai}(y,q)\}$

$= T\{ T\ \{ t_{Ai}(x,q), t_{Ai}(y,q)\}$

$= T\{T\ t_{Ai}(x,q), T\ (t_{Ai}(y,q))\}$

$= T\{\wedge\ t_{Ai}\ (x,q), \wedge\ t_{Ai}\ (y,q)\}$

$\vee\ f_{Ai}\ (xy,q) \leq \vee\{S\ \{f_{Ai}(x,q), f_{Ai}(y,q)\}$

$= S\ \{S\ \{f_{Ai}(x,q), f_{Ai}(y,q)\}$

$= S\ \{S\ (f_{Ai}(x,q)), S(\ f_{Ai}(y,q))\}$

$= S\ \{\vee\ f_{Ai}\ (x,q), \vee\ f_{Ai}\ (y,q)\}$

Hence $\cap A_i$ is Q- bifuzzy sub semi-group of G. Next for $x, y, a \in G$ we obtain

$\wedge\ t_{Ai}\ (xay,q) \geq \wedge\{\ T\{\ t_{Ai}(x,q), t_{Ai}(y,q)\ \}\}$

$= T\ \{T\ \{\ t_{Ai}(x,q), t_{Ai}(y,q)\ \}\}$

$= T\ \{T(\ t_{Ai}(x,q)), T(\ t_{Ai}(y,q))\ \}$

$= T\ \{\wedge\ t_{Ai}\ (x,q), \wedge\ t_{Ai}\ (y,q)\}$

$\vee\ f_{Ai}\ (xay,q) \leq \vee\{\ S\ \{f_{Ai}(x,q), f_{Ai}(y,q)\ \}\}$

$= S\ \{S\ \{f_{Ai}(x,q), f_{Ai}(y,q)\ \}\}$

$= S\ \{S(\ f_{Ai}(x,q)), S(f_{Ai}(y,q))\ \}$

$= S\ \{\vee\ f_{Ai}\ (x,q), \vee\ f_{Ai}\ (y,q)\}$

Hence $\cap A_i$ is Q- bifuzzy ideal of G. This completes the proof.

Conclusion

Kuroki. N [5] introduced the concept of fuzzy ideals and bi-ideals in a semi group and Lajos.S [8] investigate the concept of (1,2)-ideals of union of groups. [4] discussed the concept of Q-Vague groups and vague normal subgroups with respect to (T,S) norms. In this paper, we investigate the concept of Q-bifuzzy in several ideals of semi group and investigate some properties of such ideals.

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