International Journal of Computational Science and Mathematics.
ISSN 0974-3189 Volume 4, Number 2 (2012), pp. 159-165

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Edge Domination in Intuitionistic Fuzzy Graphs

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Abstract

In this paper we introduce the concept of edge domination and total edge domination in intuitionistic fuzzy graphs. We determine the edge domination number $\boldsymbol{\gamma'}$ and total edge domination number $\boldsymbol{\gamma'_t}$ for several classes of intuitionistic fuzzy graphs and obtain bounds for them. We also obtain Nordhaus gaddum type results for the parameters.

Keywords: Intuitionistic Fuzzy graph, Edge domination, Total edge domination, Independent edge domination.

Introduction

Atanassov [1] introduced the concept of intuitionistic fuzzy relations an intuitionistic fuzzy sets has been witnessing an rapid growth in mathematics and its applications. This ranges from traditional mathematics to information sciences. This leads to consider intuitionistic fuzzy graphs and their applications. R.Parvathy and M.G. Karunambigai's paper [2] introduced the concept of intuitionistic fuzzy graph and analysed its components. A.Nagoor Gani and S.Shajitha Begum [3] discussed the properties of various types of degrees, order and size of intuitionistic fuzzy graphs. In this paper, we introduce the concept of edge domination in intuitionistic fuzzy graphs. For graph theoretic terminology we refer to Harary 1969 [4] and for domination and edge domination in graphs we refer [5, 6]

Definitions

We review briefly some definitions in intuitionistic fuzzy graphs and introduce some new notations. Let V be a finite non empty set. Let E be the collection of all two element subsets of V. An intuitionistic fuzzy graph G = $((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is a set with four functions σ_1: V $\rightarrow$ [0,1] , $\sigma_2: V \rightarrow [0,1], \mu_1: E \rightarrow [0,1]$and $\mu_2: E \rightarrow$

[0,1] such that $0 \leq \sigma_1(x) + \sigma_2(x) \leq 1, \mu_1(x,y) \leq \sigma_1(x) \wedge \sigma_1(y), \mu_2(x,y) \leq \sigma_2(x) \vee \sigma_2(y)$ and $0 \leq \mu_1(x,y) + \mu_2(x,y) \leq 1$ for all x, y ∈ V. Here after we write $\mu_1(x,y), \mu_2(x,y)$ as $\mu_1(xy), \mu_2(xy)$

Let G = $((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph on V and $V_1 \subseteq V$. Define $\sigma_1' and \sigma_2'$ on $V_1 by$ $\sigma_1'(x)= \sigma_1(x)$ and σ_2'(x) = $\sigma_2(x)$ for all x ∈ V_1 and μ_1', μ_2' on the collection E_1 of two element subsets of V_1 by $\mu_1'(xy) = \mu_1(xy)$ and $\mu_2'(xy) = \mu_2(xy)$ for all $x, y \in V_1$. Then $H = \left((\sigma_1', \sigma_2'), (\mu_1', \mu_2')\right)$ is called an intuitionistic fuzzy sub graph of G induced by V_1 and is denoted by <V_1>.

The order p and size q of an intuitionistic fuzzy graph G = $((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ are defined to be p=$\sum_{x \in v}[\sigma_1(x) + \sigma_2(x)] and$ $q = \sum_{xy \in E}[\mu_1(xy) + \mu_2(xy)]$

Let $\sigma_1: V \to$ [0,1], $\sigma_2: V \to$ [0,1] be two fuzzy sets of V. Then the complete intuitionistic fuzzy graph on (σ_1, σ_2) is defined to be $((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ where $\mu_1(xy) = \sigma_1(x) \wedge \sigma_1(y)$ $and \mu_2(xy) = \sigma_2(x) \vee \sigma_2(y)$ for all x, y ∈ V and is denoted by$K_{(\sigma_1, \sigma_2)}$.

