

Interior Point Algorithm for Linear Programming Problem and Related inscribed Ellipsoids Applications.

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Abstract

The interior point algorithm of Karmarkar (1984) was received with great interest. His work was the route of various development in the field of linear programming and very soon in related field such as linear complimentary, quadratic, convex as well as non-linear programming problems.

In this paper we developed theoretical and as well as algorithm procedure for interior point method. We consider the optimization of non-convex function subject to linear inequalities constraints. The objective function is minimize over the interior ellipsoids to generate sequence of interior point converging to optimal solution. The decent direction is computed.

The second part of this paper contains affine scaling algorithm. This algorithm helps to solve quadratic programming problem resulting from the original problem. The computational complexity of the method is also shown through minimal norm theorem.

Keywords: IPP, QPP, Affine Scaling Algorithm, Computational complexity, Minimal Norm.

Introduction

Karmarkar (9) introduced his paper in the year 1984 about the interior point method

was a first kind of paper on which the computational complexity of the method for Linear Programming Problem was known. According to his claim the method requires $O(n^{3.5}L)$ arithmetic operations and in practice it is much faster than the Simplex method. His work stimulated tumultuous developments in the field of linear programming and soon after in related fields such as linear complementarity, quadratic and convex programming and non-linear programming in general.

Many interior point algorithms use a potential or multiplicative barrier function to measure the progress of algorithm (for complexity analysis) and / or to calculate search direction. This search direction is often a projected gradient or Newton direction with respect to the function. D Den hertog and C. Roos [7] have given a nice survey of projected gradient and Newton directions for all used potential and barrier functions.

Quadratic programming plays a unique roll in optimization theory. It is a continuous optimization that includes linear programming and fundamental subroutine for general non-linear programming and is considered one of the most challenging combinatorial optimization problems.

The combinatorial nature of quadratic programming is basically embedded in the existence of inequality constraints, which in general make inequality constrained optimization(ICD) harder to solve than equality constrained optimization (ECO). Most of the methods such as simplex type methods of Cottle and Dantzig [3], Lemke [12] and Wolfe [15], gradient projection (GP) method of Rosen [14] solve a sequence of ECO's in order to approach the optimal solution for ICO. Geometrically, they move along the boundary of the feasible region to approach the optimal feasible solution. These methods are not polynomial time. Kachiyan[11] in 1979 published a proof showing that certain LP algorithm called the ellipsoid method is polynomial.

In this chapter, we present Karmarkar's interior point method for linear programming problem based on the minimization of a non-convex potential function. The constraint set is determined by the set of Linear inequalities, and the method generates a sequence of strict interior points of this polytope and at each consecutive point, the value of the potential function is reduced. The direction used to determine the new iterate is computed by solving a non-convex quadratic program on an ellipsoid.

The Problem

We consider the dual standard form of linear programming problem :

$$\begin{aligned} & \text{(P) Minimize } C^T x \\ & \text{Subject to} \\ & Ax = b \\ & X \geq 0 \end{aligned}$$

Where $C, x \in R^n, b \in R^m, A$ is an $m \times n$ matrix. The dual problem associated with (p) is

$$\begin{aligned} & \text{(D) Maximize } b^T y \\ & \text{Subject to} \end{aligned}$$

$$A^T y \leq c$$

Where $y \in \mathbb{R}^m$, we make the following assumptions.

- The feasible region $y = \{y \in \mathbb{R}^m \mid A^T y \leq c\}$ is bounded.
- The interior, $\text{int}\{y\}$ of the feasible region is non empty.
- The maximum value of the objective function is v .

Many interior point algorithm use potential or barrier function. The potential function associated with (D) is

$$\Phi(Y) = q \ln(v - b^T Y) - \sum_{i=1}^n \ln(d_i(y))$$

Where, $d_i(y) = c_i - a_i^T y, i = 1, 2, 3, \dots, n$, The multiplicative barrier formulation of potential function is

$$F(y) = \frac{(v - b^T y)^q}{\prod_{i=1}^n (c_i - a_i^T y)}$$

Non-Convex Optimization.

