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Analyzing of Two-Lane Traffic Flow Simulation Model using Cellular Automata

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Abstract

As traffic demand on road networks steadily increases every year, the need for alleviation of global congestion arises. Our contribution lies in assisting of traffic engineers and policy makers who are faced with the problem of controlling the traffic flows. In this paper, we explore the usefulness of cellular automata to traffic flow modeling. We extend some of the existing CA models to capture characteristics of traffic flow that have not been possible to model using either conventional analytical models or existing simulation techniques. The well-known NaSch model for one lane, Deterministic CA model is discussed. NaSch model with modified cell size and variable acceleration rate is extended to two-lane cellular automaton model for traffic flow. A set of state rules is applied to provide lane-changing maneuvers. S-t-s rule given in the BJH model which describes the behavior of jammed vehicle is implemented in the present model and effect of variability in traffic flow on lane-changing behavior is studied. Flow rate between the single-lane road and two-lane road is compared under the influence of s-t-s rule and braking rule. Simulation results show the ability of this modeling paradigm to capture the most important features of the traffic flow phenomena.

Keywords: Cellular automata, Deterministic CA, NaSch Model, Modified NaSch Model, Braking probability, Slow-to-start rule, Slow-to-start probability and Two- lane CA .

Introduction

The growing of traffic volumes demand new solutions from traffic engineering and traffic science in order to be able to accommodate the changing requirements. Building new and safe routes for traffic is expensive and in many areas of the world there is already lack of space. Although these concerns primarily urbanized areas, the lining out of new, multi-lane highways may be a different problem even through thinly populated areas. In cities even widening of streets may be impossible because of the local housing stock. Consequently, often the only option is to intensify the use of the existing road networks. Most efficient usage of the existing road infrastructure is related to the management and controlling of traffic flows, not forgetting safety and convenience for the people traveling there. Traffic flow modeling is an important step in the design and control of transportation systems. In the research of traffic flow, there have been proposed simplified models that still capture the essentials of the dynamics of the transportation system .Mathematical modeling and computer simulations play important roles in studying the impacts of various policies on vehicular traffic. Modeling and simulation techniques are integral components of intelligent information systems being used in advanced countries. Recently there has been much of interest in studying traffic flow with Cellular Automata (CA) models. CA models have the distinction of being able to capture micro-level dynamics and relate these to macro level traffic flow behavior.

Traffic Flow Modeling in Literature

A cellular automata (CA) is a extremely simplified program for the simulation of complex transportation systems. The first application of the CA for simulation model of traffic flows on street and highways was introduced by Nagel and Schreckenberg popularly known as NaSch Model [1].This model is based on the homogeneous traffic flow. Chowdhury et al. made an attempt to model two kinds of vehicles using CA, This model is a two-lane traffic flow model with two different types of vehicles , characterized by two different values of the maximum speed($v_{\max}^{k}$) for k^{th} type of vehicle [2]. A simple model for two-lane traffic was investigated , but the update rules were not defined in the same manner as in NaSch model. The two-lane cellular automata model based upon the single-lane CA introduced by Rickert et al. was examined [3]. Several branches and hysteresis in flow-density graph are observed. Results relative to a simple CA model without periodic boundary condition for a highway with variable number of on-ramps were presented [4]. A 2D extended version of the 1D Fukui-Ishibashi model, elaborated by Wang et al [5], was presented for single-lane traffic to take into account the exchange of vehicles between the first and second lane. In general lane-changing rule can be symmetric or asymmetric with respect to the lanes or to the vehicles. While symmetric rules treat both lanes equally, asymmetric rule sets especially have to be applied for the simulation of German highways, where lane changes are dominated by right lane preferences and a right lane overtaking ban [6]. A new CA model by introducing the Honk effect into the basic symmetric two-lane CA model was proposed by [7]. The set of lane-changing rules suggested by Chowdhury et al. [8] was revised to take the Honk effect into account. A simple lattice-based exclusion model which can be considered as a crude representation of traffic on a two-lane motorway was introduced [9].Effect of an aggressive lane-changing behavior on a two-lane road in presence of slow vehicles and fast vehicles has been further studied [10]. A highway traffic flow model with blockage induced by an accident vehicle was introduced in which both symmetric and asymmetric lane-changing rule were adopted [11]. Further it is found that vehicles will change lane more frequently when the traffic is heterogeneous with an accident car. In presence of a signalized intersection, existence of a certain combination of density ρ and cycle time which optimizes the traffic efficiency in a two-lane model due to overtaking is studied [12].

