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i-Hamiltonian Laceability in Product Graphs

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Abstract

For a connected graph G, let h(G) be the length of a Hamiltonian walk in G and call it the Hamiltonian number of G. Let i be a non-negative integer. A connected graph G of order n is called i-Hamiltonian if h(G)=n+i. In this paper, we define i-Hamiltonian-t-laceable graphs and i-Hamiltonian-t*-laceable graphs. We explore i-Hamiltonian-t*-laceability properties in the cartesian product of graphs involving paths and cycles.

Keywords: Connected graph, Hamiltonian-t-laceable, Hamiltonian-t*-laceable, i-Hamiltonian-t-laceable, i-Hamiltonian-t*-laceability

2000 Mathematics Subject Classification: 05C45, 05C99

Introduction

Let G be a finite, simple, connected and undirected graph. Let u and v be two vertices in G. The distance between u and v denoted by $d(u,v)$ is the length of a shortest u-v path in G. In [1] Goodman and Hedetniemi introduced the concept of a Hamiltonian walk in a connected graph G, defined as a closed spanning walk of minimum length in G. They denoted the length of a Hamiltonian walk in G by $h(G)$ and called $h(G)$ as the Hamiltonian number of G. Therefore, for a connected graph of order $n \geq 3$, it follows that $h(G)=n$ if and only if G is Hamiltonian. Figure 1 below shows a connected graph G with $h(G)=6$.

Let i be a non-negative integer. A connected graph G of order n is called *i-Hamiltonian* [2] if $h(G)=n+i$. Thus a 0-Hamiltonian graph is Hamiltonian. An almost Hamiltonian graph is a graph G of order n and $h(G)=n+1$.

A graph G is *Hamiltonian-t-laceable* [3] if there exists in G a Hamiltonian path between every pair of vertices u and v with $d(u,v)=t$, $1 \leq t \leq diamG$, where t is a positive integer.

A graph G is *Hamiltonian-t^*-laceable* [4] if there exist in G a Hamiltonian path between at least one pair of distinct vertices u and v such that $d(u,v)=t$, $1\leq t\leq diamG$.

With the concepts of *i-Hamiltonicity* and *Hamiltonian Laceability,* we define the following

Definition 1: Let G be a connected graph of order n, let $h_p(G)$ be the length of a Hamiltonian path between any two distinct vertices in G. A Hamiltonian path in G is called a 0-*Hamiltonian path* if $h_p(G)=n-1$ and a path in G is called *1-Hamiltonian path* if $h_p(G)=n$.

Definition 2: Let i be a non-negative integer. A connected graph G of order n is called *i-Hamiltonian-t-laceable* if there exists in G, a *i-Hamiltonian path* between every pair of distinct vertices u and v with the property $d(u,v)=t$, $1\leq t\leq diamG$.

Definition 3: A connected graph G of order n is called *i-Hamiltonian-t*-laceable* if there exists in G, a *i-Hamiltonian path* between at least one pair of distinct vertices u and v with the property $d(u,v)=t$, $1\leq t\leq diamG$.

Figure 1 below illustrates a 1-*Hamiltonian* graph G with $h(G)=6$. With respect to the vertices v_1 *and* v_2 this graph is *1-Hamiltonian-2*-laceable.*

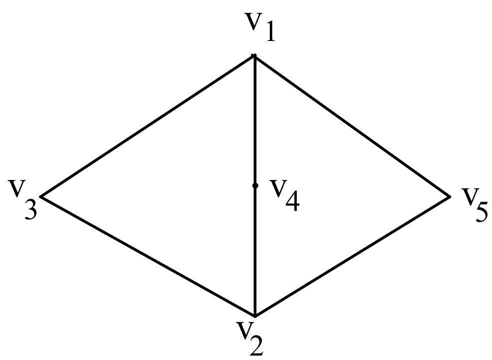

Figure 1: A graph with h(G)=6

Results

Theorem 1: Let $G=P_m$ *and* $H=P_n$. *If m and n are odd integers such that* $m,n\geq 3$, *the Cartesian-product* $G\times H$ *is 1-Hamiltonian-t*-laceable, for t=1, 3 and 5.*

Proof: Let $G_1=G\times H$. In G_1 there are mn vertices and diameter of $G\times H$ is $(m+n)-1$. Let the vertices of G_1 be denoted by a_{ij}, $1\leq i\leq m$, $1\leq j\leq n$.

Let B_i denote the m paths in G_1 given by; B_i: a_{i1}-a_{i2}-a_{i3}-………..-a_{in} and let P_j denote the n paths in G_1 given by; P_j: a_{1j}-a_{2j}-a_{3j}-………..-a_{mj}.

Then, in G_1, $d(a_{11},a_{12}) = 1$ and the path P: $\{P_1 \cup B_m \cup(a_{mn},a_{m-1n})\cup (a_{m-12}, a_{m-22}) \cup$

$(B_{m-1} - (a_{m-11}, a_{m-12})) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots . \cup (B_4 - (a_{41}, a_{42})) \cup (a_{42}, a_{32}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{3n}, a_{2n}) \cup (B_2 - (a_{2n}, a_{2n-1}) \cup \ldots\ldots \cup (a_{23}, a_{22}) \cup (a_{22}, a_{21})) \cup (B_1 - (a_{1n-1}, a_{1n-2}) \ldots\ldots . (a_{14}, a_{13}) \cup (a_{13}, a_{12})) \cup (a_{2n}, a_{1n}) \cup (a_{1n-1}, a_{2n-1}) \cup (a_{2n-2}, a_{1n-2}) \cup \ldots . \cup (a_{14}, a_{24}) \cup (a_{23}, a_{13}) \cup (a_{13}, a_{22}) \cup (a_{22}, a_{12}) \}$ is a 1-Hamiltonian path. Hence G_1 is *1-Hamiltonian-1*-laceable*.