Consider the following non-convex optimization problems :

$$\text{Minimize } \{\Phi(y)\} \quad A^T y \leq C.$$

Is solve this problem, we use and approach similar to classical Levenberg – Marquardt method. Let $y^0 \in \text{Int } y = \{y \in \mathbb{R}^m : A^T y < c\}$ be given initial interior point. The algorithm we propose generates a sequence of interior points of Y . Let y^k be in $\text{Int}(y)$ at the k th iteration. Around y^k a quadratic approximation of potential function is set up. Let $D = \text{diag}(d_i(y))$, $e = [1, 1, \dots, 1]^T$, $\Phi_0 = v - b^T y$ and k be a constant. The quadratic approximation of $\Phi(y)$ around Y^k is given by

$$Q(y) = \frac{1}{2} (y - y^k)^T H (y - y^k) + h^T (y - y^k) + K \quad \text{where the gradient,}$$

$$h = \nabla \Phi(y^k) = \frac{q}{\Phi_0} b + A D^{-1} e$$

and the hessian,

$$H = \nabla^2 \Phi(y^k) = - \frac{q b b^T}{\Phi_0^2} + A D^{-2} A^T$$

Now minimizing $Q(y)$ subject to $A^T y \leq c$ is carried in substituting the polytope by an inscribed ellipsoid defined by the hessian matrix, and the resulting approximation problem becomes easy. Ye[84] has independently proposed a polynomial time algorithm for non convex quadratic programming on an ellipsoid. Imai [8] propose the approximation of the multiplicative penalty function by a linear

function defined by the gradient on an ellipsoid, determined by the hessian at the point. His method is polynomial and requires $D(n^2L)$ arithmetic iterations, where each iterations requires $D(n^3)$ operations on numbers of L bits.

Proposition 3.3.1 Consider the polytope defined by

$$Y = \{Y \in \mathbb{R}^m : A^T y \leq C\}$$

and $y^K \in \text{Int}(Y)$. Consider the ellipsoid.

$$E(r) = \{y \in \mathbb{R}^m : (y - y^K)^T A D^{-2} A^T (y - y^K) \leq r^2\}$$

Then for $r \leq 1$, $E(r) \subset Y$, i.e. $E(r)$ is an inscribed ellipsoid in Y .

Proof. When $r=1$ $E(r) \subset E(1)$, for $0 \leq r < 1$. Assume that $w \in E(1)$.

Then $(w - Y^K)^T A D^{-2} A^T (w - Y^K) \leq 1$ and consequently $D^{-1} A^T (w - Y^K) \leq 1$.

Denoting i th row of A^T by a_i^T , we have

$$\frac{1}{c_i - a_i^T Y^K} a_i^T (w - Y^K) \leq 1 \quad \forall i = 1, 2, \dots, n$$

$$a_i^T (w - Y^K) \leq c_i - a_i^T Y^K, \quad \forall i = 1, 2, \dots, n$$

$$a_i^T w \leq c \quad \text{consequently, } w \in P.$$

Corollary 3.3.1. $E(\frac{1}{\sqrt{2}})$ is contained in the feasible region P .

$$\text{(Note } r = \frac{1}{\sqrt{2}} < 1)$$

A geometric expression derived from the LP affine scaling algorithm (Dikin [4], Barnes [1], Kortanek and Shi [10]) can be drawn as an interior ellipsoid centered at the starting interior point in the feasible region. Then the objective function can be minimized over this interior ellipsoid to generate the next interior solution point. A series of such ellipsoids can thus be constructed to generate a sequence of interior points converging to the optimal solution point that sets on the boundary. In case the solution point itself is an interior solution (which can happen if the objective function is a nonlinear function) then the series terminates as soon as the optimal point is encircled by the nearest ellipsoid. The above geometric expression can be represented by the following optimization problem.

$$\text{Minimize } \frac{1}{2} (\Delta y)^T H \Delta y + h^T \Delta y \quad (3.3.1)$$

$$\text{Subject to } (\Delta y)^T A D^{-2} A^T \Delta y \leq r^2 \quad (3.3.2)$$

Where $A D^{-2} A^T$ is invertible matrix and by assumption is positive definite and $\Delta y = y - y^K$. The optimal solution Δy^* to (3.3.1) and (3.3.2) is a descent direction of $Q(y)$ from y^K . For a given radius $r > 0$, the value of the original potential function $\Phi(y)$, may

increase by moving in the direction Δy^* because of the higher order terms ignored in the approximation. If the radius is decrease sufficiently, the value of the potential function will decrease by moving in the new Δy^* direction. We say local minimum to function $\Phi(y)$ has been found if the radius must be reduced below a tolerance level ϵ to achieve a reduction in the value of the potential function.

