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Discrete Time, Cost and Quality Trade-Off Problem with Renewable and Nonrenewable Resources

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Abstract

The discrete time –cost trade-off problem (DTCTP) is one of the main aspects of project scheduling. In DTCTP , we treat cost as a non-renewable resource and the availability of renewable resource per period has been neglected. It was recently suggested that the quality of project should also be taken into consideration. Therefore, when renewable resources and quality are considered, the traditional DTCTP is extended to a new discrete resource quality constrained time cost-trade off problem (DRQTCTP), which involves renewable resources, non renewable resources and quality constraints. The objective of DRQTCTP involves the scheduling of project activities in order of minimizing the total cost of the project while maximizing the quality of the project and also meet a given deadline. An example is given for the DRQTCTP and it is observed that when subjected to same amount of non-renewable resources, the optimal project cost in DRQTCTP is higher than that in traditional DTCTP but there is no change in quality.

Keywords: Time –cost trade-off, discrete time, Project management, cost and quality problem.

Introduction

Since the late 1950s, critical path method (CPM) techniques have become widely recognized as valuable tools for the planning and scheduling of projects. But most of the cases, project should be implemented before the date calculated by CPM method. To achieve this goal, more sophisticated equipments or employment of more human resources can be used. Finding the most cost- effective way to complete a project with in a specific completion time is desirable for schedule planners. Therefore, for the project to be completed with the least possible amount of time and cost, obtaining a logical trade-off between cost and project duration is necessary. Several mathematical and heuristic models have been developed to solve time-cost trade-off problems [1] .

In the early time-cost trade-off models the direct activity cost function was assumed to be linear non-increasing function of its duration. The objective was usually to deliver the project with In a specified dead line while minimizing the projects direct and indirect costs. Solution procedures for the linear case are given in [2-7]. Concave cost functions were considered in [8], while convex cost functions were studied in [9]. In DTCTP, cost is considered as a non renewable resources and the availability of renewable resources per period has been neglected. Quality of a completed project is also affected by project crashing [10]. Although the DTCTP has been studied for nearly thirty years, there are still some drawbacks to the demands of project management. DTCTP focuses on nonrenewable resources while the renewable resources have not been attached and the direct costs were considered while the indirect costs were neglected. Hence a new problem that considers project's time, renewable , non-renewable cost and quality simultaneously, here after referred to as DRQDCTP, need to be considered.

In this paper, a new DRQDCTP model is introduced and solution procedure for the DRQTCTP is presented. In section 2, we present the problem and present the mathematical models. In section 3, we present our solution procedure. In section 4, we present a numerical example for our model.

Problem formulation

A project is defined as a directed acyclic graph G=(V,A) in which V is the set of nodes and A is the set of arcs. In this case, the project is modeled by an activity on arc network(AOA) where its arcs represent project activities and its nodes define specific events. Each project activity say (i,j), has different modes of execution of which mode k requires t_{ijk} time , c_{ijk} cost and q_{ijk} quality respectively.

It is assumed that if k and r are two alternatives for activity (i,j) such that of k < s there exists two relationships 1) if $t_{ijk} < t_{ijs}$ then $c_{ijk} > c_{ijs}$ and $q_{ijk} < q_{ijs}$; 2) if $t_{ijk} > t_{ijs}$ then $c_{ijk} < c_{ijs}$ and $q_{ijk} > q_{ijs}$; only one set of 1 or 2 will be satisfied. The quality level ($0< q_{ijk} <1$) assigned to each activity depends on its nature. The objective is to construct the complete and efficient time , cost and quality profile to offer decision support in crashing a project. We have developed three mathematical models that maximize the quality of the project while minimizing its duration and costs by considering renewable , non-renewable costs.