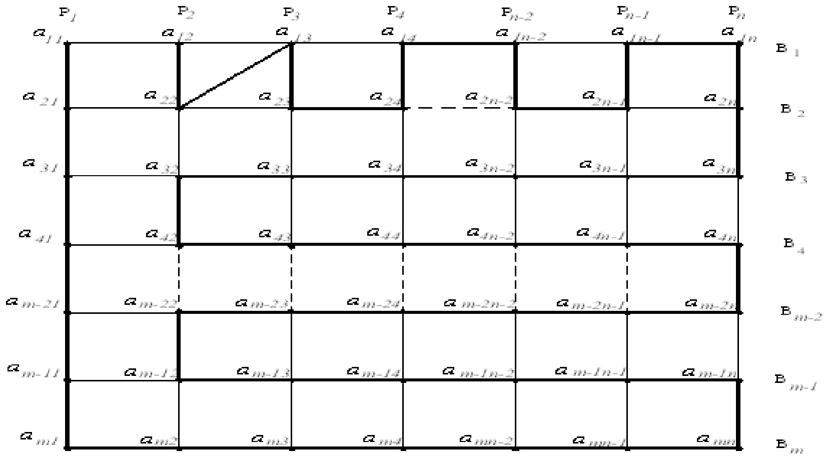

Figure 2: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{12}) = 1$

Also, in G_1, $d(a_{11},a_{14}) = 3$ and the path $P: \{P_1 \cup B_m \cup (a_{m-12}, a_{m-22}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots . \cup (B_4 - (a_{41}, a_{42})) \cup (a_{42}, a_{32}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{3n}, a_{2n}) \cup (a_{2n}, a_{1n}) \cup (a_{1n}, a_{2n-1}) \cup (B_2 - (a_{2n}, a_{2n-1}) \cup (a_{2n-1}, a_{2n-2}) \ldots . (a_{24}, a_{23})) \cup (B_1 - (a_{1n}, a_{1n-1}) \text{---} \cup (a_{11}, a_{12}))\}$ is a 1-Hamiltonian path. Hence G_1 is *1-Hamiltonian-3*-laceable*.

Figure 3: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{14}) = 3$

Further, in G_1 $d(a_{11},a_{1n-1}) = 5$ and the path $P: \{P_1 \cup B_m \cup (a_{m-12}, a_{m-22}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots\ldots \cup (B_4 - (a_{41}, a_{42})) \cup (a_{42}, a_{32}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{3n}, a_{2n}) \cup (a_{2n},a_{1n}) \cup (a_{1n},a_{2n-1}) \cup (a_{22},a_{12}) \cup (B_2-(a_{2n}, a_{2n-1}) \cup \ldots\ldots\ldots \cup (a_{21}a_{22})) \cup \{B_1-(a_{11},a_{12}) \cup \ldots\ldots \cup (a_{1n},a_{1n-1})\}$ is a 1-Hamiltonian path. Hence G_1 is *1-Hamiltonian-5*-laceable.*

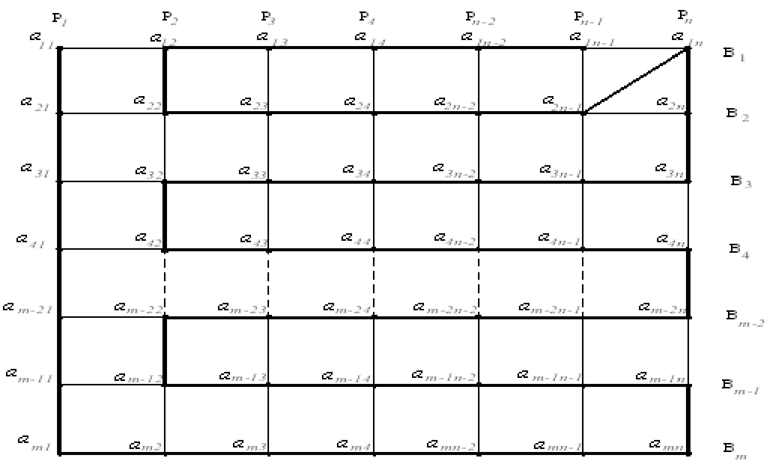

Figure 4: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11}, ,a_{1n-1}) = 5$

Hence the proof.

Theorem 2: Let *$G=P_m$ and $H=P_n$. If m and n are odd integers such that $m ,n \geq 3$, the Cartesian-product $G\times H$ is 1-Hamiltonian-t*-laceable, for t=2, 4 and 6.*

Proof: Let $G_1=G\times H$. In G_1 there are mn vertices and diameter of $G \times H$ is $(m+n)-1$. Let the vertices of G_1 be denoted by a_{ij}, $1\leq i\leq m$, $1\leq j\leq n$.

Let B_i denote the m paths in G_1 given by; B_i: a_{i1}-a_{i2}-a_{i3}-………..-a_{in} and let P_j denote the n paths in G_1 given by; P_j: a_{1j}-a_{2j}-a_{3j}-………..-a_{mj}.

Then, in G_1, $d(a_{11},a_{13}) = 2$ and the path $P: \{P_1 \cup B_m \cup (a_{mn},a_{m-1n}) \cup (a_{m-12}, a_{m-22}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots\ldots \cup (B_4 - (a_{41}, a_{42})) \cup (a_{42}, a_{32}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{3n}, a_{2n}) \cup (B_2 - (a_{2n}, a_{2n-1}) \cup \ldots\ldots \cup (a_{22}, a_{21})) \cup (a_{2n}, a_{1n}) \cup (B_1 - (a_{1n-1},a_{1n-2}) \ldots\ldots (a_{14}, a_{13}) \cup (a_{11},a_{12})) \cup (a_{1n-1},a_{2n-1}) \cup (a_{2n-2}, a_{1n-2}) \cup \ldots\ldots \cup (a_{14},a_{24}) \cup (a_{22},a_{12})\}$ is a 0-Hamiltonian path. Hence G_1 is *0-Hamiltonian-2*-laceable*.

