# A Stochastic Model for the Expected Time to Recruitment in a Single Graded Manpower System with Two Thresholds Having SCBZ Property 

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#### Abstract

In this paper, an organization subjected to random exit of personnel due to policy decisions taken by the organization is considered. There is an associated loss of manpower if a person quits. As the exit of personnel is unpredictable, a new recruitment policy involving two thresholds - one is optional and the other one mandatory is suggested to enable the organization to plan its decision on recruitment. Based on shock model approach, a mathematical model is constructed using an appropriate univariate policy of recruitment. The analytical expression for the mean and variance of the time to recruitment are obtained when i) the loss of manhour process forms a sequence of independent and identically distributed continuous random variables ii) the inter-decision times are independent and identically distributed continuous random variables and iii) the optional threshold level is an exponential random variable and the distribution of the mandatory level has SCBZ property and vice versa. The results are numerically illustrated and analyzed by assuming specific distributions.


Keywords: Manpower planning, Shock models, Univariate recruitment policy, Mean and variance of the time to recruitment.

2000 Mathematics Subject Classification: Primary: 90B70, Secondary: 91B40, 91D35.

## Introduction

Exits of personnel which is in other words known as wastage is an important aspect in
the manpower planning. Many models have been discussed using different types of distributions. Such model could be seen in [1] and [4]. In [2] the expected time to recruitment is obtained when the inter-decision times are independent and identically distributed random variables. The expected time to recruitment is obtained in [6] when the threshold distribution has SCBZ property [7]. The results of [6] are extended in [5] when the inter-decision times are exchangeable constantly correlated exponential random variables. In [8] the mean time to recruitment and the optimum cost of recruitment are obtained for different univariate policies under different conditions. In [9] the mean time to recruitment is obtained when the survival time process is geometric and the threshold distribution has SCBZ property. In all the above cited works, the problem of time for recruitment in a single graded marketing organization involves only one threshold value. Since the number of exits in a policy decision making epoch is unpredictable and the time at which the cumulative loss of man hours crossing a single threshold is probabilistic, the organization has left with no choice except making recruitment immediately upon the threshold crossing. In this paper, this limitation is removed by considering the following new recruitment policy involving two thresholds in which one is optional and the other is mandatory. If the cumulative loss of manpower crosses the optional threshold, the organization may or may not go for recruitment. However, recruitment is necessary whenever the cumulative loss of manpower crosses the mandatory threshold. In view of this policy, the organization can plan its decision upon the time for recruitment. Recently, for a single graded system involving the optional and mandatory thresholds [3], have obtained the mean and variance of the time to recruitment for exponential thresholds. In this paper, the authors extend their above cited paper when one threshold has SCBZ property and the other follows exponential distribution and vice versa.

## Model description and analysis for model I

Consider an organization taking decisions at random epochs in $[0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower if a person quits. It is assumed that the loss of manpower is linear and cumulative. Let $X_{i}$ be the loss of manpower due to the $\mathrm{i}^{\text {th }}$ decision epoch, $\mathrm{i}=1,2,3, \ldots$ forming a sequence of independent and identically distributed random variables following exponential distribution with parameter $\mu$. It is assumed that the inter-decision times are independent and identically distributed exponential random variables with parameter $\lambda$ having probability density function (distribution function) $f().(F()$.$) . Let f_{k}().\left(F_{k}().\right)$ be $k$ fold convolution of $f().(F()$.$) . Let f^{*}($.$) be$ the Laplace transform of $f($.$) . The loss of manpower process and the process of inter-$ decision times are assumed to be statistically independent. Let $Y$ be a continuous random variable denoting the optional threshold having SCBZ property. Let $Z$ be a continuous random variable denoting the mandatory threshold following exponential distribution with parameter $\beta$ such that $Z>Y$. It is assumed that $Y, Z$ and $X_{i}$, $\mathrm{i}=1,2,3, \ldots$ are independent. The univariate recruitment policy employed in this paper
is stated as follows. If the total loss of manpower exceeds the optional threshold level the organization may or maynot go for recruitment, but if the total loss of manhours exceeds the mandatory threshold recruitment is necessary. Let $\theta$ be the probability that the organization is not going for recruitment whenever the total loss of manpower crosses the optional threshold level $Y$. Let $\mathrm{T}_{1}$ be a continuous random variable denoting the time for recruitment in the organization with probability density function $1_{1}(\mathrm{t})$ and cumulative distribution function $\mathrm{L}_{1}($.$) . Let V_{k}(t)=F_{k}(t)-F_{k+1}(t)$ be the probability that there are exactly $k$-decision epochs in $(0, t]$ where $F_{0}(t)=1$. Let $\mathrm{E}\left(\mathrm{T}_{1}\right)$ be the expected time for recruitment and $\mathrm{V}\left(\mathrm{T}_{1}\right)$ be the variance of the time for recruitment.

