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Geo/Geo/c/k Interdependent Queueing Model with Controllable Arrival Rates and Feedback

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Abstract

In this paper the Geo/Geo/c/k queueing model with controllable arrival rates, c-server with identical service rates and feedback is considered. The steady state solution and system characteristics are derived for this model. The analytical results are numerically illustrated and the effect of the nodal parameters on the system characteristics are studied and relevant conclusion is presented.

Keywords: Multi server, Controllable arrival rates, bivariate Bernoulli feedback, finite capacity, system characteristics.

AMS 2000 subject classification: primary 60k25; secondary 68M20. 90B22.

INTRODUCTION

Queueing theory continues to be one of the most extensive theories of stochastic models. A large number of results in queueing theory is based on research on behavioural problems. Many practical queueing systems especially those with feedback have been widely applied to real life situations, such as the problem involving hospital emergency wards handling critical patients and unsatisfied customers in public telephone booths of coin box type etc. In day today life, one encounters numerous examples of queueing situations where all arriving customers require the main service and only some requires the secondary service provided by the server. In the words of Bhat [1], “A queueing model in which the arrivals and services are correlated is known as interdependent queueing model. Takacs [12] considered a queue with feedback customers which has applications in real life formulation of queue with feedback mechanism. Thiagarajan and Srinivasan [10] have analysed various queueing models with controllable arrival rates having interdependent interarrivals and service times. Kalyanaraman and Renganathan [6] have studied vacation queueing models with instantaneous Bernoulli feedback. In most of the research works, the authors have considered that the arrival and service patterns are independent. But in many real life situations, the arrival and service patterns are interdependent. Srinivasan and Thiagarajan [11] have also analysed a finite capacity multi-server poisson input queue with interdependent inter-arrival rates and obtained the average system size and average waiting time in the system under steady state conditions.Kalyanaraman and Sumathy [7] have studied a feedback queue with multiple servers and batch service. Recently Rani and Srinivasan [9] have studied a multiserver loss and delay interdependent queueing model with controllable arrival rates, no passing and feedback, In the earlier work, Goswami and Gupta [2] have obtained the distribution of the number of customer served during a busy period in a discrete time Geo/Geo/1 queue. Thiagarajan and Srinivasan [13] have analysed Geo/Geo/c/∞ interdependent queueing model with controllable arrived rates and obtained the steady state probabilities and the system characteristics when the joint distribution of inter-arrival and service time is a bivariate geometric distribution. In the present paper, a mathematical model for a Geo/Geo/c/k interdependent queueing model with controllable arrival rates and feedback is described, the steady state equations are derived, the steady state probabilities and the system characteristics are obtained. The analytical results are numerically illustrated and the effect of the modal parameters on the system characteristics are studied and relevant conclusion is presented.

DESCRIPTION OF THE MODEL

Consider c-server finite capacity queueing system with controllable arrival rates and feedback. Customers arrive at the service station one by one according to a bivariate Geometric stream with arrival rates $\lambda_0, \lambda_1 (> 0)$ and mean dependence rate ε. There are c-servers which provides service to all the arriving customers. Service time are independent and identically distributed Bernoulli random variables with service rate μ_n and mean dependence rate ε. After the completion of each service, the customer can either join at the end of the queue with probability p or he can leave the system with probability q with p + q = 1. Customer both newly arrived and those who require feedback are served in the order in which they join the tail of the original queue. It is assumed that there is no difference between the regular arrival and feedback arrival. The customers are served according to the first come first served rule with following assumptions.

The arrival process $\{X_1(t)\}$ and the service completion process $\{X_2(t)\}$ of the system are correlated and follow a bivariate Bernoulli process given by

$$P\{X_1(t) = x_1, X_2(t) = x_2\} = \sum_{j=0}^{\min(x_1,x_2)} \left[(\lambda_i - \varepsilon)t\right]^{x_1 - j} \left[(\mu_n - \varepsilon)t\right]^{x_2 - j} \times \left(\left[1 - (\lambda_i - \varepsilon)t\right]^{1-(x_2-j)} \left[1 - (\mu_n - \varepsilon)t\right]^{1-(x_2-j)}\right)$$

