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Optimum Replenishment Policies for Time Decaying Items with Selling Price and Stock Dependent Demand

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Abstract

In the classical inventory it is assumed that all the costs associated with the inventory system remains constant over time. Since most decision makers think that the inflation does not have significant influence on the inventory policy and most of the inventory models developed so far does not include inflation and time value of money as parameters of the system. But due to large scale of inflation the monetary situation in almost of the countries has changed to an extent during the last thirty years. Nowadays inflation has become a permanent feature in the inventory system. Inflation enters in the picture of inventory only because it may have an impact on the present value of the future inventory cost. Thus the inflation plays a vital role in the inventory system and production management though the decision makers may face difficulties in arriving at answers related to decision-making. In this paper an inventory model for deteriorating items and the effects of inflation and lifetime have been developed. An increase in the deterioration factors poses a decreases in optimal time as well as the selling price of the commodity. The backlogging parameter is found to be slightly increased the optimal time and the optimal selling price. But an increase in this parameter adversely affects the net profit of our system.

Keywords: Inflation, Time Value of Money, Partial Backlogging

Introduction

At present, it is impossible to ignore the effects of inflation and it is necessary to consider the effects of inflation on the inventory system. The initial attempt in this direction was made by Buzacott (1975). He dealt with an EOQ model under inflation subject to different types of pricing policies. In the subsequent year, Bierman *et al.* (1977**)** showed that the inflation rate does not affect the optimal order quantity perse; rather, the difference between the inflation rate and the discount rate affects on it. Misra (1975) investigated inventory systems under the effects of inflation. Economic analysis of dynamic inventory models with non-stationary costs and demand was presented by Hariga (1994**)**. The effect of inflation was also considered in this analysis. Chang (2004) proposed an inventory model under a situation in which the supplier has provided a permissible delay in payments to the purchaser if the ordering quantity is greater than or equal to a predetermined quantity. Shortage was not allowed and the effect of the inflation rate, deterioration rate and delay in payments were discussed as well. An economic order quantity inventory model for deteriorating items was developed by Bose *et al*. (1995). Authors developed inventory model with linear trend in demand allowing inventory shortages and backlogging. The effects of inflation and time-value of money were incorporated into the model. Ray and Chaudhuri (1997) developed a finite time-horizon deterministic economic order quantity inventory model with shortages, where the demand rate at any instant depends on the on-hand inventory at that instant. The effects of inflation and time value of money were taken into account. A generalized dynamic programming model for inventory items with Weibull distributed deterioration was proposed by Chen (1998). The demand was assumed to be time-proportional, and the effects of inflation and time-value of money were taken into consideration. Shortages were allowed and partially backordered. The effects of inflation and time-value of money on an economic order quantity model have been discussed by Moon and Lee (2000). The two-warehouse inventory models for deteriorating items with constant demand rate under inflation were developed by Yang (2004). Shortages were allowed and fully backlogged in the models. Some numerical examples for illustration were provided. Though a considerable number of research work has been done in this area Dey *et al.* (2004) analyzed an EOQ model with fuzzy inflation and time value of money. Jaggi *et al.* (2006) presented the optimal inventory replenishment policy of deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite planning horizon. The demand rate is assumed to be a function of inflation; shortages are allowed and completely backlogged. Jaggi *et al*. (2007) discussed the optimal inventory replenishment policy for deteriorating items under inflationary conditions using a discounted cash flow (DCF) approach over a finite time horizon with shortages and demand rate was assumed to be a function of inflation. Roy *et al*. (2008) studied an inventory model for a deteriorating item with linearly displayed stock dependent demand in imprecise environment under inflation and time value of money, only few of them have considered these as fuzzy quantities. Two stage inventory problem over finite time horizon under inflation and time value of money was discussed by Dey *et al.* (2008). Mirzazadeh *et al*. (2009) developed the inventory system with finite replenishment rate, finite time horizon and deteriorating items and the demand rate is dependent to the inflation rates.