As in [6], the distribution of $Y$ is given by

$$
\mathrm{H}(\mathrm{y})=1-\mathrm{pe}^{-\left(\alpha_{1}+\alpha_{2}\right) \mathrm{y}}-\mathrm{qe}^{-\alpha_{2} \mathrm{y}}
$$

where

$$
p=\frac{\alpha_{1}-\alpha_{3}}{\alpha_{2}+\alpha_{1}-\alpha_{3}}, q=\frac{\alpha_{2}}{\alpha_{2}+\alpha_{1}-\alpha_{3}} \text { and } p+q=1
$$

Now,
$P\left[\sum_{i=0}^{k} X_{i}<Y\right]=$ Probability that the system does not fail, after k epochs of exits

$$
=\int_{0}^{\infty} g_{k}(x)[1-H(x)] d x
$$

$$
\begin{equation*}
=p\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right]^{k}+q\left[g^{*}\left(\alpha_{3}\right)\right]^{k} \tag{1}
\end{equation*}
$$

Similarly, $P\left[\sum_{i=0}^{k} X_{i}<Z\right]=\left[g^{*}(\beta)\right]^{k}$
By the law of total probability and using (1) and (2)
$\mathrm{P}\left[\mathrm{T}_{1}>\mathrm{t}\right]=\sum_{k=0}^{\infty}$ [Probabilit thaexactly decisionsretakerin $[0, \mathrm{t}), \mathrm{k}=0,1,2, \ldots \times$
(Probability that the total number of exits in these k -decisions does not cross the optional level $Y$ or the total number of exits in these k-decisions crosses the optional level $Y$ but lies below the mandatory level $Z$ and the organization is not making recruitment)]

$$
\begin{aligned}
& =\sum_{k=0}^{\infty} V_{k}(t) P\left[\sum_{i=1}^{k} X_{i}<Y\right]+\sum_{K=0}^{\infty} V_{k}(t) P\left[\sum_{i=1}^{k} X_{i} \geq Y\right] \times P\left[\sum_{i=1}^{k} X_{i}<Z\right] \times \theta \\
& =\sum_{k=0}^{\infty}\left[F_{k}(t)-F_{k+1}(t)\right] p\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right]^{k}+q\left[g^{*}\left(\alpha_{3}\right)\right]^{k} \\
& +\theta \sum_{k=0}^{\infty}\left[F_{k}(t)-F_{k+1}(t)\right]\left[1-p\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right]^{k}-q\left[g^{*}\left(\alpha_{3}\right)\right]^{k}\right] \times\left[g^{*}(\beta)\right]^{k}
\end{aligned}
$$