$x_1, x_2 = 0,1;\ \lambda_i, \mu_n > 0,\ i = 0,1;\ \ 0 \le \varepsilon < \min(\lambda_i, \mu_n),\ i = 0,1;$
$n = 0,1,2, \ldots, c-1, c, c+1, \ldots r-1, r, r+1, \ldots, R-1, R, R+1, \ldots K-1, K$

with parameters $\lambda_0(\lambda_1), \mu_n$ and ε as mean faster (slower) rate of arrivals, mean service rate and co-variance between arrival and service processes respectively. It is also assumed that c < r. The mean service rate when the system size n is defined as

$$\mu_n = \begin{cases} nq\mu; & 0 \le n < c \\ cq\mu; & c \le n \le k \end{cases}$$

The postulates of the model are

1. the probability that there is no arrival and no service completion during any interval, when the system is in faster rate of arrivals either with feedback or without feedback, is $\left[1-(\lambda_0-\varepsilon)t\right]\left[1-\left\{[p(\mu-\varepsilon)]_n+[q(\mu-\varepsilon)]_n\right\}t\right]$
2. the probability that there is no arrival and one service completion during any internal, when the system is in faster rate of arrivals either with feedback or without feedback, is $\left[\left[1-(\lambda_0-\varepsilon)t\right]\left\{[p(\mu-\varepsilon)]_n+[q(\mu-\varepsilon)]_n\right\}t\right]$
3. the probability that there is one arrival and no service completion during any interval, when the system is in faster rate of arrivals either with feedback or without feedback, is $\left[(\lambda_0-\varepsilon)t\right]\left[1-\left\{[p(\mu-\varepsilon)]_n+[q(\mu-\varepsilon)]_n\right\}t\right]$
4. the probability that there is one arrival and one service completion during any interval, when the system is in faster rate of arrivals either with feedback or without feedback, is $\left[(\lambda_0-\varepsilon)t\right]\left[\left\{[p(\mu-\varepsilon)]_n+[q(\mu-\varepsilon)]_n\right\}t\right]$
5. the probability that there is no arrival and no service completion during any interval, when the system is in slower rate of arrivals either with feedback or without feedback, is $\left[1-(\lambda_1-\varepsilon)t\right]\left[1-\left\{[p(\mu-\varepsilon)]_n+[q(\mu-\varepsilon)]_n\right\}t\right]$
6. the probability that there is no arrival and one service completion during any interval, when the system is in slower rate of arrivals either with feedback or without feedback, is $\left[1-(\lambda_1-\varepsilon)t\right]\left[\left\{[p(\mu-\varepsilon)]_n+[q(\mu-\varepsilon)]_n\right\}t\right]$
7. the probability that there is one arrival and no service completion during any interval, when the system is in slower rate of arrivals either with feedback or without feedback, is $\left[(\lambda_1-\varepsilon)t\right]\left[1-\left\{[p(\mu-\varepsilon)]_n+[q(\mu-\varepsilon)]_n\right\}t\right]$
8. the probability that there is one arrival and service completion during any interval, when the system is in slower rate of arrivals either with feedback or without feedback, is $\left[(\lambda_1-\varepsilon)t\right]\left[\left\{[p(\mu-\varepsilon)]_n+[q(\mu-\varepsilon)]_n\right\}t\right]$

STEADY STATE EQUATIONS

We observe that only $p_n(0)$ exists when n = 0, 1, 2, . . . , c – 1, c. . . . r – 1, r; both $P_n(0)$ and $P_n(1)$ exist when n = r + 1, r + 2, . . . R – 1; $P_n(1)$ exists when n = R, R+1, . . . k, further $P_n(0) = P_n(1) = 0$ if n > k.

Let $\rho_0 = \frac{1}{\lambda_0 - \varepsilon}$, $\rho_1 = \frac{1}{\lambda_1 - \varepsilon}$, $q_0 = \frac{1}{q(\mu - \varepsilon)}$, $p_0 = \frac{1}{p(\mu - \varepsilon)}$, $\overline{\rho_0} = 1 - \rho_0, \overline{\rho_1} = 1 - \rho_1$, $\overline{q_0} = 1 - q_0$ and $\overline{p_0} = 1 - \overline{p_0}$.