Let G = $((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ be an intuitionistic fuzzy graph on V and S ⊆ V. Then the intuitionistic fuzzy cardinality of S is defined to be $\sum_{V \in S}[\sigma_1(v) + \sigma_2(v)]$.

The complement of an intuitionistic fuzzy graph G denoted by $\bar{G}$=$((\sigma_1, \sigma_2), (\overline{\mu_1}, \overline{\mu_2})$ $where$ $\overline{\mu_1}$(xy)= $\sigma_1(x) \wedge \sigma_1(y) - \mu_1(xy)$ and $\overline{\mu_2}(xy) = \sigma_2(x) \vee \sigma_2(y) - \mu_2(xy)$ An edge e = xy of an intuitionistic fuzzy graph is called an effective edge if $\mu_1(xy) = \sigma_1(x) \wedge \sigma_1(y)$ and $\mu_2(xy) = \sigma_2(x) \vee \sigma_2(y)$.

$N(x) = \{y \in V / \mu_1(xy) = \sigma_1(x) \wedge \sigma_1(y) and$ $\mu_2(xy) = \sigma_2(x) \vee \sigma_2(y)\}$ is called the neighbourhood of x. $N[x] = N(x) \cup \{x\}$ is called the closed neighbourhood of x

The degree of a vertex can be generalised in different ways for an intuitionistic fuzzy graph G = (V,X). The weight of an effective edge e = xy with labelling (μ_1, μ_2) is defined as $\mu_1 + \mu_2$.

The effective degree of a vertex u is defined to be the sum of the weights of effective edges incident at u and is denoted by dE(u). $\sum_{v \in N(u)} \sigma_1(v) + \sigma_2(v)$ is called the neighbourhood degree and is denoted by dN(u).

The minimum effective degree $\delta_E(G)$= min$\{dE(u) | u \in V(G)\}$ and the maximum degree$\Delta_E(G) = max\{dE(u) | u \in V(G)\}$.

The $effective$ $edge$ $degree$ of an $edge$ $e = uv$ is $defined$ to be

$$\text{dE(e)}= \begin{cases} dE(u) + dE(v) - 1 \text{ } if \text{ } e \text{ } is \text{ } an \text{ } effective \text{ } edge \\ dE(u) + dE(v) \text{ } if \text{ } e \text{ } is \text{ } not \text{ } an \text{ } efeffective \text{ } edge \end{cases}$$

The minimum edge effective degree $\boldsymbol{\delta'}_{\boldsymbol{E}}(e) = min\{dE(e) | e \in X\}$ and the maximum edge effective degree $\Delta'_{\boldsymbol{E}}(\boldsymbol{e}) = max\{dE(e) | e \in X\}$. N(e) is the set of all effective edge incident with the vertices of e.

In a similar way minimum neighbourhood degree and the maximum neighbourhood degree denoted by δ_N and $\Delta_N$respectively can also be defined.

An intuitionistic fuzzy graph G = $((\sigma_1, \sigma_2), (\mu_1, \mu_2))$ is said to be bipartite if the vertex set V can be partitioned into two non $empty$ $V_1$and V_2 such that every edge e of G has one end say u in V_1 and the other end say v in V_2. Further if $\mu_1(uv) = \sigma_1(u) \wedge \sigma_1(v)$ and $\mu_2(uv) = \sigma_2(u) \vee \sigma_2(v)$ for all u ∈ V_1 and $v \in V_2$ then G is called a complete bipartite graph and is denoted by $K_{(\gamma_{11},\gamma_{12}),(\gamma_{21},\gamma_{22})}$ $where\ \gamma_{11},\gamma_{12}$ are respectively the restriction of σ_1 to $V_1\ and\ V_2$ and γ_{21},γ_{22} are respectively the restrictions of $\sigma_2\ to\ V_{1,}V_2$

Line graph L (G) of a graph G is the graph whose vertices are the edges of G and two vertices in L(G) are adjacent if and only if their corresponding edges are adjacent in G.