On simplification, it can be shown that

$$
\begin{aligned}
& \mathrm{P}\left[\mathrm{~T}_{1}>\mathrm{t}\right]=\left[\begin{array}{l}
1+p\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right)-1\right] \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right]^{k-1} \\
+q\left[g^{*}\left(\alpha_{3}\right)-1\right] \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}\left(\alpha_{3}\right)\right]^{k-1}+\theta\left[g^{*}(\beta)-1\right] \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}(\beta)\right]^{k-1} \\
-\theta p\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)-1\right] \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)\right]^{k-1} \\
-\theta q\left[g^{*}\left(\alpha_{3}\right) g^{*}(\beta)-1\right] \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}\left(\alpha_{3}\right) g^{*}(\beta)\right]^{k-1}
\end{array}\right] \\
& \therefore \mathrm{L}_{1}(\mathrm{t})==1-\mathrm{P}\left[\mathrm{~T}_{1}>\mathrm{t}\right] \\
& L_{1}(t)=\left[\begin{array}{l}
p\left[1-g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right] \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right]^{k-1} \\
+q\left[1-g^{*}\left(\alpha_{3}\right)\right] \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}\left(\alpha_{3}\right)\right]^{k-1}+\theta\left[1-g^{*}(\beta)\right] \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}(\beta)\right]^{k-1} \\
-\theta p\left[1-g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)\right) \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)\right]^{k-1} \\
-\theta q\left[1-g^{*}\left(\alpha_{3}\right) g^{*}(\beta)\right] \sum_{k=0}^{\infty} F_{k}(t)\left[g^{*}\left(\alpha_{3}\right) g^{*}(\beta)\right]^{k-1}
\end{array}\right]
\end{aligned}
$$

and

$$
\begin{align*}
& 1_{1}(\mathrm{t})=\left[\begin{array}{l}
p\left[1-g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right] \sum_{k=0}^{\infty} f_{k}(t)\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right]^{k-1} \\
+q\left[1-g^{*}\left(\alpha_{3}\right)\right] \sum_{k=0}^{\infty} f_{k}(t)\left[g^{*}\left(\alpha_{3}\right)\right]^{k-1}+\theta\left[1-g^{*}(\beta)\right] \sum_{k=0}^{\infty} f_{k}(t)\left[g^{*}(\beta)\right]^{k-1} \\
-\theta p\left[1-g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)\right] \sum_{k=0}^{\infty} f_{k}(t)\left[g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)\right]^{k-1} \\
-\theta q\left[1-g^{*}\left(\alpha_{3}\right) g^{*}(\beta)\right] \sum_{k=0}^{\infty} f_{k}(t)\left[g^{*}\left(\alpha_{3}\right) g^{*}(\beta)\right]^{k-1}
\end{array}\right] \\
& \therefore 1_{1}^{*}(\mathrm{~s})=  \tag{3}\\
& \quad \frac{p\left[1-g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right] f^{*}(s)}{1-g^{*}\left(\alpha_{1}+\alpha_{2}\right) f^{*}(s)}+\frac{q\left[1-g^{*}\left(\alpha_{3}\right)\right] f^{*}(s)}{1-g^{*}\left(\alpha_{3}\right) f^{*}(s)}+\frac{\theta\left[1-g^{*}(\beta)\right] f^{*}(s)}{1-g^{*}(\beta) f^{*}(s)} \\
&
\end{align*}
$$

Since

$$
f^{*}(s)=\frac{\lambda}{\lambda+s}
$$

From (3) we have

$$
\mathrm{I}_{1}^{*}(\mathrm{~s})=\begin{align*}
& \frac{p \lambda\left[1-g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right]}{\lambda+s-\lambda g^{*}\left(\alpha_{1}+\alpha_{2}\right)}+\frac{q \lambda\left[1-g^{*}\left(\alpha_{3}\right)\right]}{\lambda+s-\lambda g^{*}\left(\alpha_{3}\right)}+\frac{\theta \lambda\left[1-g^{*}(\beta)\right]}{\lambda+s-\lambda g^{*}(\beta)}  \tag{4}\\
& \\
& -\frac{\theta p \lambda\left[1-g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)\right]}{\lambda+s-\lambda g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)}-\frac{\theta q \lambda\left[1-g^{*}\left(\alpha_{3}\right) g^{*}(\beta)\right]}{\lambda+s-\lambda g^{*}\left(\alpha_{3}\right) g^{*}(\beta)}
\end{align*}
$$