In the present study the cell size is reduced and variable acceleration rate (rather than 1) is taken into account [13]. A slow-to-start (s-t-s) rule used in the Benjamin-Johnson-Hui (BJH) CA model for single lane traffic simulation [14] is implemented to two-lane traffic simulation. We investigate the effect of s-t-s and braking probability rule over lane-changing maneuver among vehicles in two-lane road and a detailed comparison of effect of braking probability and s-t-s probability over two-lane traffic flow is carried out using simulation.

Traffic Model Classifications

In general, there are two types of traffic models: Macroscopic and Microscopic.

Macroscopic models describe traffic with aggregate variables such as traffic density, mean speed, and volume. The use of such variables reduces the computation requirements for macroscopic modeling, making real-time calculation quite feasible

Microscopic modeling considers the individual vehicle's physical status and the factors that control human driving behavior. The movement of individual vehicles is governed by the driver's behavior, the road topology, the status of surrounding vehicles, and the headway distribution. Each vehicle in the traffic may be described by a set of parameters that includes position, actual speed, desired speed, route choice, and willingness to pass the other vehicles.

Definition of CA

Cellular Automata are dynamical systems in which space and time are discrete. A cellular automaton consists of a regular grid of cells, each of which can be in a finite number of k possible states, updated synchronously in discrete time steps according to local, identical interaction rules. The state of a cell is determined by the previous states of surrounding neighbor-hood of the cell.

Mathematicians View

Notation: d = dimension, k = states per site, r = radius. For simplicity, assume d = 1 for the moment.

Formally, a CA is represented by the 4-tuple $\{Z, S, N, f\}$ where: Z is the finite or infinite lattice S is a finite set of cell states or *values* , N is the finite neighborhood , f is the local transition function defined by the transition table or the rule .

A ``local (or neighborhood) function'' f is defined on a finite region as $f = S^{2r+1} \rightarrow S$

Both the domain and range of f are finite. The global function $F : S^z \rightarrow S^z$ arises from f, is defined as $F(c) = f(c_{i-r}, \ldots\ldots, c_{i+r})$

Cellular Automata for One-Lane Traffic Flow

CA are mathematical idealizations of physical systems in which space and time are discrete, and physical quantities take on a finite set of discrete values. A cellular automaton consists of a regular uniform lattice, usually finite in extent, with discrete variables occupying the various sites. The state of a cellular automaton is completely specified by the values of the variables at each site. The variables at each site are updated simultaneously, based on the values of the variables in their neighborhood at the preceding time step, and according to a definite set of "local rule."

Our initial traffic model is defined as a one dimensional array with L cells with closed (periodic) boundary conditions. This means that the total number of vehicles N *in* the system is maintained constant. Each cell (site) may be occupied by one vehicle, or it may be empty. Each cell corresponds to a road segment with a length l equal to the average headway in a traffic jam. Traffic density is given by $\rho = N/L$. Each vehicle can have a velocity from 0 to v_{max}. The velocity corresponds to the number of sites that a vehicle advances in one iteration. The movement of vehicles through the cells is determined by a set of updating rules. These rules are applied in a parallel fashion to each vehicle at each iteration. The length of iteration can be arbitrarily chosen to reflect the desired level of simulation detail. The choice of a sufficiently small iteration interval can thus be used to approximate a continuous time system. The state of the system at iteration is determined by the distribution of vehicles among the cells and the speed of each vehicle in each cell.

We use the following notation to characterize each system state:

x(n): position of the nth vehicle,

v(n): speed of nth vehicle, and

d(n): gap between the nth and the (n+1)th vehicle (i.e., vehicle immediately ahead) and is given by d(n) = x(n + 1) - x(n) - 1.

Deterministic CA

In the deterministic single lane model, vehicle motion is determined by the following set of updating rules:

1. Acceleration of free vehicles: If v(n) < v_{max} and d(n) ≥ v(n) + 1, then

   v(n) = v(n) + 1.