Notation

x_{ijk} = 1 if activity (i,j) is executed in mode k
= 0 otherwise
m= number of modes
t_{ijk} = duration of activity (i,j) in mode k

r_{ijk} = the amount of renewable resource r
P_r = price of the renewable resource r
c_{ijk} = cost of the activity (i,j) = $t_{ijk} \times r_{ijk} \times P_r$ + the cost of non renewable resources
q_{ijk} = quality of activity (i,j) in mode k
N_i = { E_i , E_i+1, …, L_i}, where E_i is the earliest time of occurrence for event i and L_i is the latest time of occurrence for event i.
y_{iu} = 1 if event i occurs in time u
= 0 otherwise
M_T = maximum time to complete the project
B_{max} = maximum available budget
l_{ij} = lower bound for quality of activity (i,j)

The two mathematical models using 0-1 integer linear programming are

i. Min $Z_1 = \sum_{k=1}^{m} c_{ijk} x_{ijk}$

Subject to

$$\sum_{k=1}^{m} x_{ijk} = 1$$

$$\sum_{u \in N_j} u y_{ju} - \sum_{w \in N_i} w y_{iw} \geq \sum_{k=1}^{m} t_{ijk} x_{ijk}$$

$$\sum_{u \in N_n} u y_{nu} \leq M_T ,$$

$$\sum_{k=1}^{m} q_{ijk} x_{ijk} \geq l_{ij}$$

where (i,j) is an activity , x_{ijk} = 0 or 1 for each activity(i,j) executed in each mode k and y_{iu}= 0 or 1 for each event i and for each u $\in N_i$,

ii. Max $z_2 = \left(\prod_{(i,j) \in A} \left(\sum_{k=1}^{m} q_{ijk} x_{ijk} \right) \right)^{\frac{1}{|A|}}$,

Subject to constraints

$$\sum_{k=1}^{m} x_{ijk} = 1$$

$$\sum_{u \in N_j} u y_{ju} - \sum_{w \in N_i} w y_{iw} \geq \sum_{k=1}^{m} t_{ijk} x_{ijk}$$

$$\sum_{u\in N_n} u y_{nu} \le M_T \,,$$

$$\sum_{(i,j)\in A}\sum_{k=1}^{m} c_{ijk} x_{ijk} \le B_{\max} \,,,$$

$$\sum_{k=1}^{m} q_{ijk} x_{ijk} \ge l_{ij}$$

where (i,j) is an activity , x_{ijk} = 0 or 1 for each activity(i,j) executed in each mode k and y_{iu}= 0 or 1 for each event i and for each $u \in N_i$,

Solution Procedure

The method of solving DRQTCTP is more complicated than the traditonal DTCTP. Considering the constraints involving budget and time, we use a preprocessing procedure to reduce the solution space in two integer linear programming models, this procedure is introduced by Sprecher et al [11]. If the constraint involving budget in one model is neglected the size of complete solution space is reduced. Here, we have used software TORA to solve the linear programming models.

Numerical Example

An activity on the arc (AOA) network G= (V,A) is given in Fig.1 where V={1,2,3,4,5,6,7} is the set of nodes and A is the set of arcs representing the activities. The table I represents time, nonrenewable cost, quality of each activity of Fig.1. with 5 modes. Each activity can be executed in five modes. In order to validate the DRQTCTP model, we simply reprocess the project data based on the original activity (time, renewable resource, cost , quality). In each execution mode (k=1,2,3,4,5), cost = activity duration × renewable resource requirements × price + requirements for non renewable resources. Here we consider renewable resource per period is 4 (i.e., r_{ijk}= 4) and price per period is 6(i.e., P_r=6). We have calculated the cost using above requirements and is presented in Table II. In the proposed model budget is considered as B_{max} = 2500 and l_{ij} =0. For traditional DTCTP using table I , Minimum of Z_1 is 124 and for DRQTCTP using table II, Minimum of Z_1 is 1579. For traditional DTCTP using table I , Maximum of Z_2 is 0.91 and for DRQTCTP using table II, Maximum of Z_2 is 0.91. . The result obtained in this example, when subjected to same amount of non-renewable resources, the optimal project cost in DRQTCTP is higher than that in traditional DTCTP but there is no change in quality for two time –cost trade-off problems.