In place of the ellipsoid.

$$\{\Delta y \in \mathbb{R}^M : \Delta y^T A D^{-2} A^T \Delta y \leq r^2\} \tag{3.3.3}$$

may be replaced by the sphere

$$\{\Delta y \in \mathbb{R}^M : \Delta y^T \Delta y \leq r^2\} \tag{3.3.4}$$

Without loss of generality, since $A D^{-2} A^T$ is be assumption positive definite and (3.3.3) can be converted to (3.3.4) by means of non singular transformation space.

Computing the descent direction

ΔY^* is optimal to (3.3.1) and (3.3.2) if and only if there exists $\mu \geq 0$ such that

$$(H + \mu A D^{-2} A^T) \Delta y^* = -h \tag{3.4.1}$$

Or,

$$(H + \mu I) \Delta y^* = -h \tag{3.4.1a}$$

$$\mu (\Delta y^{*T} A D^{-2} A^T \Delta y^* - r^2) = 0 \tag{3.4.2}$$

Or

$$\mu (\|\Delta y^*\| - r) = 0 \tag{3.4.2a}$$

and

$$H + \mu A D^{-2} A^T \tag{3.4.3}$$

Is positive semi definite. With the change of variable $\gamma = \frac{1}{\mu+1}$ and substituting h and H in to (3.4.1) we obtain an expression for ΔY^* satisfying (3.4.1)

$$\begin{aligned} \Delta Y^* &= -\left(A D^{-2} A^T - \frac{q b b^T}{\Phi_0} + \mu A D^{-2} A^T \right)^{-1} h \\ &= -\left[A D^{-2} A^T - \frac{q \gamma b b^T}{\Phi_0^2} \right]^{-1} \gamma \left[\frac{q b}{\Phi_0} + A D^{-1} e \right] \end{aligned} \tag{3.4.4}$$

In (3.3.4), r does not appear however (3.3.4) is not defined for all values of γ . But if radius r of the ellipsoid is kept within certain bound, then there exists an interval

$0 \leq \gamma \leq \gamma_{\max}$ such

That $AD^{-2}A^T - \frac{q\gamma b b^T}{\phi_0^2}$ is non-singular.

Proposition 3.4.1. There exists $\gamma > 0$ such that the direction ΔY^* , given in (3.4.4) is a descent direction of $\Phi(Y)$.

$$\begin{aligned}
 \Delta y^* &= - \left[AD^{-2}A^T - \frac{q\gamma b b^T}{\phi_0^2} \right]^{-1} \gamma(h) \\
 &= - \left[AD^{-2}A^T \left(1 - \frac{\gamma q (AD^{-2}A^T) b b^T}{\phi_0^2} \right) \right]^{-1} \gamma(h) \\
 &= -\gamma \left[\left(1 - \frac{\gamma q (AD^{-2}A^T) b b^T}{\phi_0^2} \right) \right]^{-1} (AD^{-2}A^T)^{-1} h \\
 &= \gamma \left[\left(1 - \frac{\gamma q (AD^{-2}A^T) b b^T}{\phi_0^2} \right) \right]^{-1} (AD^{-2}A^T)^{-1} (-h)
 \end{aligned} \tag{3.4.5}$$

Let $\gamma = \varepsilon > 0$ and considering

$\lim_{\varepsilon \rightarrow 0} +h^T \Delta Y^*$ we have

$$\lim_{\varepsilon \rightarrow 0} +\Delta Y^* = s(AD^{-2}A^T)^{-1} (-h)$$

And therefore,

$$\lim_{\varepsilon \rightarrow 0} +h^T \Delta Y^* = s h^T (AD^{-2}A^T)^{-1} - h$$

Since by assumption, $\varepsilon > 0$ and $h^T (AD^{-2}A^T)^{-1} h > 0$ then

$$\lim_{\varepsilon \rightarrow 0} +h^T \Delta Y^* < 0$$

Let

$$H_c = AD^{-2}A^T$$

$$H_0 = -\frac{q b b^T}{\phi_0^2}$$

And define $M = H_c + \gamma H_0$

Given that the current iterate Y^K , we first find the value of γ such that $M \Delta Y = \gamma h$ has a solution ΔY^* . This can be done by binary search method. Once the parameter γ is found out the linear system.