Then the stationary equations which are written through the matrix of densities are given by

$$(\rho_0 \overline{q_0} + \rho_0 \overline{p_0}) P_0(0) = \overline{\rho_0} q_0 P_1(0) \quad \ldots (1)$$

$$\left(\overline{\rho_0} q_0 + \rho_0 \overline{q_0} + \rho_0 \overline{p_0}\right) P_1(0) = \left(\rho_0 \overline{p_0} + \rho_0 \overline{q_0}\right) P_0(0) + 2\overline{\rho_0} q_0 P_2(0) \quad (2)$$

$$\left(2\overline{\rho_0} q_0 + 2\rho_0 \overline{q_0} + \rho_0 \overline{p_0}\right) P_2(0) = \left(\rho_0 \overline{q_0} + \rho_0 \overline{p_0}\right) P_1(0) + 3\overline{\rho_0} q_0 P_3(0) \quad (3)$$

$$\left(n \overline{\rho_0} q_0 + n \rho_0 \overline{q_0} + n \rho_0 \overline{p_0}\right) P_n(0) = (n-1)\left(\rho_0 \overline{q_0} + \rho_0 \overline{p_0}\right) P_{n-1}(0)$$
$$+(n+1)\rho_0 \overline{q_0} P_{n+1}(0), \qquad n = 2, 3, 4, \ldots c-1 \quad \ldots (4)$$

$$\left(c \overline{\rho_0} q_0 + c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0}\right) P_c(0) = \left[(c-1)\rho_0 \overline{q_0} + (c-1)\rho_0 p_0\right] P_{c-1}(0)$$
$$+ c \overline{\rho_0} q_0 P_{c-1}(0) \quad \ldots (5)$$

$$\left(c \overline{\rho_0} q_0 + c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0}\right) P_n(0) = \left(c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0}\right) P_{n-1}(0) + c \rho_0 q_0 P_{n+1}(0) \quad \ldots (6)$$
$$n = c+1, c+2, c+3, \ldots r-1$$

$$\left(c \overline{\rho_0} q_0 + c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0}\right) P_r(0) = \left(c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0}\right) P_{r-1}(0) + c \overline{\rho_0} q_0 P_{r+1}(0)$$
$$+ c \rho_1 q_0 P_{r+1}(1) \quad \ldots (7)$$

$$\left[c \overline{\rho_0} q_0 + c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0}\right] P_n(0) = \left(c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0}\right) P_{n-1}(0) + c \overline{\rho_0} q_0 P_{n+1}(0) \quad \ldots (8)$$
$$n = r+1, r+2, \ldots R-2$$

$$\left(c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0} + c \overline{\rho_0} q_0\right) P_{R-1}(0) = \left(c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0}\right) P_{R-2}(0) \quad (9)$$

$$\left(c \rho_1 \overline{q_0} + c \overline{\rho_1} q_0 + c \rho_1 \overline{p_0}\right) P_{r+1}(1) = c \overline{\rho_1} q_0 P_{r+2}(1) \quad (10)$$

$$\left(c \rho_1 \overline{q_0} + c \overline{\rho_1} q_0 + c \rho_1 \overline{p_0}\right) P_n(1) = \left(c \rho_1 \overline{q_0} + c \rho_1 \overline{p_0}\right) P_{n-1}(1) + c \overline{\rho_1} q_0 P_{n+1}(1) \quad \ldots (11)$$

$$\left(c \overline{\rho_1} q_0 + c \rho_1 \overline{q_0} + c \rho_1 \overline{p_0}\right) P_R(1) = \left(c \rho_0 \overline{q_0} + c \rho_0 \overline{p_0}\right) P_{R-1}(0)$$
$$+ \left(c \rho_1 \overline{q_0} + c \rho_1 \overline{p_0}\right) P_{R-1}(1) + c \overline{\rho_1} q_0 P_{R+1}(1) \quad \ldots (12)$$

$$\left(c \overline{\rho_1} q_0 + c \rho_1 \overline{q_0} + c \rho_1 \overline{p_0}\right) P_n(1) = \left(c \rho_1 \overline{q_0} + c \rho_1 \overline{p_0}\right) P_{n-1}(1) + c \overline{\rho_1} q_0 P_{n+1}(1) \quad \ldots (13)$$
$$n = R+1, R+2, R+3, \ldots k-1$$

$$\left(c \overline{\rho_1} q_0 + c \rho_1 \overline{q_0} + c \rho_1 \overline{p_0}\right) P_k(0) = \left(c \rho_1 \overline{q_0} + c \rho_1 \overline{p_0}\right) P_{k-1} \quad \ldots (14)$$

Let $\rho(0)=\dfrac{\lambda_0-\varepsilon}{q(\mu-\varepsilon)}$ and $\rho(1)=\dfrac{\lambda_1-\varepsilon}{q(\mu-\varepsilon)}$

where $\rho(0)=\left(\dfrac{q_0}{\rho_0}\right)$ is faster rate of arrivals intensity and $\rho(1)=\left(\dfrac{q_0}{\rho_1}\right)$ is slower rate of arrivals intensity.