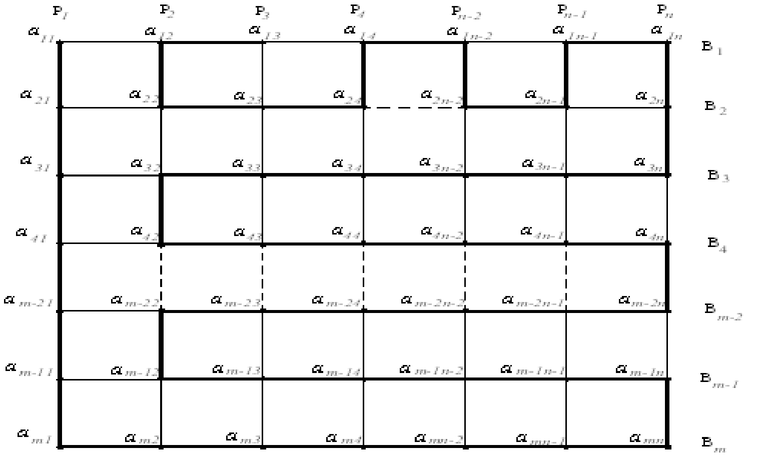

Figure 5: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{13}) = 2$

Also, in G_1, $d(a_{11},a_{1n-2}) = 4$ and the path $P: \{P_1 \cup B_m \cup (a_{mn}, a_{m-1n}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})) \cup (a_{m-12}, a_{m-22}) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots\ldots \cup (B_4 - (a_{41}, a_{42})) \cup (a_{42}, a_{32}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{3n}, a_{2n}) \cup (B_2-(a_{21}, a_{22}) \cup \ldots\ldots \cup (a_{2n},a_{2n-1}) \cup (B_1-(a_{11},a_{12}) \cup (a_{1n-2},a_{1n-1}) \cup (a_{11},a_{12}))\}$ is a 0-Hamiltonian path. Hence G_1 is *0-Hamiltonian-4*-laceable.*

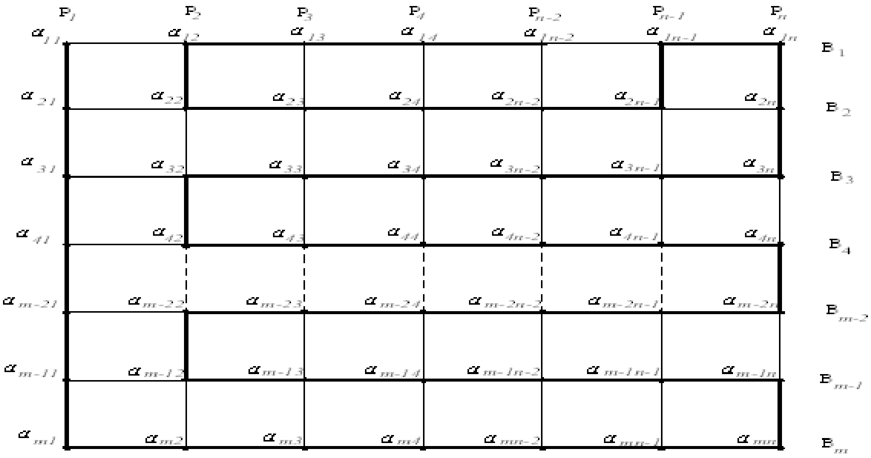

Figure 6: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{1n-2}) = 4$

Further, in G_1, $d(a_{11},a_{1n}) = 6$ and the path $P: \{P_1 \cup B_m \cup (a_{m-12}, a_{m-22}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots\ldots \cup (B_4 - (a_{41}, a_{42})) \cup (a_{42}, a_{32}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{3n}, a_{2n}) \cup (B_2-(a_{21}, a_{22})) \cup \{B_1-(a_{11},a_{12}))\}$ is a 0-Hamiltonian path. Hence G_1 is *0-Hamiltonian-6*-laceable*.

Figure 7: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{1n}) = 6$

Hence the proof

Theorem 3: *Let $G=P_m$ and $H=P_n$. If m and n are even integers such that $m, n \geq 3$, the Cartesian-product $G\times H$ is 1-Hamiltonian-t*-laceable, for t=2, 4 and 6.*

Proof: Let $G_1= G\times H$. In G_1 there are mn vertices and diameter of $G \times H$ is $(m + n) - 1$. Let the vertices of G_1 be denoted by a_{ij}, $1\leq i\leq m$, $1\leq j\leq n$. Let B_i denote the m paths in G_1 given by B_i: a_{i1}-a_{i2}-a_{i3}-……….-a_{in} and let P_j denote the n paths in G_1 given by P_j: a_{1j}-a_{2j}-a_{3j}-……….-a_{mj}.

Then in G_1, $d(a_{11},a_{13}) = 2$ and the path $P: \{P_1 \cup B_m \cup (a_{mn}, a_{m-1n}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})) \cup (a_{m-12}, a_{m-22}) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots\ldots \cup (B_4 - (a_{41}, a_{42})) \cup (a_{42}, a_{32}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{3n}, a_{2n}) \cup (B_2 - (a_{2n}, a_{2n-1}) \cup \ldots\ldots \cup (a_{21}, a_{22})) \cup (B_1 - (a_{1n-1},a_{1n-2}) \ldots\ldots(a_{11}, a_{12})) \cup (a_{2n},a_{1n}) \cup (a_{1n-1},a_{2n-1}) \cup (a_{2n-2},a_{1n-2}) \cup \ldots\ldots \cup (a_{14},a_{24}) \cup (a_{22},a_{12})\}$ is a *1-Hamiltonian path*. Hence G_1 is *1-Hamiltonian-2*-laceable*.