$$M \Delta Y^* = \gamma h$$

Is solved for $\Delta Y^* = \Delta Y^*(\gamma)$. The length $u(\Delta Y^*(\gamma))$ is a monotonically increasing function of γ in interval $0 \leq \gamma \leq \gamma_{\max}$. From the optimality condition (3.4.2)

implies that $\gamma = \sqrt{\iota(\Delta Y^*(\gamma))}$ if $\mu > 0$. Small lengths results in small change in potential function, since γ is small and the optimal solution lies on the surface of the ellipsoid. A very large length may not correspond to an optimal solution of (3.3.1) and (3.3.2), since this may requires $\gamma > 1$. Let the acceptable length of interval be (\underline{l}, \bar{l}) of $\iota(\Delta Y^*(\gamma))$ if $(\underline{l} \leq \iota(\Delta Y^*(\gamma)) \leq \bar{l})$. When $\iota(\Delta Y^*(\gamma)) < \underline{l}$, γ is increased and the system $M\Delta Y^* = \gamma h$ is resolved with new M and h . On the other hand, if $\iota(\Delta Y^*(\gamma)) > \bar{l}$, γ is reduced and $M\Delta Y^* = \gamma h$ is resolved. Once the acceptable length is produced, we use $\Delta Y^*(\gamma)$ as the descent direction.

Affine scaling algorithm

We propose affine scaling algorithm for the problem.

Lemma 3.5.1 Let H be symmetric rational matrix and L be its encoding length and λ be any eigen value of H . Then

$$|\lambda| \leq \max_{i,j} |h_{ij}| \leq 2^{0(L)} \tag{3.5.1}$$

And either

$$\lambda = 0 \text{ or } |\lambda| > 2 \cdot 2^{0(L)} \tag{3.5.2}$$

Proof. $\lambda \leq \max_{\|\Delta Y\|=1, \Delta Y \neq 0} \Delta Y^T H \Delta Y$

$$= \max_{\|\Delta Y\|=1, \Delta Y \neq 0} \left(\sum_{i=1}^n \sum_{j=1}^n \Delta Y_i h_{ij} \Delta Y_j \right)$$

$$\leq \max_{i,j} |h_{ij}| \left(\max_{\|\Delta Y\|=1, \Delta Y \neq 0} \sum_{i=1}^n \sum_{j=1}^n |\Delta Y_i \Delta Y_j| \right)$$

$$\leq \max_{i,j} |h_{ij}| \left(\max_{\|\Delta Y\|=1, \Delta Y \neq 0} \sum_{i=1}^n \sum_{j=1}^n \frac{\Delta Y_i^2 + \Delta Y_j^2}{2} \right)$$

$$= \max_{i,j} |h_{ij}| \left(\frac{n+n}{2} \right)$$

$$= n \max_{i,j} |h_{ij}|$$

Similarly, we have

$$\lambda \geq -n \max_{i,j} |h_{ij}|$$

As λ is one of the roots of the characteristic polynomial corresponding to H :

$$= \lambda^n + a_{n-1}\lambda^{n-1} + a_{n-2}\lambda^{n-2} + \dots + a_1\lambda + a_0 = 0$$

Where a_j is are all rationales of the minor of H and there sizes are all bounded by 1. If $\lambda \neq 0$ and $|\lambda| \leq 2^{0(L)}$ and suppose for simplicity $a_0 \neq 0$, then

$$0 < |a_0| = |\lambda^n + a_{n-1}\lambda^{n-1} + \dots + a_1\lambda|$$

$$\leq |\lambda| |\lambda^{n-1} + a_{n-1}\lambda^{n-2} + \dots| \leq |\lambda| \left(1 + \sum_{j=1}^{n-1} |a_j| \right) \leq 2^{-L}$$

Which contradict the fact a_0 is a non zero rational number whose size is bounded by L.

Lemma 3.5.2. If P_\wedge by orthogonal projection matrix on to the null space $\{x \in \mathbb{R}^n : Ax = 0\}$, and B be an orthonormal basis spanning the null space. Then $\lambda(B^T H B) \in \{\lambda(P_\wedge H P_\wedge)\}$ and the columns of A^T are the eigen vector of $P P_\wedge H P_\wedge$ corresponding to the zero eigen values of $P_\wedge H P_\wedge$.