It is known that

$$
\begin{align*}
& \mathrm{E}\left(\mathrm{~T}_{1}\right)=\left[\frac{-\mathrm{d}}{\mathrm{ds}} \mathrm{l}^{*}(\mathrm{~s})\right]_{\mathrm{s}=0}  \tag{5}\\
& \mathrm{E}\left(\mathrm{~T}_{1}^{2}\right)=\left[\frac{\mathrm{d}^{2}}{\mathrm{ds}^{2}} \mathrm{l}^{*}(\mathrm{~s})\right]_{\mathrm{s}=0}  \tag{6}\\
& \mathrm{~V}\left(\mathrm{~T}_{1}\right)=\mathrm{E}\left(\mathrm{~T}_{1}^{2}\right)-\left[\mathrm{E}\left(\mathrm{~T}_{1}\right)\right]^{2} \tag{8}
\end{align*}
$$

Since

$$
\begin{equation*}
g^{*}(\alpha)=\frac{\mu}{\mu+\alpha} \tag{9}
\end{equation*}
$$

using (9), (5), (6) and (7) in (4), we have
$\mathrm{E}\left(\mathrm{T}_{1}\right)=\frac{1}{\lambda}\left[\begin{array}{l}\frac{\mathrm{p}\left(\mu+\alpha_{1}+\alpha_{2}\right)}{\alpha_{1}+\alpha_{2}}+\frac{\mathrm{q}\left(\mu+\alpha_{3}\right)}{\alpha_{3}}+\frac{\theta(\mu+\beta)}{\beta}-\frac{\theta \mathrm{p}\left(\mu+\alpha_{1}+\alpha_{2}\right)(\mu+\beta)}{\mu\left(\alpha_{1}+\alpha_{2}\right)+\beta\left(\mu+\alpha_{1}+\alpha_{2}\right)} \\ -\frac{\theta \mathrm{q}\left(\mu+\alpha_{3}\right)(\mu+\beta)}{\mu\left(\alpha_{3}+\beta\right)+\alpha_{3} \beta}\end{array}\right]$
$\mathrm{E}\left(\mathrm{T}_{1}^{2}\right)=\frac{2}{\lambda^{2}}\left[\begin{array}{l}\frac{p\left(\mu+\alpha_{1}+\alpha_{2}\right)^{2}}{\left(\alpha_{1}+\alpha_{2}\right)^{2}}+\frac{q\left(\mu+\alpha_{3}\right)^{2}}{\alpha_{3}^{2}}+\frac{\theta(\mu+\beta)^{2}}{\beta^{2}}-\frac{\theta p\left[\left(\mu+\alpha_{1}+\alpha_{2}\right)(\mu+\beta)\right]^{2}}{\left[\mu\left(\alpha_{1}+\alpha_{2}\right)+\beta\left(\mu+\alpha_{1}+\alpha_{2}\right)\right]^{2}} \\ -\frac{\theta q\left[\left(\mu+\alpha_{3}\right)(\mu+\beta)\right]^{2}}{\left[\mu\left(\alpha_{3}+\beta\right)+\alpha_{3} \beta\right]^{2}}\end{array}\right]$
$V\left(T_{1}\right)=E\left(T_{1}^{2}\right)-\left[E\left(T_{1}\right)\right]^{2}$
(10) gives the mean time for recruitment. (11) together with (12) gives the variance of the time to recruitment.