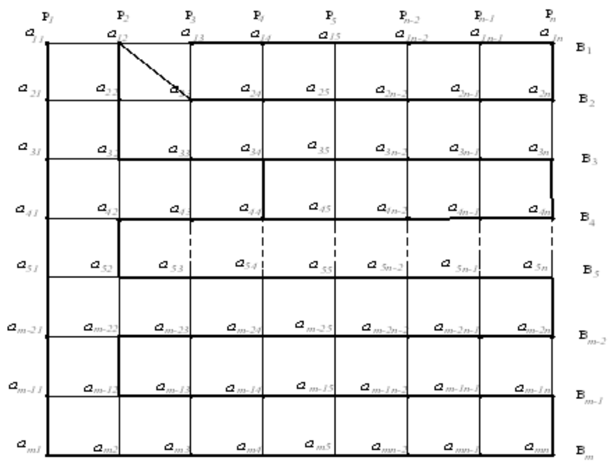

Figure 8: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{13}) = 2$

Also, in G_1, $d(a_{11},a_{1n-2}) = 4$ and the path $P: \{P_1 \cup B_m \cup (a_{mn}, a_{m-1n}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})) \cup (a_{m-12}, a_{m-22}) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots \cup (B_4 - (a_{41}, a_{42})) \cup (a_{42}, a_{32}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{3n}, a_{2n}) \cup (B_2 - (a_{2n}, a_{2n-1}) \cup \ldots \cup (a_{21}, a_{22})) \cup (B_1 - (a_{1n-1},a_{1n-2}) \ldots \cup (a_{11}, a_{12})) \cup (a_{2n},a_{1n}) \cup (a_{1n-1},a_{2n-1}) \cup \ldots \cup (a_{22},a_{12})\}$ is a *1-Hamiltonian path*. Hence G_1 is *1-Hamiltonian-4*-laceable*.

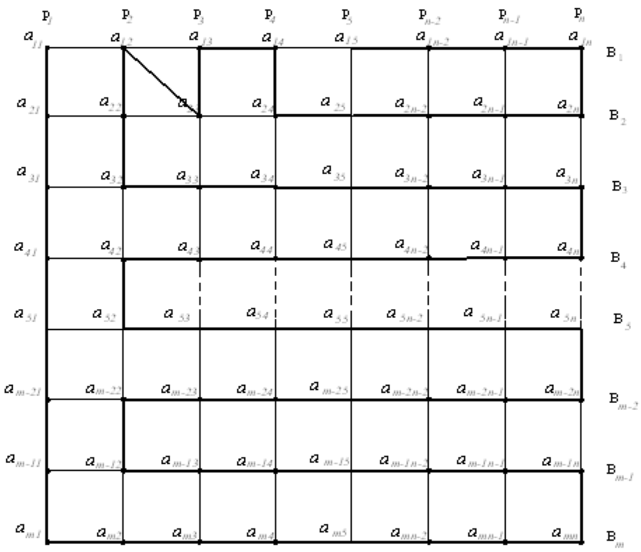

Figure 9: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{1n-2}) = 4$

Further in G_1, $d(a_{11},a_{1n}) = 6$ and the path $P: \{P_1 \cup B_m \cup (a_{mn}, a_{m-1n}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})) \cup (a_{m-12}, a_{m-22}) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots \cup (B_4 - (a_{41}, a_{42})) \cup (a_{42}, a_{32}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{3n}, a_{2n}) \cup (B_2 - (a_{21}, a_{22})) \cup (a_{22}, a_{12}) \cup (B_1-(a_{11}, a_{12}))\}$ is a 1-Hamiltonian path. Hence G_1 is 1-Hamiltonian-6*-laceable.

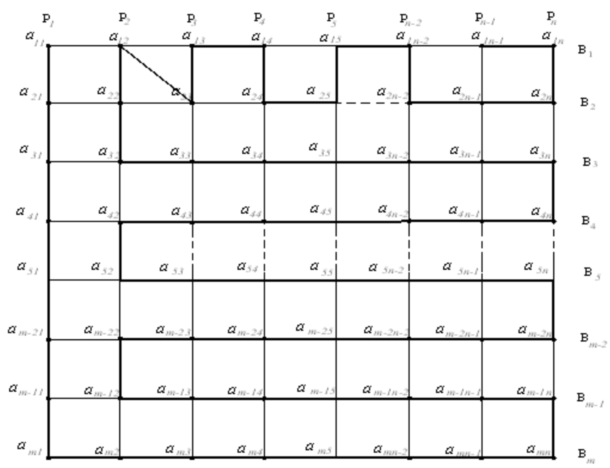

Figure 10: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{1n}) = 6$.

Hence the proof. ■

Theorem 4: *Let $G=P_m$ and $H=P_n$. Then the Cartesian-product $G \times H$ is 0-Hamiltonian-t*-laceable, for t=1, 3, 5 such that $1 \leq t \leq (m+n)-2$ where m and n be even for $m, n \geq 3$.*

Proof: Let $G_1= G \times H$. In G_1 there are mn vertices and diameter of $G \times H$ is $(m + n) - 1$. Let the vertices of G_1 be denoted by a_{ij}, $1\leq i\leq m$, $1\leq j\leq n$. Let B_i denote the m paths in G_1 given by B_i: a_{i1}-a_{i2}-a_{i3}-………..-a_{in} and P_j denote the n paths in G_1 given by; P_j: a_{1j}-a_{2j}-a_{3j}-………..-a_{mj}.

Then in G_1, $d(a_{11},a_{12}) = 1$ *and the path P:* $\{P_1 \cup B_m \cup(a_{mn},a_{m-1n})\cup (a_{m-12}, a_{m-22}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})\cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup\ldots\cup(B_5 - (a_{51}, a_{52})) \cup(a_{52}, a_{42}) \cup(B_4 - (a_{41}, a_{42})) \cup(a_{4n}, a_{3n}) \cup(B_3 - (a_{31}, a_{32}))\cup(a_{32}, a_{22}) \cup(B_2 - (a_{21}, a_{22})) \cup(a_{2n}, a_{1n})) \cup (B_1 - (a_{11},a_{12}))\}$ *is a 0-Hamiltonian path. Hence G_1 is 0-Hamiltonian-1*- laceable.*