This paper is concerned with the development of inventory models for deteriorating items and the effects of inflation and life time. The demand is dependent on selling price and inventory level. Shortages are allowed and partial backlogging of unsatisfied demand is considered. The partial backlogging depends on the length of the waiting time for the next replenishment. Numerical illustration also proves the validity of the model.

Assumptions and Notations

The following assumptions have been adopted for the proposed model to be discussed:

1. Single item inventory is considered.
2. Replenishment is instantaneous.
3. Deterioration of the items starts after a definite time.
4. Deterioration rate taken as linear time dependent function.
5. There is no repair or replenishment of deteriorating items during the period under consideration.
6. The demand rate is a function of on hand inventory and depend on the selling price.
7. Inflation and time value of money are considered.
8. Product transactions are followed by instantaneous cash flow.
9. Shortages are allowed with partial backlogging.
10. The model has been developed for a finite planning horizon.

Notations

$I(t)$	Inventory level at any time t,
$(a+bt)$	Linear time dependent deterioration rate function
λ	Lifetime of the commodity.
$D(t)=\eta-\alpha s+\gamma I(t)$	Demand rate (units/unit time), η, α and γ are positive constants.
$1/\left[1+\delta(T-t)\right]$	Backlogging parameter of the items, $,\delta\geq 0$, t is waiting time and $T_1 \leq t \leq T$
A	Ordering cost per cycle, \$/cycle.
$(c_1+\phi t)$	Inventory holding cost per unit item per unit time, \$/unit item/unit time
C_2	Shortage cost per unit item backordered per unit time, \$/unit item/unit time
C_3	Lost sale cost per unit item per unit item, \$/unit item
s	Per unit selling price of the item \$/unit item
r	Constant representing the difference between the real interest rate and nominal interest rate
c	Purchasing cost per unit item, \$/unit item
H	Planning horizon of the model

T	Replenishment cycle time
N	Number of replenishments of the inventory, where N = H/T.

Model Formulation and Solution

The planning horizon has been divided into *N* equal cycles, such that the length of each cycle is exactly $T = H/N$. Hence, the re-order points are *kT*, where, $k=0, 1, 2,\ldots, (N-1)$. Each replenishment cycle can be further divided into four periods. The cycle starts with an initial inventory level of I_0 units. From initial time $t = 0$ to $t = \lambda$, any infinitesimal change in the inventory level is due to demand only, as during this period, there is no deterioration or decay of the inventory. After λ time period has elapsed, deterioration sets in, and the inventory level now reduces more rapidly due to both demand and deterioration, until it reaches zero level at time $t = T_1$. Now shortages are accumulated which are partially backlogged depending upon the waiting time for the next replenishment. At the end of the cycle, the inventory reaches a maximum shortage level I_s, and the next order is placed to clear the backlog and again raise the inventory level to I_0.

Mathematically, the system can be represented by the following system of differential equations:

$$I'(t) = -(\eta - \alpha s + \gamma I(t))$$
$$0 \le t \le \lambda \tag{1}$$

$$I'(t) + (a + bt)I(t) = -(\eta - \alpha s + \gamma I(t))$$
$$\lambda \le t \le T_1 \tag{2}$$

$$I'(t) = \frac{-(\eta - \alpha s + \gamma I(t))}{1 + \delta(T - t)}$$
$$T_1 \le t \le T \tag{3}$$