2. Slowing down due to other vehicles: If v(n) > d(n) - 1, then v(i) = d(n).
3. Vehicle motion: Vehicle is advanced v(n) sites.

These updating rules were first suggested by Nagel.

Figure 1 shows the application of these three updating rules to an example system with 24 cells and 7 vehicles:

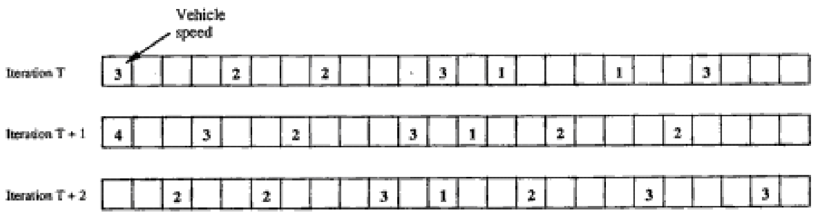

Figure 1: Example CA Model Evaluation

Under these rules, all vehicles have identical behaviors and obey the same maximum speed. These assumptions can be easily relaxed. For example, different vehicles could be assigned different maximum speeds. The tolerated gap between vehicles could also be made vehicle-dependent. Erratic acceleration and deceleration may also be included by introducing random accelerations and decelerations. Throughout the simulation, we use a maximum speed v_{max} = 5 cells/iteration. We let each iteration correspond to one second. The length of each cell is taken to be 7.5 meters, which includes the average length of a vehicle and the gap between two neighboring vehicles in a traffic jam. Therefore, vehicles assume the discrete speeds v_0 = 0 km/h, v_1 = 27 km/h, v_2 = 54 km/h, …, and v_{max} = 135 km/hr. This scaling is, however, not unique. Deterministic CA can also be useful as a modeling paradigm for automated highway systems, where vehicle speeding and vehicle deceleration are externally controlled

NaSch Model of Cellular Automaton for Two- Lane Traffic Flow

The Model and its rules

Designing a simulation model as simple as possible, the most radical way is to use integer variables for space, time and speed. Such a simulation model is called a cellular automaton [ ]. In cellular automata traffic flow model, the road is divided into *L* cells, and a vehicle has a length of *l* cell(s).It is usually assumed that the length of a vehicle is 7.5 m, and then the length of a cell corresponds to 7.5/*l* m. In This model, *l* = 1 is selected.. The length of a cell is given by the minimum space headway between vehicles in jam. This philosophy is represented by the following four rules in the Nasch Model.

Rule 1. Acceleration : $v_n^{(t+\Delta t/3)} = \min\{v_n^{(t)} + 1,\ v_{max}\}$;

Rule 2. Deceleration : $v_n^{(t+2\Delta t/3)} = \min\{v_n^{(t+\Delta t/3)},\ d_n\}$;

Rule 3. Randomization : $v_n^{(t+\Delta t)} = \max\ \{v_n^{t+2\Delta t/3)} - 1, 0\}$ with probability p;

Rule 4. Movement : $x_n^{(t+\Delta t)} = x_n^{t} + v_n^{(t+\Delta t)}$.

Where $x_n^{\ t}$ and $v_n^{\ t}$ denote the position and speed of the n[th] vehicle at a time t respectively; v_{max} is the maximum velocity ; $d_n = x_{n+1}^t - x_n^t - \ell$, (here $\ell = 1$) the number of empty cells in front of n[th] vehicle at a time t, is called distance headway ; p is the randomization probability. Generally, when comes to description of highway traffic, length of a cell is about 7.5m, which is the length of a car during heavy congestion. A time step of Δt = 1 sec and maximum speed v_{max}= 5 cells/time step, that is, 135 km / hr is taken in this model. This indicates that changing speed will only be 7.5 m/sec, 15 m/sec, and 22.5 m/sec and so on. To overcome this problem, different cell sizes are modeled and a reduced cell size of 0.5 m and variable acceleration rate that depends upon the speed of the particular vehicle, are taken into account. Under this fine discretization, we can describe the vehicle moving process more properly. A light vehicle occupies 12 cells with v_{max}= 60 cells which correspond to 108 km/hr where as heavy vehicle occupies 20 cells with v_{max}= 40 cells which correspond to 72 km/hr. For this discretization of cell size, Rule 1 of NaSch Model is modified as below.