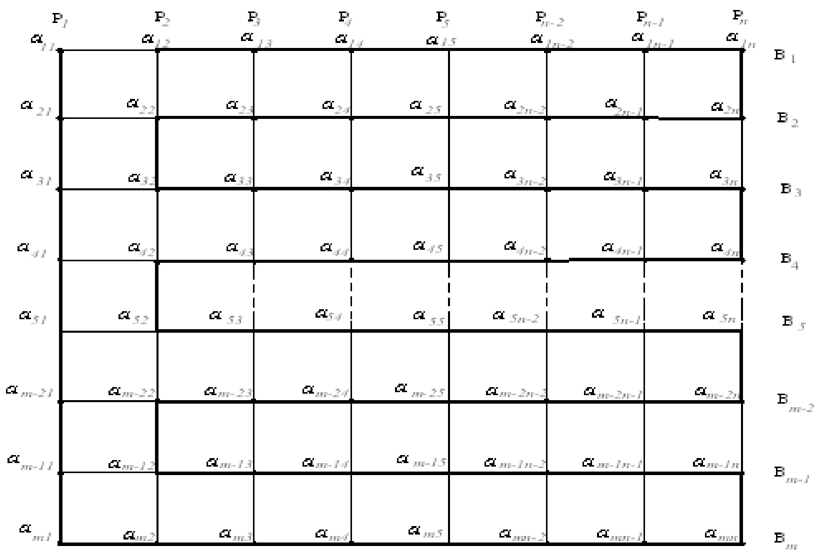

Figure 11: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{12}) = 1$

Also, in G_1, $d(a_{11},a_{14}) = 3$ *and the path P:* $\{P_1 \cup B_m \cup(a_{mn},a_{m-1n})\cup (a_{m-12}, a_{m-22}) \cup (B_{m-1} - (a_{m-11}, a_{m-12})\cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup\ldots\cup (B_5 - (a_{51}, a_{52})) \cup (a_{52}, a_{42}) \cup (B_4 - (a_{41}, a_{42})) \cup (a_{4n}, a_{3n}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{32}, a_{22}) \cup(a_{22},a_{12}) \cup(a_{13},a_{23}) \cup (B_2 - (a_{21}, a_{22}) \cup(a_{22},a_{23})) \cup (a_{2n}, a_{1n})\cup (B_1 - (a_{11},a_{12}) \cup(a_{13},a_{14}))\}$ *is a 0-Hamiltonian path. Hence G_1 is 0-Hamiltonian-3*- laceable.*

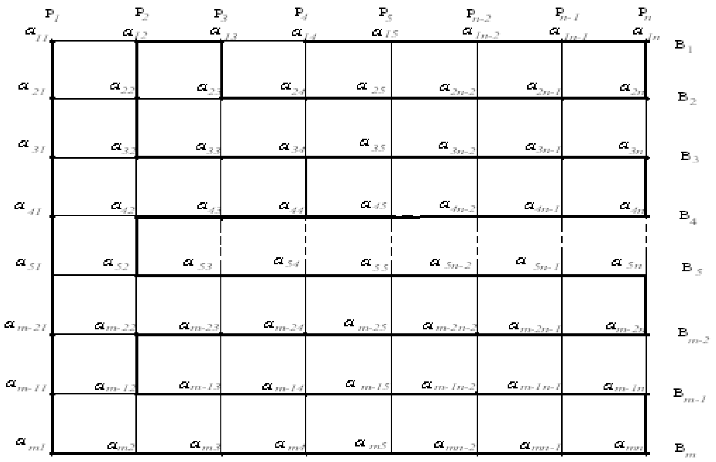

Figure 12: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{14}) = 3$

Further in G_1, $d(a_{11},a_{1n-1}) = 5$ *and the path* P: $\{P_1 \cup B_m \cup (a_{mn},a_{m-1n}) \cup (a_{m-12}, a_{m-22}) \cup (B_{m-1} - (a_{m-11}, a_{m-12}) \cup (B_{m-2} - (a_{m-21}, a_{m-22})) \cup \ldots . \cup (B_5 - (a_{51}, a_{52})) \cup (a_{52}, a_{42}) \cup (B_4 - (a_{41}, a_{42})) \cup (a_{4n}, a_{3n}) \cup (B_3 - (a_{31}, a_{32})) \cup (a_{32}, a_{22}) \cup (B_2 - (a_{21}, a_{22}) \cup (a_{22}, a_{23}) \cup (a_{24}, a_{25}) \ldots \ldots \cup (a_{2n-3}, a_{2n-2})) \cup (a_{2n}, a_{1n}) \cup (B_1 - (a_{11}, a_{12}) \cup (a_{13}, a_{14}) \cup \ldots \ldots \cup (a_{1n-3}, a_{1n-2})) \cup (a_{12}, a_{22}) \cup (a_{22}, a_{13}) \cup (a_{24},a_{14}) \cup (a_{15},a_{25}) \ldots \ldots \cup (a_{2n-3},a_{1n-3})\}$ *is a 0-Hamiltonian path. Hence* G_1 *is 0-Hamiltonian-*5^**- laceable.*

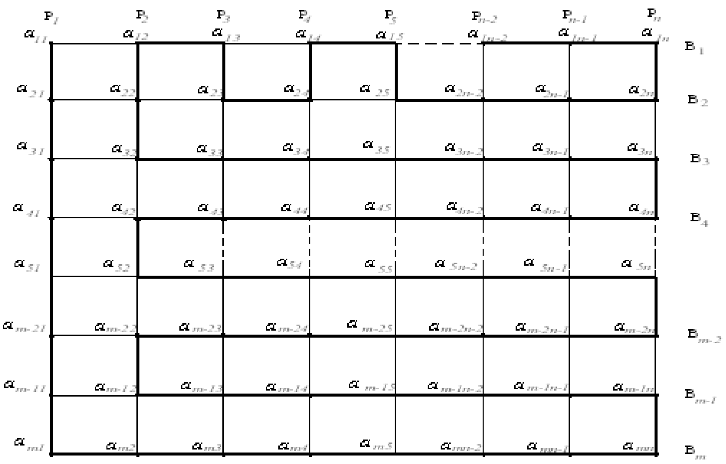

Figure 13: Cartesian product of $G=P_m$ and $H= P_n$, $d(a_{11},a_{1n-1}) = 5$

Hence the proof

Theorem 5: *Let* $G=C_m$ *and* $H=P_n$. *If* $n \geq 2$ *is an integer and* $m \geq 3$ *is an odd integer, the Cartesian-product* $G\times H$ is 0-Hamiltonian-t*-laceable for t=1,2 and 3.