Proof. Let λ be an eigen value of $B^T H B$. Then there exists a v such that

$$B^T H B v = \lambda v$$

Define u such that $B u = P_\wedge H B v$ then

$$B^T P_\wedge H B v = B B^T u = u$$

Which together with $B^T P_\wedge = B^T$ imply

$$u = B^T H B v = \lambda u$$

Therefore

$P_\wedge H P_\wedge (Bv) = P_\wedge H B v = B u = \lambda B v$ which shows that λ is an eigen values of $P_\wedge H P_\wedge$ with Bv being the eigen vector. Moreover,

$$P_\wedge H P_\wedge A^T = 0 \in \mathbb{R}^{n \times m}$$

Each column of A^T is an eigen vector of $P_\wedge H P_\wedge$ corresponding to zero eigen values.

Optimality condition for problem

The first order necessary condition for problem (3.5.1) and (3.3.2).

$$(H + \mu i)\Delta Y = -h \tag{3.6.1}$$

and

$$\|\Delta Y\| \leq r, \mu \geq 0 \text{ and } \mu (r - \|\Delta Y\|) = 0 \tag{3.6.2}$$

The second order condition for problem (3.3.1) and (3.3.2) is $\lambda (h + \mu 1) \geq 0$

Denoted λ is the least eigen values of H. Then the second order condition can be expressed as

$$\mu \geq \max (0, -\lambda)$$

When $\lambda < 0$ we must have $\|\Delta Y\| = r$ in (3.6.2). Since $\mu > |\lambda| > 0$ from (3.6.3).

Lemma 3.6.1 if $\lambda_1 < 0$ then $g(\Delta \mathcal{F}) - g(0) \leq -1/2 r^2 |\lambda|$

Proof. Using (3.6.1), (3.6.2) and (3.6.3) noting $\|(\Delta \mathcal{F})\| = r$ in the non convex case

$$\begin{aligned} G(0) - g((\Delta \mathcal{F})) &= -1/2 \Delta \mathcal{F}^T H \Delta \mathcal{F} - h^T (\Delta \mathcal{F}) \\ &= -1/2 \Delta \mathcal{F}^T H \Delta \mathcal{F} + \Delta \mathcal{F} (H + \mu I) \Delta \mathcal{F} \\ &= \Delta \mathcal{F}^T (1/2 H + \mu I) \Delta \mathcal{F} \\ &\geq r^2 (\mu + 1/2 \lambda) \\ &\geq 1/2 r^2 |\lambda|. \end{aligned}$$

Lemma 3.6.2 Let $(\mu_1, \Delta Y_1)$, and $(\mu_2, \Delta Y_2)$ satisfy (3.6.1), (3.6.2) and (3.6.3). Then $\mu_1 = \mu_2$ and $g(\Delta Y_1) = g(\Delta Y_2)$.

Proof. Lemma (3.6.2) is obviously true if H is convex since the solution of the problem is unique. Now suppose H is non-convex and $\mu_1 \neq \mu_2$. Without loss of generality, we assume that $\mu_1 > \mu_2 > 0$ Note that $\|\Delta Y_1\| = \|\Delta Y_2\| = r$ and $(H + \mu_1 I)$ is positive definite and $(H + \mu_1 I)^{-1} (H + \mu_2 I) \Delta Y_2 = -(H + \mu_1 I)^{-1} h = \Delta Y_1$

$$\begin{aligned} &\text{Which } > \|(H + \mu_1 I)^{-1} (H + \mu_2 I)\| \\ &= \|(H + \mu_1 I)^{-1} (H + \mu_2 I)\| \|\Delta Y_2\| / r \\ &\geq \|(H + \mu_1 I)^{-1} (H + \mu_2 I) \Delta Y_2\| / r \\ &= \frac{\|\Delta Y_1\|}{r} = 1 \end{aligned}$$

Results a contradiction

To prove the uniqueness of the minimal objective, value, we only need to concerned with the case $\mu_1 = \mu_2 = |\lambda|$, since, otherwise the minimal solution for the problem is again unique. For any solution satisfying (3.6.1) with $\mu = |\lambda|$

$$\Delta Y = v + b$$

Where b is a particular solution and v is a homogenous solution to (3.6.1) (As V is also a sub space of all eigen vectors corresponding to the least eigen value of H). Thus

$$\begin{aligned} G(\Delta Y) &= -(v+b)^T (1/2 H + |\lambda| I) (v+b) \\ &= -1/2 |\lambda| (\|v\|^2 + \|b\|^2) - 1/2 b^T (H + |\lambda| I) b \\ &= -1/2 |\lambda| (\|\Delta Y\|^2) - 1/2 B^T (H + |\lambda| I) b \\ &= -1/2 r^2 |\lambda| - 1/2 B^T (H + |\lambda| I) b \end{aligned}$$

Which is independent of v .