Rule 1. Acceleration : $v_n^{(t+\Delta t/3)} = \min\{ v_n^{(t)} + a ,\ v_{max} \}$,
where acceleration a is defined as follows:

$$a = \begin{cases} 4, & if \quad v_n \le 12 , \\ 3, & if \ 12 \le v_n \le 22, \\ 2, & if \quad v_n > 22 \end{cases} \tag{1}$$

Vehicle parameters are all discrete, as position, speed, acceleration, time and so on. When time is from $t \to t+1$, model will be updated by rules. Although it is one of the simplest traffic flow models, it is nevertheless capable of reproducing properties of real traffic flow, like the density-flow relation and the spatial-temporal evolution of jams.

According to theses rules the speed and the acceleration/deceleration ratio of a vehicle are independent of speed of other vehicles at any time. They are only functions of the gap in the front. Thus, these rules can be updated in parallel for any vehicle. However, the acceleration and deceleration ratio can take infinite large value if a vehicle changes its speed according to these rules. The average deceleration ratio over the driver population is p_{brake}. The average acceleration ratio over the driver population yields 1-p_{brake}. Despite its extreme simplicity, this model shows many features which agree with the real-world traffic.

Two-Lane CA Model for Traffic Flow

The single lane model is very inadequate for realistic modeling purposes. In reality, vehicles would be moving in a multi-lane road. The next logical step after modeling a single lane traffic is to model a two-lane traffic. Nagel and Schreckenberg introduced

a two lane model consisting of two parallel single lane models with periodic boundary conditions and four additional rules defining the exchange of vehicles between the lanes. A basis for modeling the lane changing is as follows:
Rule (i) : Check ahead your current lane if another car is in your way.
Rule (ii) : Check ahead on the other lane if it is better there
Rule (iii) : Check back on the other lane if you would get in the way of another vehicle.
Rule (iv) : Based on the result of the first three rules, decide whether to remain on the same lane or change to the other lane.

Using these rules and the algorithm for the one-lane model, we can add the following algorithm for lane changing. A vehicle will only be allowed to change to other lane if the following conditions are satisfied.

Incentive Criteria (Rule (i)) :

$$d_n < \min\{ v_n + a , v_{max} \} , \tag{2}$$

Improvement Criteria (Rule (ii)) :

$$d_{n,other} > d_n , \tag{3}$$

Safety Criteria (Rule (iii)) :

$$d_{n,back} > v_{max} \tag{4}$$

where $d_{n,other}$, $d_{n,back}$ denote the number of empty cells between the nth and its two neighbor vehicles in the other lane at time t, respectively. If all rules are satisfied then the vehicle will change lanes.

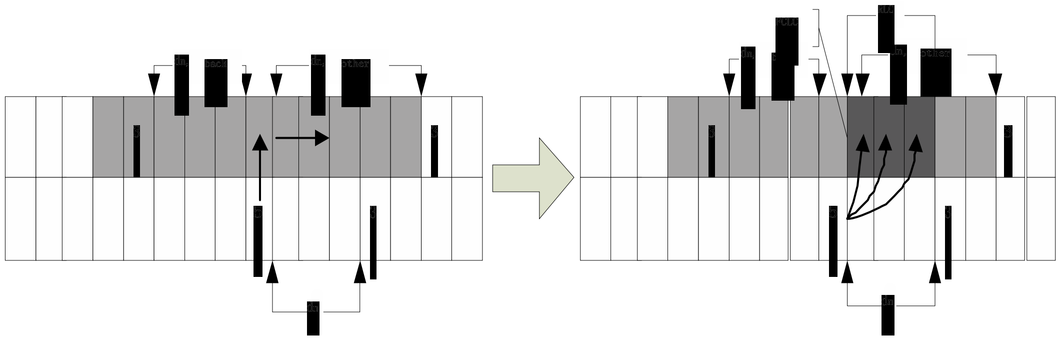

Figure 2: Extensions for lane changing behavior

(Here, RLC is the abbreviation of " Region for Lane Changing", FCLC for "First Cell for Land Changing")