## Model description and analysis for model II

The model description is same as that of model I except for the following change, Let $Y$ be a continuous random variable denoting the optional threshold following exponential distribution with parameter $\beta$. Let $Z$ be a continuous random variable denoting the mandatory threshold having SCBZ property such that $Z>Y$. Let $\mathrm{T}_{2}$ be a continuous random variable denoting the time for recruitment in the organization with probability density function $1_{2}(t)$ and cumulative distribution function $L_{2}($.$) .$ Proceeding as in model I, it can be shown that

$$
1_{2}^{*}(s)=\begin{align*}
& \frac{\lambda\left[1-g^{*}(\beta)\right]}{\lambda+s-\lambda g^{*}(\beta)}+\frac{\theta p \lambda\left[1-g^{*}\left(\alpha_{1}+\alpha_{2}\right)\right]}{\lambda+s-\lambda g^{*}\left(\alpha_{1}+\alpha_{2}\right)}+\frac{\theta q \lambda\left[1-g^{*}\left(\alpha_{3}\right)\right]}{\lambda+s-\lambda g^{*}\left(\alpha_{3}\right)}  \tag{14}\\
& -\frac{\theta p \lambda\left[1-g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)\right]}{\lambda+s-\lambda g^{*}\left(\alpha_{1}+\alpha_{2}\right) g^{*}(\beta)}-\frac{\theta q \lambda\left[1-g^{*}\left(\alpha_{3}\right) g^{*}(\beta)\right]}{\lambda+s-\lambda g^{*}\left(\alpha_{3}\right) g^{*}(\beta)}
\end{align*}
$$

The mean and variance of the time to recruitment is found to be

$$
\begin{align*}
& \mathrm{E}\left(\mathrm{~T}_{2}\right)=\frac{1}{\lambda}\left[\begin{array}{l}
\frac{(\mu+\beta)}{\beta}+\frac{\theta \mathrm{p}\left(\mu+\alpha_{1}+\alpha_{2}\right)}{\alpha_{1}+\alpha_{2}}+\frac{\theta \mathrm{q}\left(\mu+\alpha_{3}\right)}{\alpha_{3}}-\frac{\theta \mathrm{p}\left(\mu+\alpha_{1}+\alpha_{2}\right)(\mu+\beta)}{\mu\left(\alpha_{1}+\alpha_{2}\right)+\beta\left(\mu+\alpha_{1}+\alpha_{2}\right)} \\
-\frac{\left.\theta \mathrm{q}\left(\mu+\alpha_{3}\right)(\mu+\beta)\right)^{2}}{\mu\left(\alpha_{3}+\beta\right)+\alpha_{3} \beta}
\end{array}\right]  \tag{15}\\
& \mathrm{E}\left(\mathrm{~T}_{2}{ }^{2}\right)=\frac{1}{\lambda}\left[\begin{array}{l}
\left.\frac{(\mu+\beta)^{2}}{\beta^{2}}+\frac{\theta \mathrm{p}\left(\mu+\alpha_{1}+\alpha_{2}\right)^{2}}{\left(\alpha_{1}+\alpha_{2}\right)^{2}}+\frac{\theta \mathrm{q}\left(\mu+\alpha_{3}\right)^{2}}{\alpha_{3}^{2}}-\frac{\theta \mathrm{p}\left[\left(\mu+\alpha_{1}+\alpha_{2}\right)(\mu+\beta)\right]^{2}}{\left[\mu\left(\alpha_{1}+\alpha_{2}\right)+\beta\left(\mu+\alpha_{1}+\alpha_{2}\right)\right]^{2}}\right] \\
\mathrm{V}\left(\mathrm{~T}_{2}\right)=\mathrm{Eq}\left(\left(\mu+\alpha_{2}{ }^{2}\right)-\left[\mathrm{E}\left(\mathrm{~T}_{2}\right)\right]^{2}\right. \\
{\left[\mu\left(\alpha_{3}+\beta\right)+\alpha_{3} \beta\right]^{2}}
\end{array}\right. \tag{16}
\end{align*}
$$

## Numerical Illustration and Conclusions

The following table gives the mean and variance of the time to recruitment for both the models when $\alpha_{1}=100, \alpha_{2}=50, \alpha_{3}=30, \lambda=0.2$ are fixed for both the models and fixing $\beta=0.01$ for model I and $\beta=0.0166$ for model II and varying $\theta$ and $\mu$ simultaneously.