Proof: Let $G_1 = G \times H$. Let the vertices of G_1 be denoted by a_{ij}, $1 \le i \le m$, $1 \le j \le n$. Let B_i denote the m paths in G_1 given by B_i: a_{i1}-a_{i2}-a_{i3}-………..-a_{in} and P_j denote the n paths in G_1 given by; P_j: a_{1j}-a_{2j}-a_{3j}-………..-a_{mj}. Where n is an integer and m is odd.

Then in G_1, $d(a_{11},a_{1n})=1$ and the path P: $P_1 \cup B_m \cup (a_{mn},a_{m-1n}) \cup (B_{m-1}-(a_{m-11},a_{m-12})) \cup (a_{m-12},a_{m-22}) \cup B_{m-2}-(a_{m-21},a_{m-22})) \cup \ldots.. \cup (B_4-(a_{41},a_{42})) \cup (a_{42},a_{32}) \cup (B_3-(a_{31},a_{32})) \cup (a_{3n},a_{2n}) \cup (B_2-(a_{22},a_{12})) \cup (a_{2n},a_{1n}) \cup (B_1-(a_{11},a_{12}))$ *is a 0-Hamiltonian path. Hence G_1 is a 0-Hamiltonian-1*- laceable.*

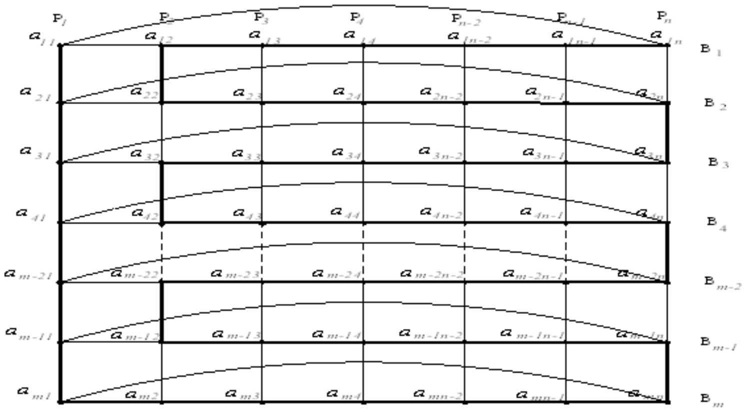

Figure 14: Cartesian product of $G=C_m$ and $H= P_n$, $d(a_{11},a_{1n})=1$

Also, in G_1, $d(a_{11},a_{1n-1})=2$ and the path P: $(a_{11},a_{1n}) \cup (a_{1n},a_{2n}) \cup (a_{2n},a_{21}) \cup (a_{21},a_{31}) \cup (a_{31},a_{3n}) \cup (a_{3n},a_{4n}) \cup (a_{4n},a_{41}) \cup \ldots\ldots.. \cup (a_{m-21},a_{m-2n}) \cup (a_{m-2n},a_{m-1n}) \cup (a_{m-1n},a_{m-11}) \cup (a_{m-11},a_{m1}) \cup (a_{m1},a_{mn}) \cup (B_m-(a_{mn},a_{mn-1}) \cup (a_{m1},a_{m2})) \cup (P_2-(a_{m2},a_{m-12})) \cup (a_{12},a_{13}) \cup (a_{2n},a_{1n})) \cup (P_3- (a_{m3},a_{m-13})) \cup (a_{m-13},a_{m-14}) \cup (P_4-(a_{m-14},a_{14})) \cup \ldots\ldots.. \cup (P_{n-2}-(a_{m-1n-2},a_{mn-2})) \cup (a_{m-1n-2},a_{m-1n-1}) \cup (P_{n-1}-(a_{m-1n-1},a_{mn-1}))$ *is a 0-Hamiltonian path. Hence G_1 is a 0- Hamiltonian-2*- laceable.*

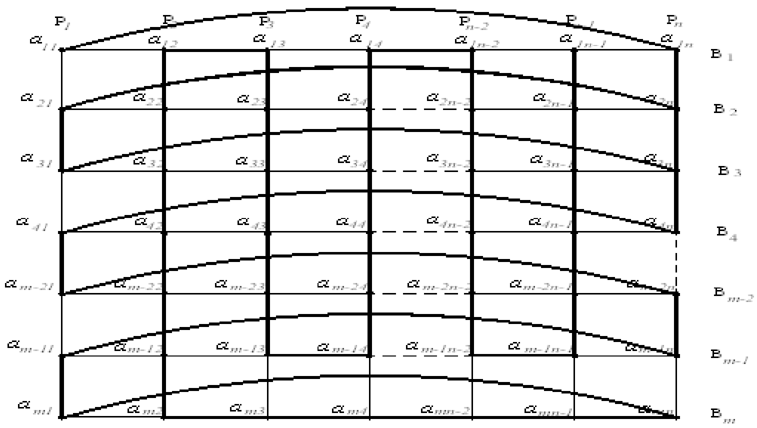

Figure 15: Cartesian product of $G=C_m$ and $H= P_n$, $d(a_{11},a_{1n-1})=2$

Further, in G_1, $d(a_{11},a_{1n-2})=3$ and the path

P: $(a_{11},a_{1n}) \cup (a_{1n},a_{2n}) \cup (a_{2n},a_{21}) \cup (a_{21},a_{31}) \cup (a_{31},a_{3n}) \cup (a_{3n},a_{4n}) \cup (a_{4n},a_{41}) \cup \ldots\ldots\ldots \cup (a_{m-21},a_{m-2n}) \cup (a_{m-2n},a_{m-1n}) \cup (a_{m-1n},a_{m-11}) \cup (a_{m-11},a_{m1}) \cup (a_{m1},a_{mn}) \cup (B_m\text{-}(a_{mn},a_{mn-1}) \cup (a_{m1},a_{m2}) \cup (a_{m2},a_{m-12}) \cup (B_{m-1}\text{-}(a_{m-11},a_{m-12}) \cup (a_{m-1n-1},a_{m-1n})) \cup (a_{m-1n-1},a_{m-2n-1}) \cup (B_{m-2}\text{-}(a_{m-2n},a_{m-2n-1}) \cup (a_{m-21},a_{m-22})) \cup \ldots\ldots \cup (B_4\text{-}(a_{41},a_{42}) \cup (a_{4n-1},a_{4n})) \cup (B_2\text{-}(a_{21},a_{22}) \cup (a_{22},a_{23})) \cup \ldots\ldots\ldots \cup (a_{2n-1},a_{2n}) \cup (a_{2n-1},a_{1n-1}) \cup (B_1\text{-}(a_{1n},a_{1n-1}) \cup \ldots\ldots \cup (a_{13},a_{14}) \cup (a_{11},a_{12}))$ *is a 0-Hamiltonian path. Hence* G_1 *is 0-Hamiltonian-*3^**-laceable.*