Benjamin-Johnson-Hui CA Model

The BJH model is a fairly straightforward extension of the NaSch model. The authors attempt to more accurately simulate the behavior of drivers which have come to a complete stop in traffic jams on the highway. Cars which have velocity 0 either accelerate at their first available opportunity (as soon as there is an empty space ahead of them) with probability $1 - p_{slow}$, or on the time step immediately after that with probability p_{slow} .Otherise, they follow the NaSch model. This scheme is intended

To reflect the fact that drivers take longer to accelerate from a complete stop, perhaps because they do not immediately notice the car ahead of them moving or because of the slow pick-up of their car's engine. So the BHJ model is essentially the NaSch model with the addition of 'slow-to-start' rule [15].

Slow-to-Start Rule (S-t-s rule)

S-t-s rule is defined mathematically as follows:

$$\text{If } v_n^{(t)} = 0, \quad d_n = 0, \quad \text{then } v_n^{(t+\Delta t/3)} = v_n^{(t+2\Delta t/3)} = v_n^{(t+\Delta t)} = 0, \tag{5}$$

where $v_n^{(t)}$ is the speed of the nth vehicle at time t and Δt is the time interval.

This s-t-s rule is applied only to the stopped vehicles having 0 headway in the previous time step with s-t-s probability q. It implies that s-t-s rule has no effect on the vehicles stopped due to randomization in the previous time step.

Modified Stochastic NaSch Model for Two-lane

The updating rules of the modified stochastic NaSch model with reduced cell size and variable acceleration rate and with implementation of s-t-s rule are given as follows:

Rule1

$$\text{If } v_n^{(t)} = 0, \quad d_n = 0, \quad \text{then } v_n^{(t+\Delta t/3)} = v_n^{(t+2\Delta t/3)} = v_n^{(t+\Delta t)} = 0 \tag{6}$$

with s-t-s probability q.

Rule2

$$v_n^{(t+\Delta t/3)} = \min\{ v_n^{(t)} + a , \ v_{max} \} \tag{7}$$

Rule 3

$$v_n^{(t+2\Delta t/3)} = \min\{ v_n^{(t+\Delta t/3)} , \ d_n \} \tag{8}$$

Rule 4

$$v_n^{(t+\Delta t)} = \max \ \{ v_n^{t+2\Delta t/3)} - 1 , 0 \} \text{ with braking probability p.} \tag{9}$$

Rule 5

$$x_n^{(t+\Delta t)} = x_n^{t} + v_n^{(t+\Delta t)}. \tag{10}$$

Together with lane-changing rules:

$$d_n < \min\{ v_n^{(t+\Delta t/3)} , v_{max} \} \tag{11}$$

$d_{n,other} > d_n$ and $d_{n,back} > v_{max}$ with lane changing probability s, (12)

where $d_n = x_{n+1}^t - x_n^t - \ell$, (here $\ell = 12$).

Simulation Results and discussion

Description of Simulation Procedure

We carry out the simulation of the model under the periodic boundary condition.

The numerical simulation is carried out with randomly generated initial configurations on a closed track containing 10,000 cells which represents a simulated road section of 5 km. The periodic boundary condition is that N vehicles were randomly distributed on both lanes. For each initial configuration of vehicles, results are obtained by averaging over 3600 time steps. For each density ρ , results are averaged over 10 different initial configurations.

The computational formula used numerical simulation are given as follows:

$$\overline{\rho_j} = \frac{1}{T} \sum_{t=1}^{t=T} n_j(t), \tag{13}$$

$$\overline{q_j} = \frac{1}{T} \sum_{t=1}^{t=T} m_j(t), \tag{14}$$

where equation (13) represents the density of the vehicles on the jth site over a time period T; $n_j(t) = 0$ if the jth site is empty and $n_j(t) = 1$ if the jth site is occupied by a vehicle at time t. Equation (14) represents the flow of vehicles on jth site; $m_j(t) = 1$, if at time t-1, there was a vehicle behind or at the jth site, and at time t, it is found after jth site. Density and flow are measured and averaged out over a time period T.