## Table 1

| $\theta / \mu$ |  | 0.04 | 0.05 | 0.0667 | 0.10 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.1 | $E\left(T_{1}\right)$ | 7.0033 | 7.5041 | 8.3387 | 10.0081 |
|  | $V\left(T_{1}\right)$ | 121.0197 | 168.7704 | 269.4626 | 550.0002 |
|  | $E\left(T_{2}\right)$ | 17.0483 | 20.0604 | 25.0804 | 35.1206 |
|  | $V\left(T_{2}\right)$ | 290.6394 | 402.4110 | 629.0190 | $1.233 \mathrm{e}+003$ |
| 0.3 | $E\left(T_{1}\right)$ | 11.0027 | 12.5033 | 15.0044 | 20.0065 |
|  | $V\left(T_{1}\right)$ | 288.9946 | 418.7334 | 691.6229 | $1.449 \mathrm{e}+003$ |
|  | $E\left(T_{2}\right)$ | 17.0485 | 20.0606 | 25.0807 | 35.1209 |
|  | $V\left(T_{2}\right)$ | 290.6365 | 402.4066 | 629.0118 | $1.233 \mathrm{e}+003$ |
| 0.5 | $E\left(T_{1}\right)$ | 15.0021 | 17.5026 | 21.6700 | 30.0048 |
|  | $V\left(T_{1}\right)$ | 424.9787 | 618.7111 | $1.024 \mathrm{e}+003$ | $2.149 \mathrm{e}+003$ |
|  | $E\left(T_{2}\right)$ | 17.0487 | 20.0608 | 25.0809 | 35.1211 |
|  | $V\left(T_{2}\right)$ | 290.6335 | 402.4021 | 629.0047 | $1.233 \mathrm{e}+003$ |


| 0.7 | $E\left(T_{1}\right)$ | 19.0016 | 22.5019 | 28.3357 | 40.0032 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $V\left(T_{1}\right)$ | 528.9719 | 768.7037 | $1.269 \mathrm{e}+003$ | $2.649 \mathrm{e}+003$ |
|  | $E\left(T_{2}\right)$ | 17.0489 | 20.0610 | 25.0811 | 35.1214 |
|  | $V\left(T_{2}\right)$ | 290.6306 | 402.3977 | 628.9976 | $1.233 \mathrm{e}+003$ |

From the above table, we observe the following:

1. When the probability ( $\theta$ ) for not going for recruitment when the optional threshold level is crossed increases as well as the average loss of manhour $(1 / \mu)$ increases simultaneously (keeping other parameters fixed) the mean and variance of the time to recruitment increase for models I and II.
2. As $\theta$ alone increases, the mean and variance of the time to recruitment increase for models I and II.
3. As $\mu$ alone increases, the mean and variance of the time to recruitment increase for models I and II.

In table 2 given below, the mean and variance of the time to recruitment for both the models are tabulated when $\alpha_{1}=100, \alpha_{2}=50, \alpha_{3}=30, \mu=0.2$ are fixed for both the models and fixing $\beta=0.01$ for model I and $\beta=0.0166$ for model II and varying $\theta$ and $\lambda$ simultaneously.