From (1), (2) and (3), (4) it can be shown that

$$P_n(0)=\frac{1}{n}(R+S)^n P_0(0)\text{, where } R=\frac{\rho_0\overline{q_0}}{\overline{\rho_0}\,q_0} \text{ and } S=\frac{\rho_0\overline{p_0}}{\overline{\rho_0}\,q_0} \tag{15}$$

n = 1, 2, 3, 4, . . . c,

using the result (15) in (5) and (6) we get

$$P_n(0)=\frac{1}{c}(R+S)^n P_0(0)\text{, n + c + 1, c + 2, c + 3, . . . r} \tag{16}$$

From equation (7), (8) and (16) we get

$$P_n(0)=\frac{1}{c}[R+S]^n P_0(0)-\frac{\left[1-(R+S)^{n-r}\right]}{1-(R+S)}\frac{\overline{\rho_1}}{\overline{\rho_0}}P_{r+1}(1) \tag{17}$$

n = r + 1, r + 2, . . . R – 1

using the result (17) in (9) we get

$$P_{r+1}(1)=\frac{1}{c}[R+S]^{R+r}\,\frac{\left(1-(R+S)\right)}{(R+S)^r-(R+S)^R}\,\frac{\overline{\rho_0}}{\overline{\rho_1}}P_0(0) \tag{18}$$

From equations (10) and (11), we get

$$P_n(1)=\left[\frac{1-(T+U)^{n-r}}{1-(T+U)}\right]P_{r+1}(1)\text{, where } T=\frac{\rho_1\overline{q_0}}{\overline{\rho_1}\,q_0} \text{ and } U=\frac{\rho_1\overline{p_0}}{\overline{\rho_1}\,q_0} \tag{19}$$

n = r + 2, r + 3, r + 4, . . . R – 1,

using the result (19) in (12) and (13), we get

$$P_n(1)=\left[\frac{1-(T+U)^{n-r}}{1-(T+U)}-\left(\frac{\alpha'+\beta'}{R+S}\right)\frac{\left[1-(T+U)^{n-R}\right]}{1-(T+U)}\frac{\overline{\rho_1}}{\overline{\rho_0}}\right]P_{r+1}(1)$$

where $p_{r+1}(1)$ is given by (18)

$$P_n(1)=\left[\frac{1-(T+U)^{n-r}}{1-(T+U)}-\left(\frac{\alpha'+\beta'}{R+S}\right)\frac{\left[1-(T+U)^{n-R}\right]}{1-(T+U)}\frac{\overline{\rho_1}}{\overline{\rho_0}}\right]P_{r+1}(1) \tag{20}$$

n = R, R + 1, R + 2, . . . k – 1, k

CHARACTERISTICS OF THE MODEL

In this section analytical expression for the systems characteristics are derived, and the steady state probabilities are expressed interms of $P_0(0)$.

1. The probability P(0) that the system is in faster rate of arrivals either with feedback or without feedback.

$$P(0) = \sum_{n=0}^{c} P_n(0) + \sum_{n=c+1}^{r} P_n(0) + \sum_{n=r+1}^{R-1} P_n(0) \tag{21}$$

using the result (15), (16), (17) and (18) in (21), we get,

$$P(0) = \left[\sum_{n=0}^{c} \frac{1}{n}(R+S)^n + \frac{A}{c(1-(R+S))}\right] P_0(0) \tag{22}$$

Where

$$A = \left[(R+S)^{c+1} - (R+S)^R\right] - \left[(R-r)(1-(R+S)) - \left(1-(R+S)^{R-r}\right)\right]\frac{(R+S)^{R-r}}{(R+S)^r - (R+S)^R}$$