Solving the above equations by using the boundary value conditions, $I(0) = I_0$, $I(T_1) = 0$ and $I(T_1) = 0$ we find:

$$I(t) = e^{-\gamma t}\left[\frac{-(\eta - \alpha s)}{\gamma}\left(e^{\gamma t} - 1\right) + I_0\right]$$
$$0 \le t \le \lambda \tag{4}$$

$$I(t) = \left[(\eta - \alpha s)\left(T_1 - t + \frac{(a+\gamma)}{2}\left(T_1^2 - t^2\right) + \frac{b}{6}\left(T_1^3 - t^3\right)\right)\right]e^{-(a+\gamma)t - \frac{bt^2}{2}}$$
$$\lambda \le t \le T_1 \tag{5}$$

$$I(t) = \frac{-(\eta - \alpha s)}{1 + \gamma(T - T_1)}\left(t - T_1\right)$$
$$T_1 \le t \le T \tag{6}$$

Equating the equations (4) and (5) at $t=\lambda$ we obtain :

$$I_0 = e^{-\alpha\lambda^{\beta}}(-\eta+\theta s)\left[\lambda - T_1 + \frac{\gamma}{2}\left(\lambda^2 - T_1^2\right) + \frac{\alpha}{\beta+1}\left(\lambda^{\beta+1} - T_1^{\beta+1}\right)\right] - (-\eta+\theta s)\frac{\left(e^{\lambda\gamma}-1\right)}{\gamma} \tag{7}$$

Present worth ordering cost

Order is placed at the beginning of each cycle and hence for every cycle,

$$OC = A \tag{8}$$

Present worth item cost

Inventory is bought at two instances, one at the beginning of the cycle and one at t = T, when shortages need to be cleared. Hence,

$$IC = cI_0 + ce^{-rT}\int_{T_1}^{T} D(t)dt \tag{9}$$

Present worth holding cost

Inventory is available during $[0,\lambda]$ and $[\lambda, T_1]$. Hence the holding cost needs to be computed during these two time periods.

$$HC = (c_1 + \phi t)\int_0^{\lambda} I(t)e^{-rt}dt + (c_1 + \phi t)\int_{\lambda}^{T_1} I(t)e^{-r(\lambda+t)}dt \tag{10}$$

Present worth shortage cost

Shortages are accumulated in the system during $[T_1, T]$. The maximum level of shortages are present at t = T. The total present worth of shortages during this time is,

$$SC = C_2\int_{T_1}^{T}(-I(t))e^{-r(T_1+t)}dt \tag{11}$$

Present worth lost sales cost

The backlogging parameter *B* renders a lost sale fraction of (1-B) to the system during the stock-out period. The total opportunity cost for one cycle becomes,

$$LC = C_3\int_{T_1}^{T}\left(1 - \frac{1}{1+\delta(T-t)}\right)D(t)e^{-rT}dt \tag{12}$$

Present worth sales profit

Since the inventory is available for sale during $[0,\lambda]$ and $[\lambda, T_1]$ and at time T, profit can be gained in this time only. The present worth of profit gained during this time is obtained by the following expression,

$$SP = s\int_0^{\lambda} D(t)e^{-rt}dt + se^{-r\lambda}\int_{\lambda}^{T_1} D(t)e^{-rt}dt + se^{-rT}\int_{T_1}^{T} D(t)e^{-rt}dt \tag{13}$$

Present worth total profit

The present worth of net profit is found by deducting various costs from the sales profit. Using the equations from (8) to (13), we get,

$$P = SP - OC - IC - HC - SC - LC \tag{14}$$

The total planning horizon *H* has been divided into *N* equal cycles. The inventory starts at an initial level of I_0 units and ends at zero in the final cycle. Hence, to satisfy the unfulfilled backlogged demand at the end of the last cycle, an extra replenishment is required at $t = H$. The total number of replenishments becomes $N+1$.

The first lot size is equal to I_0

$2^{nd}, 3^{rd}, ..., N^{th}$ lot size is equal $I_0 + \int_{T_1}^{T} D(t)dt$

$(N+1)^{th}$ lot size is equal to $\int_{T_1}^{T} D(t)dt$

$$TP = P\left(1 + e^{-rT} + e^{-2rT} + ... + e^{-(N-1)rT}\right)$$

$$= P\frac{1 - e^{-rNT}}{1 - e^{-rT}} \tag{15}$$

This is our objective function, which needs to be maximized.