Let G = $((\sigma_1,\sigma_2),(\mu_1,\mu_2))$ be an intuitionistic fuzzy graph. Intuitionistic fuzzy line graph of G is the intuitionistic fuzzy graph $L_{IF}(G)$(or IFL(G)) = $((\sigma_{1f},\sigma_{2f}),(\mu_{1f},\mu_{2f}))$ is a set with four functions $\sigma_{1f}: V(L(G)) \rightarrow$ [0,1], $\sigma_{2f}: V\big(L(G)\big) \rightarrow$ [0,1] such that $\sigma_{1f}(e) = \mu_1(e)$,$\sigma_{2f}(e) = \mu_2(e)$ and $0 \leq \sigma_{1f}(e) + \sigma_{2f}(e) \leq 1$ and $\mu_{if}: E(L(G)) \rightarrow$ [0,1], $\mu_{2f}: E\big(L(G)\big) \rightarrow$ [0,1] such that $\mu_{1f}(x) \leq \mu_1(e_i) \wedge \mu_1\big(e_j\big), \mu_{2f}(x) \leq \mu_2(e_i) \vee \mu_2(e_j)$ and $0 \leq \mu_{1f}(x) + \mu_{2f}(x) \leq 1$ where x=e_ie_j.

Theorem 2.1 [7]: IF G is $k_{1,3}$- free then $\gamma(G) = \gamma_i(G)$

Edge Domination in Graph

Definition 3.1: Let G = $((\sigma_1,\sigma_2),(\mu_1,\mu_2))$ be an intuitionistic fuzzy graph on (V,X). A subset S of X is said to be an edge dominating set in G if for every edge not in X, there is an effective edge and is adjacent to some edge in X. The minimum intuitionistic fuzzy cardinality of an edge dominating set in G is called the edge domination number of G and is denoted by γ'.

The above definition of edge domination in intuitionistic fuzzy graph is motivated by the following sitiuation.. Let G be a graph which represents the road network of a city. Let the vertices denote the junctions and the edges denote the connecting junctions. From the statistical data that represents the number of vehicles passing through various junctions and the number of vehicles passing through various roads during a peak hour, the membership functions $(\sigma_{1,}\sigma_2)$ and (μ_1,μ_2) on the vertex set and edge set of G can be constructed by using the standard technique given in (Bobroewicz etal.,1990; Reha Civanlar and Joel Trussel,1986). In this fuzzy graph an edge dominating set S can be interpreted as a set of roads which are busy in the sense that every road not in S is connected to a member in S by having a common junction in which the traffic flow is full.

Example 3.2: If p is even then set of independent edges $e_{1,}e_2 \ldots e_{p/2}$ form an edge dominating set of K_{σ_1,σ_2}. If p is odd, then set of independent edges $e_1, e_2, \ldots., e_{(p-1)/2}$ together with one more edge say $e_{(p+1)/2}$ forms an edge domination set of K_{σ_1,σ_2} . We have $\gamma\big(K_{\sigma_1,\sigma_2}\big) = min\{\sum_e \mu_1(e) + \mu_2\,(e)\}\ where\ minimum\ is\ taken\ over$ all sets of independent edges of G if $|V|$ = p is even and $\gamma'(k_{\sigma_1,\sigma_2})$=$min\{\sum_e \mu_1(e) + \mu_2(e) \ni\}$ where minimum is taken over all independent edges of G together with one more edges of G if $|V|$ = p is odd.

Example 3.3: Let G be any intuitionistic fuzzy graph. Then $\gamma'(G)$=$\frac{p(p-1)}{2}$ if $\mu_1(xy) < \sigma_1(x) \wedge \sigma_2(y)$ and μ_2(xy) $<$ σ_2(x) $\vee$ σ_2(y) for all x,y $\in V$

In particular $\gamma'(\overline{k_p}) = \frac{p(p-1)}{2}$.

***Example*3.4**: $\gamma\left(K_{(\gamma_{11},\gamma_{12}),(\gamma_{21},\gamma_{22})}\right) = min\{\sum_e \mu_1(e) + \mu_2(e)\}$ where minimum is taken overall sets of independent edges of $K_{(\gamma_{11},\gamma_{12}),(\gamma_{21},\gamma_{22})}$.