2. The probability P(1) that the system is in slower rate of arrivals either with feedback or without feedback.

$$P(1) = \sum_{n=r+1}^{R} P_n(1) + \sum_{n=R+1}^{K} P_n(1) \tag{23}$$

using the result (19) and (20) in (23), we get,

$$P(1) = \left\{\frac{B}{1-(T+U)} + \frac{C+D}{\left[1-(T+U)\right]^2}\right\} P_{r+1}(1) \tag{24}$$

where

$$B = (k-r) - \left(\frac{\alpha' + \beta'}{R+S}\right)\frac{\overline{\rho_1}}{\overline{\overline{\rho_0}}}(k-R)$$

$$C = \left(\frac{\alpha' + \beta'}{R+S}\right)\frac{\overline{\rho_1}}{\overline{\overline{\rho_0}}}(T+U)(1-(T+U)^{k-R})$$

$D = (T+U)^{R-r+1} + (T+U)^{k-R+1} - 2(T+U)$ and $P_{r+1}(1)$ is given by (18)

3. The probability $P_0(0)$ that the system is empty can be calculated from the normalizing condition P(0) + P(1) = 1. Therefore from equation (22) and (24) we get

$$P_0(0)=\left\{\sum_{n=0}^{c}\frac{1}{n}(R+S)^n+\frac{A}{c\left(1-(R+S)\right)}+\left[\frac{B}{\left[1-(T+U)\right]}+\frac{C+D}{\left[1-(T+U)\right]^2}\right]E\right\}^{-1} \quad (25)$$

where

$$E=\frac{1}{c}(R+S)^{R+r}\frac{\left(1-(R+S)\right)}{(R+S)^r-(R+S)^R}\frac{\overline{\rho_0}}{\overline{\overline{\rho_1}}}$$

4. Expected number of customers in the system Ls_0, when the system is in faster rate of arrivals either with feedback or without feedback.

$$Ls_0=\sum_{n=1}^{c}nP_n(0)+\sum_{n=c+1}^{r}nP_n(0)+\sum_{n=r+1}^{R-1}nP_n(0) \quad (26)$$

using the result (15), (16), (17) and (18) in (26), we get

$$Ls_0=\frac{1}{1-(R+S)}\left[\frac{F}{c}+GE\frac{\overline{\rho_1}}{\overline{\overline{\rho_0}}}\right]P_0(0) \quad (27)$$

where

$$F=c\left[R+S\right]-R\left[R+S\right]^R+\frac{(R+S)^{c+1}-(R+S)^{R+1}}{\left[1-(R+S)\right]}$$

$$G=(R-r-1)\frac{(R+r)}{2}-\frac{\left[r\left[R+S\right]-R\left[R+S\right]^{R-r}\right]}{1-(R+S)}-\frac{\left[\left[R+S\right]-(R+S)^{R-r+1}\right]}{\left[1-(R+S)\right]^2}$$

5. Expected number of customers in the system Ls_1, when the system is in the slower rate of arrivals either with feedback or without feedback.

$$Ls_1=\sum_{n=r+!}^{R-1}nP_n(1)+\sum_{n=R}^{k}nP_n(1) \quad (28)$$

From equation (18), (19), (20) and (28), we get

$$Ls_1=\left[\frac{H}{2\left(1-(T+U)\right)}+\frac{I}{\left[1-(T+U)\right]^2}+\frac{J}{\left[1-(T+U)\right]^3}\right]P_{r+1}(1) \quad \ldots (29)$$

where

$$H=(R-r)(R+r+1)+(k-R)(k+R+1)\left[1-\left(\frac{\alpha'+\beta'}{R+S}\right)\frac{\overline{\rho_1}}{\overline{\overline{\rho_0}}}\right]$$

$$I=(T+U)\left[k(T+U)^{k-r}-r\right]-\left(\frac{\alpha'+\beta'}{R+S}\right)\frac{\overline{\rho_1}}{\overline{\overline{\rho_0}}}(T+U)\left[k(T+U)^{k-R}-R\right]$$

$J=\left(\frac{\alpha'+\beta'}{R+S}\right)\frac{\overline{\rho_1}}{\rho_0}\left[(T+U)-(T+U)^{k-R+1}\right]-\left[(T+U)-(T+U)^{k-r+1}\right]$ and $P_{r+1}(1)$ is given by (18)