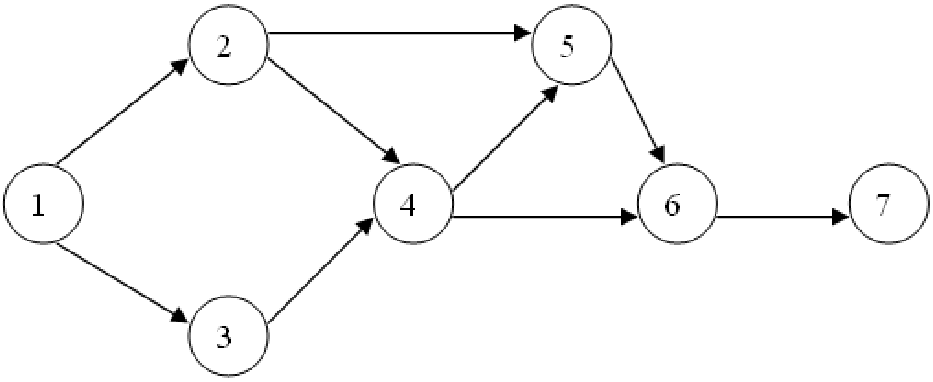

Fig. 1 : Project Network

Table I : Time, Nonrenewable costs, quality of each activity

Activity	Mode-1			Mode-2			Mode-3			Mode-4			Mode-5		
	t	c	q	t	c	q	t	c	q	t	c	q	t	c	q
1-2	7	16	0.90	6	18	0.85	5	19	0.80	4	20	0.70	3	23	0.85
1-3	8	14	0.85	7	15	0.82	6	17	0.80	5	18	0.75	4	20	0.80
2-4	10	11	0.88	9	12	0.90	8	14	0.85	7	15	0.75	6	17	0.80
2-5	14	10	0.92	13	13	0.90	12	14	0.86	11	15	0.70	10	16	0.80
3-4	8	16	0.90	7	17	0.85	6	18	0.84	5	20	0.80	4	22	0.90
4-5	11	13	0.91	10	14	0.88	9	15	0.85	8	17	0.75	7	19	0.96
4-6	11	15	0.87	10	18	0.90	9	19	0.85	8	20	0.90	7	26	0.85
5-6	8	14	0.85	7	15	0.82	6	16	0.80	5	17	0.85	4	20	0.90
6-7	11	15	0.90	10	17	0.88	9	18	0.85	8	20	0.90	7	18	0.85

Table II : Time, quality and cost using renewable resource for each activity

Activity	Mode-1			Mode-2			Mode-3			Mode-4			Mode-5		
	T	c	q	t	c	q	t	c	q	t	c	q	t	c	q
1-2	7	184	0.90	6	162	0.85	5	139	0.80	4	116	0.70	3	95	0.85
1-3	8	206	0.85	7	183	0.82	6	161	0.80	5	138	0.75	4	116	0.80
2-4	10	251	0.88	9	228	0.90	8	206	0.85	7	183	0.75	6	161	0.80
2-5	14	346	0.92	13	325	0.90	12	302	0.86	11	279	0.70	10	256	0.80
3-4	8	208	0.90	7	185	0.85	6	162	0.84	5	140	0.80	4	118	0.90
4-5	11	277	0.91	10	254	0.88	9	231	0.85	8	209	0.75	7	187	0.96
4-6	11	279	0.87	10	258	0.90	9	235	0.85	8	212	0.90	7	194	0.85
5-6	8	206	0.85	7	183	0.82	6	160	0.80	5	137	0.85	4	116	0.90
6-7	11	279	0.90	10	257	0.88	9	234	0.85	8	212	0.90	7	186	0.85

Conclusions

The traditional DTCTP only treats cost as a nonrenewable resource constraints. In this paper, we have developed two integer linear programming models considering renewable resource constraints and quality constraints, so the traditional DTCTP has been extended to a new discrete resource quality constrained time cost-trade off problem (DRQTCTP), which involves renewable resources, non renewable resources and quality constraints. After a preprocessing procedure used to reduce the solution space of DRQTCTP, we adopt the enumeration arithmetic to find the optimal solution for an example. The result is, when subjected to same amount of non-renewable resources, the optimal project cost in DRQTCTP is higher than that in traditional DTCTP but there is no change in quality.

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