## Table 2

| $\lambda / \theta$ |  | 0.1 | 0.3 | 0.5 | 0.7 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.1 | $E\left(T_{1}\right)$ | 30.0322 | 70.0254 | 110.0186 | 150.0119 |
|  | $V\left(T_{1}\right)$ | $8.0994 \mathrm{e}+003$ | $2.1697 \mathrm{e}+004$ | $3.2097 \mathrm{e}+004$ | $3.9297 \mathrm{e}+004$ |
|  | $E\left(T_{2}\right)$ | 130.4822 | 130.4827 | 130.4833 | 130.4838 |
|  | $V\left(T_{2}\right)$ | $1.7025 \mathrm{e}+004$ | $1.7025 \mathrm{e}+004$ | $1.7025 \mathrm{e}+004$ | $1.7025 \mathrm{e}+004$ |
| 0.2 | $E\left(T_{1}\right)$ | 15.0161 | 35.0127 | 55.0093 | 75.0059 |
|  | $V\left(T_{1}\right)$ | $2.0248 \mathrm{e}+003$ | $5.4244 \mathrm{e}+003$ | $8.0242 \mathrm{e}+003$ | $9.8242 \mathrm{e}+003$ |
|  | $E\left(T_{2}\right)$ | 65.2411 | 65.2414 | 65.2416 | 65.2419 |
|  | $V\left(T_{2}\right)$ | $4.2564 \mathrm{e}+003$ | $4.2563 \mathrm{e}+003$ | $4.2563 \mathrm{e}+003$ | $4.2563 \mathrm{e}+003$ |
| 0.3 | $E\left(T_{1}\right)$ | 10.0107 | 23.3418 | 36.6729 | 50.0040 |
|  | $V\left(T_{1}\right)$ | 899.9288 | $2.4108 \mathrm{e}+003$ | $3.5663 \mathrm{e}+003$ | $4.3663 \mathrm{e}+003$ |
|  | $E\left(T_{2}\right)$ | 43.4941 | 43.4942 | 43.4944 | 43.4946 |
|  | $V\left(T_{2}\right)$ | $1.8917 \mathrm{e}+003$ | $1.8917 \mathrm{e}+003$ | $1.8917 \mathrm{e}+003$ | $1.8917 \mathrm{e}+003$ |
| 0.4 | $E\left(T_{1}\right)$ | 7.5080 | 17.5063 | 27.5047 | 37.5030 |
|  | $V\left(T_{1}\right)$ | 506.2099 | $1.3561 \mathrm{e}+003$ | $2.0060 \mathrm{e}+003$ | $2.4561 \mathrm{e}+003$ |


|  | $E\left(T_{2}\right)$ | 32.6206 | 32.6207 | 32.6208 | 32.6210 |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | $V\left(T_{2}\right)$ | $1.0641 \mathrm{e}+003$ | $1.0641 \mathrm{e}+003$ | $1.0641 \mathrm{e}+003$ | $1.0641 \mathrm{e}+003$ |

From table2, we observe the following:

1. When the probability ( $\theta$ ) for not going for recruitment when the optional threshold level is crossed increases as well as the average inter-decision time ( $1 / \lambda$ ) decreases simultaneously mean and variance of the time to recruitment increase for model I and decreases for model II.
2. As $\theta$ alone increases, mean and variance of the time to recruitment increase for model I and II.
3. As $\lambda$ alone increases, mean and variance of the time to recruitment decrease for model I and II.

The following table gives the mean and variance of the time to recruitment for both the models when $\alpha_{1}=100, \alpha_{2}=50, \alpha_{3}=30, \theta=0.5$ fixed for both the models and fixing $\beta=0.01$ for model I and $\beta=0.0166$ for model II and varying $\lambda$ and $\mu$ simultaneously