Theorem 3.5: For any intuitionistic fuzzy graph G, $\gamma' + \overline{\gamma'} \leq p(p-1)$ where $\overline{\gamma'}$ is the edge domination number of $\bar{G}$ and equality holds if and only if $0 < \mu_1(xy) < \sigma_1(x) \wedge \sigma_1(y)\ and\ 0 < \mu_2(xy) < \sigma_2(x) \vee \sigma_2(y)$ for all x,y $\in$ V.

Proof: The equality is trivial. Further $\gamma' = \frac{p(p-1)}{2}\ if\ and\ only\ if\ \mu_1(xy) < \sigma_1(x) \wedge \sigma_1(y)\ and\ \mu_2(xy) < \sigma_2(x) \vee \sigma_2(y)$ for all x,y $\in$ V and $\overline{\gamma'} = \frac{p(p-1)}{2}$ if and only if $\sigma_1(x) \wedge \sigma_1(y) - \mu_1(xy) < \sigma_1(x) \wedge \sigma_1(y), \sigma_2(x) \vee \sigma_2(y) - \mu_2(xy) < \sigma_2(x) \vee \sigma_2(y)$ for all x,y $\in V$ which is equivalent to $\mu_1(xy) > 0$ and $\mu_2(xy) > 0$. Hence $\gamma' + \overline{\gamma'} = p(p-1)$ if and only if $0 < \mu_1(xy) < \sigma_1(x) \wedge \sigma_1(y)\ and\ 0 < \mu_2(xy) < \sigma_2(x) \vee \sigma_2(y)$ for all x,y $\in$ V.

Definition 3.6: An edge dominating set S of an intuitionistic fuzzy graph G is said to be minimal edge dominating set if no proper subset of S is an edge dominating set.

The following theorem gives the characterisation of minimal edge dominating set.

Theorem 3.7: An edge dominating set S is minimal if and only if for edge e $\in$ S, one of the following two conditions holds

a) N(e) $\cap\ S = \varphi$.

b) there exists an edge f $\in X - S$ such that N(f) $\cap\ S$ = {e} and f is an effective edge.

Proof: Let S be a minimal edge dominating set and e$\in S$. Then $S_e = S - \{e\}$ is not an edge dominating set and hence there exists f $\in X - S_e$ such that f is not dominated by any element of S_e. If f = e we get (a) and if f ≠ e we get (2). The converse is obvious.

Definition 3.8: An edge e of an intuitionistic fuzzy graph G is said to be an isolated edge if no effective edges incident with the vertices of e.

Thus an isolated edge does not dominate any other vertex in G.

Theorem 3.9: If G is a fuzzy graph without isolated edges then for every minimal edge dominating set S, X-S is also an edge domination set.

Proof: Let f be any edge in S. Since G has no isolated edges, there is an edge x$\in N(f)$. It follows from Theorem 3.6 that $c \in X - S$. Thus every element of S is dominated by some element of X - S.

Corollary 3 10: For any graph G without isolated edges, $\gamma' \leq \left\lceil \frac{p}{3} \right\rceil$.

Theorem 3.11: Let G be an intuitionistic fuzzy graph such that both G and $\bar{G}$ has no isolated edges. Then (i) $\gamma'(G) + \gamma'(\bar{G} \leq 2 \left\lceil \frac{p}{3} \right\rceil$ (ii) $\gamma'(G)\gamma'\overline{(G)} \leq \left\lceil \frac{p}{3} \right\rceil^2$

Definition 3.12: A set S of edges of an intuitionistic fuzzy graph is said to be independent if for every edge e$\in S$, no efficient edge of S is incident with the vertices of e.

Definition 3.13: An edge dominating S is said to be an independent edge dominating set if <S> is independent.