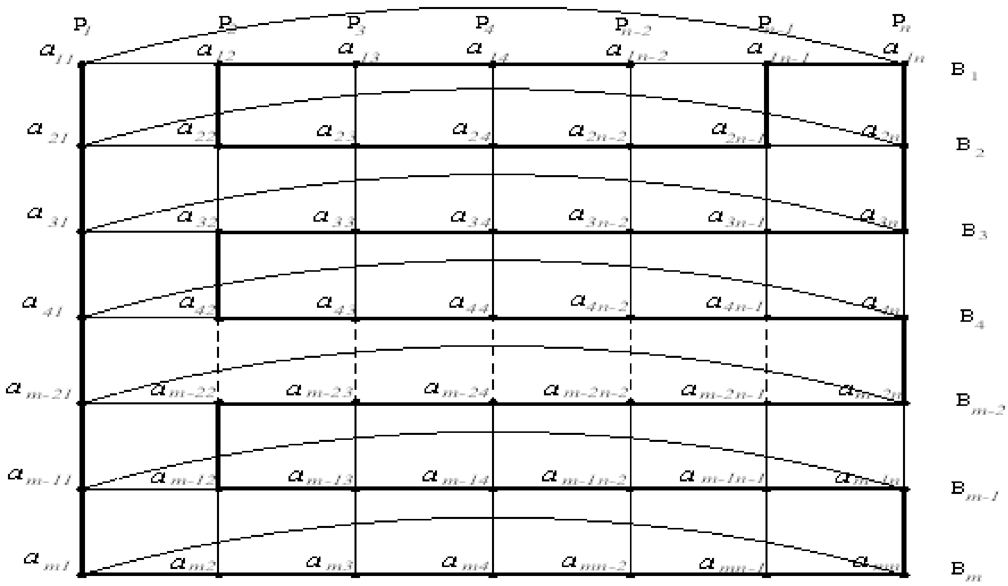

Figure 16: Cartesian product of $G=C_m$ and $H= P_n$, , $d(a_{11},a_{1n-2})=3$

Hence the proof

Theorem 6: *Let* $G=C_m$ *and* $H=P_n$*. If* $n \geq 2$ *is an integer and* $m \geq 3$ *is an even integer, the Cartesian-product* $G \times H$ is (i) *0-Hamiltonian-t*-laceable* for t=1 and 3 (ii) *1-Hamiltonian-t*-laceable* for t=2 and 4.

Proof: Let $G_1= G \times H$. Let the vertices of G_1 be denoted by a_{ij}, $1\leq i\leq m$, $1\leq j\leq n$. Let B_i denote the *m* paths in G_1 given by B_i: $a_{i1}\text{-}a_{i2}\text{-}a_{i3}\text{-}\ldots\ldots\ldots\ldots\text{-}a_{in}$ and P_j denote the n paths in G_1 given by; P_j: $a_{1j}\text{-}a_{2j}\text{-}a_{3j}\text{-}\ldots\ldots\ldots\ldots\text{-}a_{mj}$. Where *n* is any integer and *m* is even.

Then in G_1, $d(a_{11},a_{1n})=1$ and the path

P: $P_1 \cup (a_{m1},a_{m2}) \cup P_2 \cup (a_{12},a_{13}) \cup P_3 \cup (a_{m3},a_{m4}) \cup P_4 \cup \ldots\ldots \cup P_{n-1} \cup (a_{mn-1},a_{mn}) \cup P_n$ *is a 0-Hamiltonian path. Hence* G_1 *is 0-Hamiltonian-*1^**- laceable.*

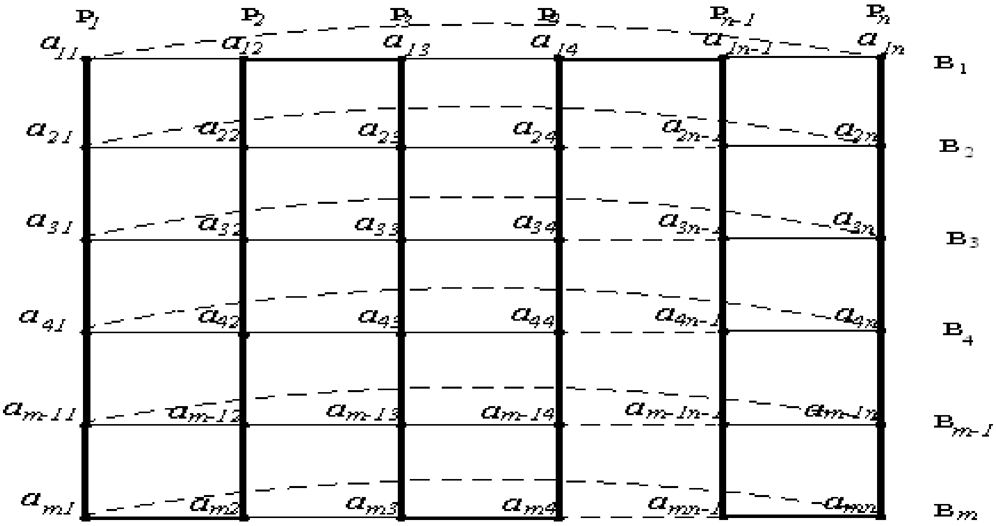

Figure 17: Cartesian product of $G=C_m$ and $H=P_n$, $d(a_{11},a_{1n})=1$

Also, in G_1, $d(a_{11},a_{2n})=2$ and the path

P: $P_1 \cup (a_{m1},a_{m2}) \cup P_2 \cup (a_{12},a_{13}) \cup P_3 \cup (a_{m3},a_{m4}) \cup P_4 \cup \ldots\ldots \cup (P_{n-1} \cup (a_{mn-1},a_{mn}) \cup (P_n-(a_{3n},a_{2n})) \cup (a_{3n},a_{1n})$ *is a 1-Hamiltonian path. Hence* G_1 *is 1-Hamiltonian-2**- laceable.*