Lane- Changing Behavior

The behavior of lane-changing criteria can be explained if two criteria are fulfilled to initiate lane change. First, the situation on the other lane must be more convenient ad second , the safety rules must be followed. We analyses the effect of two parameter p (braking probability) and q (slow-to-start probability) on lane changing behavior of vehicles. It can be shown by simulation that initially vehicles change lanes frequently and the lane- changing rate drops rapidly at time evolves. Figures 3 and 4 show independent effects of s-t-s probability and braking probability when acting alone, respectively, over the lane-changing behavior of a periodic two-lane system. From figures 3 and 4, it can be observed that an introduction of non-zero q has a strong influence than non-zero p on lane-changing rate of vehicles in two-lane road. With smaller values of p and q, vehicles rarely change lanes. With increase in the value of p and q, the lane-changing tendency among the vehicle increases. With higher values of p and q, there will always be cluster formation and between the two clusters there is sufficient space on the right lane for vehicles to change the lane, and lane change becomes more likely. Hence, maximum lane changes occur even at higher values of traffic density. Here, we choose the lane-changing probability as s = 0.8. As described in BJH model, an introduction of non-zero s-t-s probability makes the queues less fragmented and the inter-queue regions widen, as a result vehicles find enough gaps in the target lane to change the lane. Figure 5 shows the lane-changing behavior of vehicles when both parameters act together. With higher values of both parameters, safety criteria are not fulfilled and lane-changing behavior rate again reduces drastically. Figure 6 describes the lane changing behavior of the NaSch model when simulated with implementation of s-t-s rule. Parameter q has more effect than parameter q on lane-changing behavior among vehicles. When both parameters are working together, the lane-changing tendency among vehicles increases. It can be noted that the results simulated from the NaSch model and modified cell model are close to each other except the magnitude of lane-change rate/cell, because of the reduced cell size.

Single Lane Versus Two-Lane

Figure 7 describes the comparison of the single-lane model with the corresponding two-lane model with effects of parameters p and q. The graph shows rise in maximum flow q_{max}, when simulating two-lane traffic as compared to one-lane system. Since parameter p affects all the vehicles with equal probability, whereas parameter q affects only static vehicle that is, vehicles blocked by leading vehicles in the previous time step, the parameter p is more responsible than parameter q in reducing the throughput. However, when values of p and q are high, this reduction can be significant. In the low-density region ($\rho < \rho_c$), the average velocity of the traffic system is close to maximum velocity v_{max}. Hence there is not much difference in throughput of the system with non-zero p as shown in figure 7. But with zero value of parameter p , flow rises even in low-density region because parameter q comes to action only in high density region ($\rho > \rho_c$) as it affects only jammed vehicles. Figure 8 describes the effect of parameters p and q over average velocity of the two-lane system with maximum speed v_{max} = 60 cells / Δt. In high-density region, average speed becomes the decreasing function of density.

Fundamental Diagrams

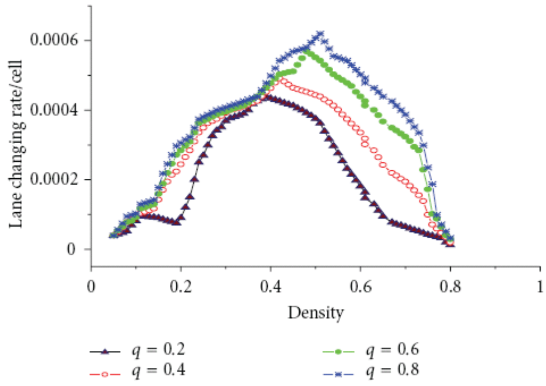

Figure 3: Relation between lane changing rate and density at braking probability p = 0.0

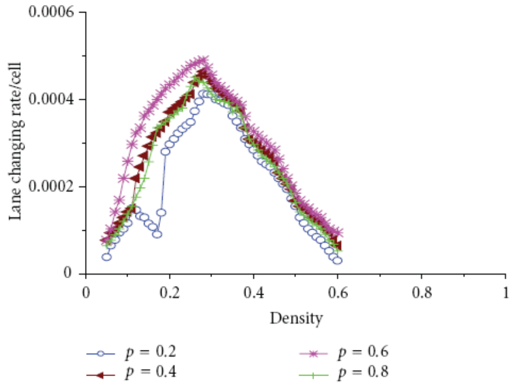

Figure 4: Relation between lane changing rate and density at s-t-s probability q = 0.0