Lemma. 3.6.3. $\bar{\mu} \leq \frac{\|h\|}{r} + n_{i,j}^{\max} |h_{i,j}|$

Proof. We only need to be concerned with the case $\bar{\mu} > |\lambda|$. In this case

$$\|H + \bar{\mu} I\| = r$$

Which implies

$$\frac{1}{\bar{\mu} + \lambda} \|h\| \geq r$$

Which further implies

$$\bar{\mu} + \lambda \leq \|h\| / r$$

Hence

$$\bar{\mu} \leq \frac{\|h\|}{r} + |\lambda|$$

Therefore from lemma (3.6.1)

$$\bar{\mu} \leq \frac{\|h\|}{r} + n_{i,j}^{\max} |h_{i,j}|$$

Lemma (3.6.1) and (3.6.2) establish a theoretical bade of using the binary search for $\bar{\mu}$. In particular, we propose the following procedure similar to the one of Ye [86].

Procedure

Step 1. Set $\mu_1 = 0$ and $\mu_3 = \frac{\|h\|}{r} + n_{i,j}^{\max} |h_{i,j}|$

Step 2. Set $\mu_2 = 1/2(\mu_1 + \mu_3)$

Step 3. Let $\mu = \mu_2$ and then solve (3.6.1)

Step 4. If $\mu_3 - \mu_1 < \epsilon$ then stop and return else if $H + \mu I$ is indefinite or negative definite or (3.6.1) has no solution, or the norm of the minimal-norm solution of (3.6.1)

is greater than r , then $\mu_1 = \mu_2$ and go to step 2; else if the norm of the solution of (3.6.1) is less than r , then $\mu_3 = \mu_4$ and go to step (2)

The above procedure can be terminated $O(n^3)$ arithmetic operation by using standard gauss elimination method.

Theorem (3.6.1) (Minimal norm theorem the algorithm is terminated in $O(n^3 \ln(1/\epsilon))$ arithmetic operation and the solutions resulting from the procedure for $\mu = \mu_3$ satisfy (3.6.1), (3.6.2) and (3.6.3) with $0 \leq \mu_3 - \bar{\mu} < O(\epsilon)$

$$\| \Delta y \| - r < O(\epsilon)$$

Proof. The checking of definiteness of $H + \mu I$ and solving (3.6.1) requires $O(n^3)$ arithmetic operations. The intervals of $\mu_3 - \mu_1$ is bounded by 2^L , and binarily shrinks to zero. Therefore, the total $O(n^3 \ln(1/\epsilon))$ arithmetic operation is sufficient to terminate the procedure.

In order to proof the second part of the theorem we consider the case $\lambda < 0$. The procedure ends with

$$\mu_1 < \bar{\mu} < \mu_3 \text{ and } \mu_3 - \mu_1 < \epsilon \leq 2^{O(L)}$$

if $\bar{\mu} = |\lambda|$ in which case we also have,

$0 \leq (\mu_3 - \bar{\mu}) / |\bar{\mu}| < O(\epsilon) \leq 2^{O(L)}$ from lemma (3.5.1) then μ_3 can be viewed as a rational approximation to $\bar{\mu}$ with relative error $2^{O(L)}$. Therefore, a solution $\Delta y = \Delta y + v$ with $\| \Delta y \| = r$ can be obtained where Δy is the minimal norm particular solution (3.6.1) and v is orthogonal to Δy is homogeneous solution to (3.6.1). Otherwise $\mu_3 - |\lambda| \geq 2^{O(L)}$ in which case $H + \mu_1$ is positive definite $\mu = \mu_3$ and $\| (H + \mu I)^{-1} \|$ is bounded by $2^{O(L)}$, and

$$\| \Delta y \| - r < 2^{O(L)} (\mu_3 - \mu_1) \leq O(\epsilon).$$

Conclusion

In this chapter we have introduced interior point algorithm for linear programming problem. We have described in detail the procedure for generating a descent direction for potential function we also used affine scaling algorithm for solving quadratic program resulting from the original problem. Its time complexity is a polynomial in $\log(1/\epsilon)$ (where ϵ is the error tolerance).

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