6. Expected number of customers in the system either with feedback or without feedback Ls is from (27) and (29), we have $Ls = Ls_0 + Ls_1$

$$Ls=\left\{\frac{1}{1-(R+S)}\left[\frac{F}{c}+GE\frac{\overline{\rho_1}}{\rho_0}\right]+\left[\frac{H}{2(1-(T+U))}+\frac{I}{\left[1-(T+U)\right]^2}+\frac{J}{\left[1-(T+U)\right]^3}E\right]\right\}P_0(0) \quad (30)$$

7. Expected waiting time Ws of the customer in the system is $Ws=\frac{Ls}{\lambda}$,

where $\overline{\lambda}=(\lambda_0-\varepsilon)P(0)+(\lambda_1-\varepsilon)P(1)$ (31)

Note:

This model includes the certain models as particular cases.

For example, when either, $p_0 = 0$, $q = 1$ or, $q_0 = 0$ and $p = 1$ and $k \to \infty$, this model reduces to Geo/Geo/c/∞ interdependent queueing model with controllable arrival rates which was discussed by Srinivasan and Thiagarajan (13); when $c = 1$, $\lambda_0 = \lambda_1 = \lambda$, either $q = 1$, $q_0 = 0$ or $p = 1$, $p_0 = 0$ and $\varepsilon = 0$, this model reduces to the conventional Geo/Geo/1/∞ model, discussed by Hunter (4)

NUMERICAL ILLUSTRATIONS

For various values of λ_0, λ_1, μ, ε, k while c, r, R are fixed values, computed and tabulated the values of $P_0(0)$, P(0) and P(1) by taking $p = q = \frac{1}{2}$

Table – 1

c	r	R	k	λ_0	λ_1	μ	ε	$P_0(0)$	P(0)	P(1)
3	6	12	20	8	6	10	0. 5	0. 009089077121	0. 063686616	0. 939270516
3	5	10	15	6	5	8	0. 5	0. 025222421000	0. 724488432	0. 275511546
3	5	10	15	5	4	10	0. 5	0. 000284923450	0. 022389009	0. 977610980
3	6	10	15	6	5	8	0. 5	0. 060321500000	0. 440601348	0. 559398652
3	5	10	15	8	6	12	0. 5	0. 006608903100	0. 113606266	0. 886393732
3	6	10	15	6	5	8	0. 0	0. 032942985000	0. 436245023	0. 563754973
3	6	10	15	8	6	8	0. 5	0. 441743553000	0. 884393101	0. 115606897
3	6	12	15	8	6	10	0. 5	0. 123597162000	0. 82582552	0. 174174458

For various values of λ_0, λ_1, μ, ε, k while c, r, R are fixed values,computed and tabulated the values of Ls_0, Ls_1, Ls and Ws by taking $p = q = \frac{1}{2}$.

Table – 2

c	r	R	k	λ_0	λ_1	μ	ε	Ls_0	Ls_1	Ls	Ws
3	6	12	20	8	6	10	0. 5	0. 343139625	168. 8373873	169. 1805269	30. 0759348
3	5	10	15	6	5	8	0. 5	0. 693761109	43. 9945817	44. 6883430	8. 5526131
3	6	10	15	6	5	8	0. 5	2. 214278319	82. 9849970	85. 1992750	17. 2447178
3	5	10	15	8	6	12	0. 5	0. 594683276	563. 4234572	564. 0181405	98. 4803931
3	6	10	15	6	5	8	0. 0	0. 820844427	36. 6947648	37. 5156092	5. 4362450
3	6	12	15	8	6	10	0. 5	8. 422985563	12. 1724014	20. 5953870	2. 8798087
3	6	10	15	8	6	8	0. 5	2. 471950163	10. 3739084	12. 8458586	1. 7666455

CONCLUSION

It is observed from table 1 and 2 that

- When the mean dependence rate increases and the other parameters are kept fixed Ls and Ws increase (either with feedback or without feedback).
- When the service rate increases and the other parameters are kept fixed, Ls and Ws increase (either with feedback or without feedback).
- When the arrival rate decreases and the other parameters are kept fixed, Ls and Ws decrease (either with feedback or without feedback).
- When the arrival size decreases and the other parameters are kept fixed, Ls and Ws decrease (either with feedback or without feedback).

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