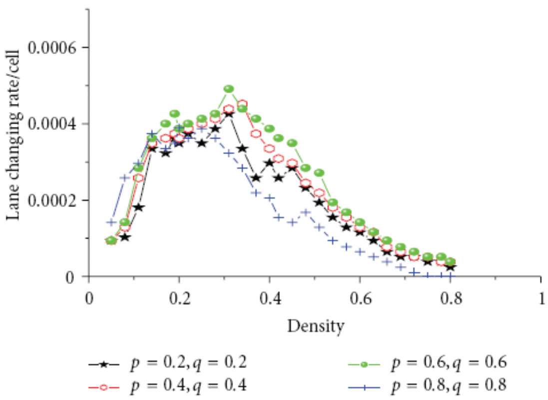

Figure 5: Relation between lane changing rate and density at non-zero values of p and q.

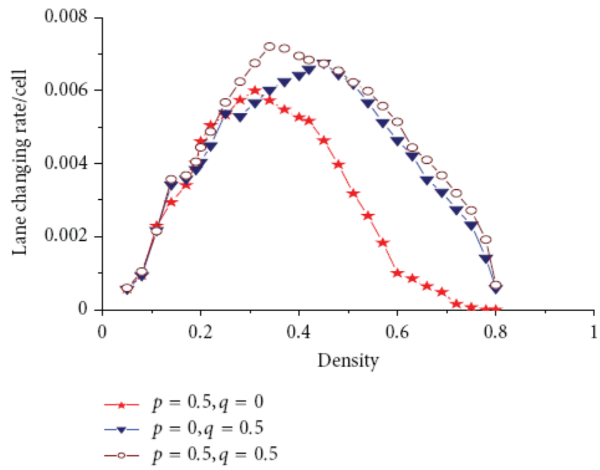

Figure 6: Relationship between lane-changing rate and density obtained from the NaSch model at various values of p and q.

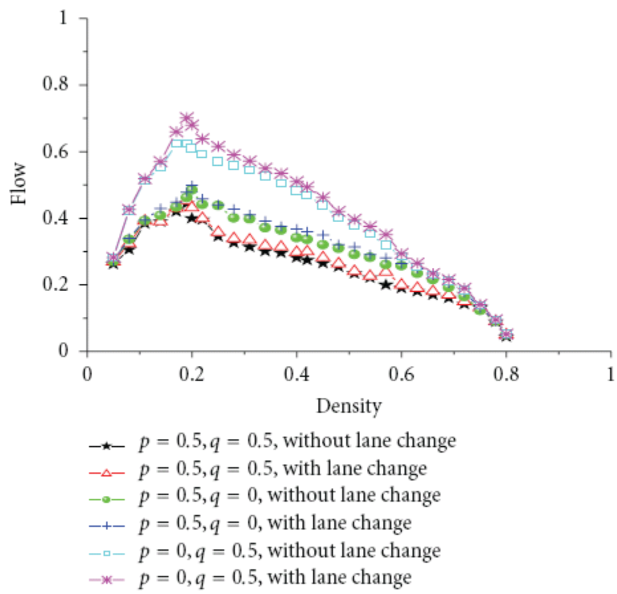

Figure 7: Relationship between density and flow with lane change and without lane change at various values of parameters p and q

Conclusion

In this paper, we have extended the BJH model to two-lane model with a reduced cell size and a variable acceleration rate.The modified NaSch model with reduced cell size is more appropriate to describe the finer variability in traffic flow rather than the NaSch model with cell size 7.5 m. We have observed the effects of braking probability and s-t-s probability over lane-changing maneuver. We studied through simulation how vehicles fulfill both the incentive and safety criteria with high values of braking and s-t-s probabilities. Combined effect of braking rule and s-t-s rule increases the effectiveness of the lane changes, because gap acceptance is increased between the vehicles. We also compared the flow and average velocity of the two-lane system with single lane under the influence of braking probability and s-t-s probability. S-t-s probability has more effect than braking probability on lane-changing rate. It is also observed that the maximum lane changing frequency occurs long after the critical density ρ_c of maximum throughput. This model reveals all the features of two-lane traffic flow.

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