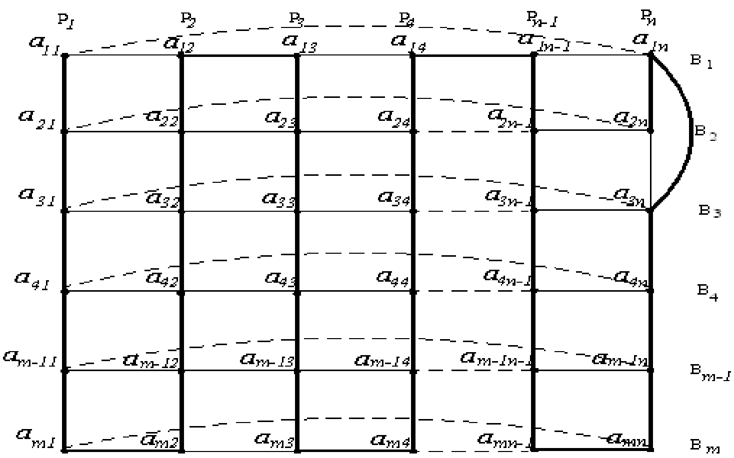

Figure 18: Cartesian product of $G=C_m$ and $H=P_n$, $d(a_{11},a_{2n})=2$

Further in G_1, $d(a_{11},a_{3n})=3$ and the path *P*: $P_1 \cup (a_{m1},a_{m2}) \cup P_2 \cup (a_{12},a_{13}) \cup P_3 \cup (a_{m3},a_{m4}) \cup P_4 \cup \ldots\ldots \cup (P_{n-1} -(a_{1n-1},a_{2n-1})) \cup (a_{1n-1},a_{1n}) \cup (a_{2n-1},a_{2n}) \cup (a_{mn-1},a_{mn}) \cup (P_n-(a_{3n},a_{2n}))$ *is a 0-Hamiltonian path. Hence* G_1 *is 0-Hamiltonian-3**- laceable.*

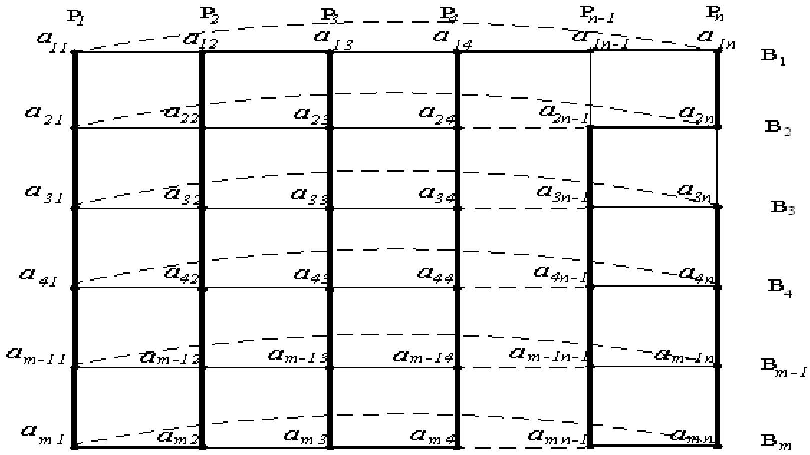

Figure 19: Cartesian product of $G=C_m$ and $H= P_n$, $d(a_{11},a_{3n})=3$

Next in G_1, $d(a_{11},a_{4n})=4$ and the path

$P: P_1 \cup (a_{m1},a_{m2}) \cup P_2 \cup (a_{12},a_{13}) \cup P_3 \cup (a_{m3},a_{m4}) \cup P_4 \cup \ldots\ldots \cup (P_{n-1} -(a_{1n-1},a_{2n-1})) \cup (a_{2n-1},a_{3n-1}) \cup (a_{mn-1},a_{mn}) \cup (P_n-(a_{1n},a_{2n}) \cup (a_{3n},a_{4n})) \cup (a_{1n},a_{2n-1})$ *is a 1-Hamiltonian path. Hence* G_1 *is 1-Hamiltonian-*4^**- laceable.*

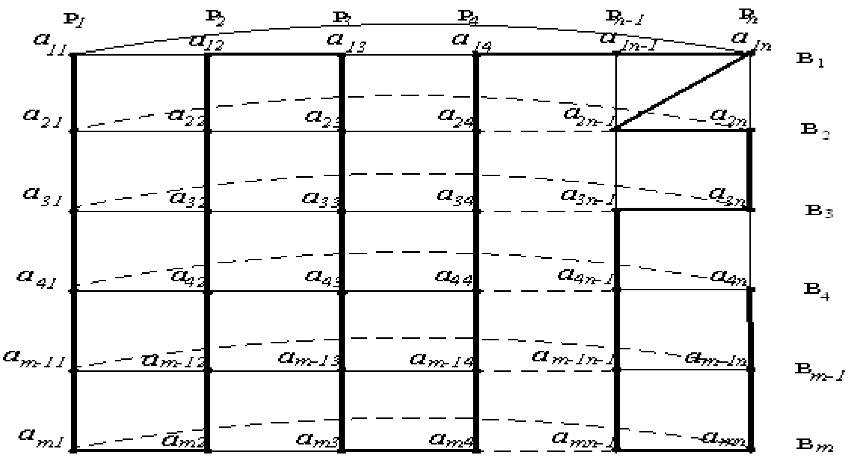

Figure 20: Cartesian product of $G=C_m$ and $H= P_n$, $d(a_{11},a_{4n})=4$

Hence the proof

Acknowledgements

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