Table 3

| $\lambda / \mu$ |  | 0.04 | 0.05 | 0.0667 | 0.1 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| 0.1 | $E\left(T_{1}\right)$ | 30.0043 | 35.0052 | 43.3400 | 60.0097 |
|  | $V\left(T_{1}\right)$ | $1.6999 \mathrm{e}+003$ | $2.4748 \mathrm{e}+003$ | $4.0997 \mathrm{e}+003$ | $8.5992 \mathrm{e}+003$ |
|  | $E\left(T_{2}\right)$ | 34.0974 | 40.1216 | 50.1618 | 70.2422 |
|  | $V\left(T_{2}\right)$ | $1.1625 \mathrm{e}+003$ | $1.6096 \mathrm{e}+003$ | $2.5160 \mathrm{e}+003$ | $4.9337 \mathrm{e}+003$ |
| 0.2 | $E\left(T_{1}\right)$ | 15.0021 | 17.5026 | 21.6700 | 30.0048 |
|  | $V\left(T_{1}\right)$ | 424.9787 | 618.7111 | $1.0249 \mathrm{e}+003$ | $2.1498 \mathrm{e}+003$ |
|  | $E\left(T_{2}\right)$ | 17.0487 | 20.0608 | 25.0809 | 35.1211 |
|  | $V\left(T_{2}\right)$ | 290.6335 | 402.4021 | 629.0047 | $1.2334 \mathrm{e}+003$ |
| 0.3 | $E\left(T_{1}\right)$ | 10.0014 | 11.6684 | 14.4467 | 20.0032 |
|  | $V\left(T_{1}\right)$ | 188.8794 | 274.9827 | 455.5208 | 955.4694 |
|  | $E\left(T_{2}\right)$ | 11.3658 | 13.3739 | 16.7206 | 23.4141 |
|  | $V\left(T_{2}\right)$ | 129.1705 | 178.8454 | 279.5576 | 548.1851 |
| 0.4 | $E\left(T_{1}\right)$ | 7.5011 | 8.7513 | 10.8350 | 15.0024 |
|  | $V\left(T_{1}\right)$ | 106.2447 | 154.6778 | 256.2305 | 537.4515 |
|  | $E\left(T_{2}\right)$ | 8.5244 | 10.0304 | 12.5405 | 17.5606 |
|  | $V\left(T_{2}\right)$ | 72.6584 | 100.6005 | 157.2512 | 308.3541 |

From table 3, we observe the following:

1. When the average inter-decision times $(1 / \lambda)$ decreases as well as the average loss of manhours $(1 / \mu)$ decreases simultaneously, mean and variance of the time to recruitment decrease for model I and II.
2. As $\mu$ alone increases, mean and variance of the time to recruitment increase for model I and II.
3. As $\lambda$ alone increases, mean and variance of the time to recruitment decrease for model I and II.

## References

[1] Barthlomew.D.J and Forbes,A.,"Statistical techniques for manpower planning", John Wiely and Sons, 1979.
[2] Elangovan,R. and Sathiyamoorthi, R. "A Shock model approach to determine the expected time for recruitment", Journal of Decision and Mathematical Sciences,Vol 2,No.1-3, pp67-68,1998.
[3] Esther Clara.J.B. and Srinivasan,A, "Expected time for recruitment in a single graded manpower system with two thresholds", Proceedings of the AICTE Sponsored National Conference On Recent Developments and Applications of Probability theory, Random Process and Random Variables in Computer Science, pp. 98-102, 2008.
[4] Grinold,R.C and Marshall,K.J, "Manpower planning", North Holland, New York, 1977.
[5] Kasturri.K and Srinivasan.A,"Expected time to recruitment for correlated inter- decision time of exits when threshold distribution has SCBZ property", Acta Ciencia Indica.Vol. XXXI M. No.1, pp 277-283, 2005.
[6] Parthasarathy,S and Sathiyamoorthi, R., "On the expected time to recruitment when threshold distribution has SCBZ property", International Journal of Management and systems,Vol.19,No.3, pp 233-240, 2003.
[7] Rao, B.R. and Talwalker,S., "Setting the Clock Back to Zero Property for a class of life distribution", Journal of Statistical Planning and Inference,Vol. 24, pp 347-352, 1990.
[8] Saavithri.V and Srinivasan.A, "Expected time to recruitment on some univariate policies of recruitment in manpower models", Proceedings of the First National Conference on Mathematical \& Computational Models, PSG Tech, Coimbatore, pp 41-46, 2001.
[9] Srinivasan.A, Uma.K.P, Udaya Chandrika.K and Saavithri.V, "Expected time to recruitment for an univariate policy when threshold distribution has SCBZ property" Proceedings of the Third National Conference on Mathematical and Computational Models, PSG College of Technology, Coimbatore, pp 242246,2005.