Solution Procedure

As is quite clear from the equation, *TP* is a function of T_1 and s. Here, both T_1 and s are continuous variables, and as a result for optimizing the total present value of net profit, there are two conditions,

$$\frac{\partial TP}{\partial T_1} = 0 \text{ and } \frac{\partial TP}{\partial s} = 0 \tag{16}$$

These two equations are highly non linear hence are solved with the help of mathematical software MATHEMATICA 5.2**.** The equations are solved for different values of *N* and *T* for a fixed planning horizon *H*.

Numerical Illustrations

The following data has been considered for solving the equations of the model:
Demand parameter: $\eta = 250$, $\alpha = 4$, $\gamma = 2.5$,
Backlogging parameter: $\delta = 2$,
Purchasing cost: $c = 5$,
Holding cost: $c_1 = 0.4$,
Shortages cost: $C_2 = 1$,
Lost sale cost: $C_3 = 8$,
Planning horizon: $H = 10$ months,
Inflation rate: $r = 0.08$,
Deterioration rate: $a = 0.05$, $b = 2$,

λ = 1.5 months.

Table 1. Sensitivity Analysis of Optimal Solution with respect to various System Parameters

Variation Parameter		Percentage Variation in Parameter				
		-10%	-5%	0%	+5%	+10%
a	T_1	1.5145	1.4783	1.4625	1.4524	1.4492
	s	30.1293	30.0943	29.7276	29.6923	29.2587
	I_0	465.14	480.78	510.66	513.449	525.87
	TP	32734	32937	33853	34472	35642
b	T_1	1.6872	1.6036	1.5243	1.4631	1.3845
	s	29.2531	29.0639	28.7173	28.2326	27.8271
	I_0	475.91	478.05	485.64	494.72	502.64
	TP	31425	31782	32353	32846	33187
δ	T_1	1.7320	1.7376	1.8223	1.8761	1.880
	s	27.1271	27.4368	28.7173	29.2336	29.7494
	I_0	495.65	492.46	481.32	472.40	471.15
	TP	34591	34547	34453	34385	34301

Conclusion

This study generalized the effect of life time on inventory system for time dependent linear deterioration rate with deterministic demand. Demand rate is dependent on both stock level and selling price. In the case of selling price of demand it is used to see how sensitive the demand for a good is to a price change. The higher the price elasticity, the more sensitive consumers are to price changes. Very high price elasticity suggests that when the price of a good goes up, consumers will buy a great deal less of it and when the price of that good goes down, consumers will buy a great deal more. Very low price elasticity implies just the opposite, that changes in price have little influence on demand. Supply and demand is an economic model based on price and quantity in a market. It predicts that in a competitive market, price will function to equalize the quantity demanded by consumers, and the quantity supplied by producers, resulting in an economic equilibrium of price and quantity. In traditional EOQ models it has been considered that inflation has no role to play. But in today's scenario there is no economy of the world, which is not affected by the nasty effect of the inflation. Even few economies are slowing down their growth rates to curb the creeping inflation, which is acting as devil for the common man. Presently whole world is in the claws of inflation and to manage inventory of items without considering effect of inflation is not possible, therefore we have also taken the effect of inflation in this study. Shortages are occurring with partial backlogging. Backlogging rate is waiting time for the next replenishment. Profit maximization technique is used to get the expressions for profit function and other parameters. The problem is illustrated numerically and significant features of the result due to different parameters are discussed.

From the table 1, an increase in the deterioration factors *a* and *b*, decreases the optimal time T_1 as well as the selling price of the commodity. But the profit keeps increasing substantially with an increase in α and β. The backlogging parameter slightly increases the optimal time and the optimal selling price. But an increase in this parameter adversely affects the net profit of our system. For further study, stochastic demand and stock dependent demand can be considered.

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