The minimum intuitionistic fuzzy cardinality of an independent edge dominating set of G is called the independent edge domination number of G and is denoted by $\gamma_i{}'$.

Theorem 3.14: For any intuitionistic fuzzy graph G, $\gamma' = \gamma_i{}'$.

Proof: Obviously $\gamma'(G) = \gamma(L_{IF}(G))$. As L(G) is $K_{1,3} -$ free graph , by Theorem. 2.1 $\gamma(L_{\mathrm{IF}}(G) = \gamma_i(L_{\mathrm{IF}}(G)$. Clearly $\gamma_i(L_{\mathrm{IF}}(G) = \gamma_i'(G)$. Hence $\gamma' = \gamma_i{}'$

Theorem 3.15: For any intuitionistic fuzzy graph G, $\gamma' \leq q \rightarrow \Delta_E{}'$ where q is the number. of edges of G.

Proof: Let e be an edge of maximum effective degree $\Delta_E{}'$. Let X be the edge set of G s.t **|X| = q** clearly X - N(e) is an edge dominating set of G so that $\boldsymbol{\gamma' \leq q - \Delta_E{}'}$**.**

Definition 3.16: Let G be an intuitionistic fuzzy graph without isolated edges. An edge dominating set X is said to be a total edge dominating set if <S> has no isolated edge.

The minimum intuitionistic fuzzy cardinality of a total edge dominating set is called the total edge domination number of G and is denoted by $\gamma_t{}'$.

Theorem 3.17: For any intuitionistic fuzzy graph G, $\boldsymbol{\gamma \leq \gamma_t{}'}$

Theorem 3.18: For any fuzzy graph G with q edges $\boldsymbol{\gamma_t{}' = q}$ if and only if every edge of G has a unique neighbour.

Proof: If every edge of G has a unique neighbour then X is the only total edge dominating set of G so that $\gamma_t{}' = q$. Conversely suppose $\gamma_t{}' = q$. If there exists an edge with neighbours x and y then X-{x} is a total edge dominating set of G. So that $\gamma_t{}' < q$ which is contradiction.

Corollary 3.19: If $\gamma_t{}' = q$, then the number of edges of G is even.

Theorem 3.20: Let G be an intuitionistic fuzzy graph without isolated edges. Then $\gamma_t{}' + \overline{\gamma_t} \leq 2q$ and equality holds if and only if

- The number of edges in G is even say 2n.
- There is a set S_1 of n mutually disjoint $P_3{}'s$ ((P_n *denotes the path on n vertices*) in G.
- There is a set S_2 of n mutually disjoint $P_3{}'s$ in $\bar{G}$ and
- For any edge xy $\notin S_1 \cup S_2$, $0 < \mu_1(xy) < \sigma_1(x) \wedge \sigma_1(y)$ *and* $0 < \mu_2(xy) < \sigma_2(x) \vee \sigma_2(y)$.

Proof: Since $\gamma_t{}' \leq q$ and $\overline{\gamma_t{}'} \leq 2q$, the inequality follows. Further $\gamma_t{}' + \overline{\gamma_t{}'} = 2q$ if and only if $\gamma_t{}' = q$ and $\overline{\gamma_t{}'} = q$ and hence by corollary 3.19. The number of edges in G is even say 2n. Since $\gamma_t{}' = q$, there is a set S_1 of n disjoint $P_3{}'s$ in G. Similarly there is a set s_2 of n disjoint $P_3{}'$ in $\bar{G}$. Further xy $\notin S_1 \cup S_2$ then $0 < \mu_1(xy) < \sigma_1(x) \wedge \sigma_1(y)$ *and* $0 < \mu_2(xy) < \sigma_2(x) \vee \sigma_2(y)$. The converse is obvious.

Conclusion

The concept of edge domination in graphs is very rich both in theoretical developments and applications. Many edge domination parameters have been investigated by different authors and in this paper we have introduced the concept of edge domination, total edge domination and independent edge domination in fuzzy graphs. Work on other edge domination parameters will be reported in forthcoming papers.

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