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Common Fixed Point Theorem using Implicit Function with EA like Property in Modified Intuitionistic Fuzzy Metric Space

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Abstract

The present paper deals with introduction of "E. A. Like" property and its application in proving common fixed point theorem in a fuzzy metric space. In his paper we have generalized the result of Jain et al. [4]

Keywords: Fuzzy metric space; E. A. property; E. A. Like property; weakly compatible maps.

AMS Subject Classification: 47H10, 54H25.

Introduction

The evolution of fuzzy mathematics commenced with the introduction of the notion of fuzzy sets by Zadeh [9], in 1965, as a new way to represent the vagueness in everyday life. Atanassov [2] introduced the idea of intuitionistic fuzzy set. Park [5] introduces the intuitionistic fuzzy metric space using the concept of intuitiotnistic fuzzy set. Since then, various concepts of fuzzy metric and intuitionistic fuzzy metric spaces were considered. Recently, in 2006 Saadati [6] introduced the notion of L-fuzzy metric spaces with the help of continuous t norms as a generalization of fuzzy metric space due to George and Veeramani [3] and Intuitionistic Fuzzy Metric Space due to Park [5]. Since the Intuitionistic Fuzzy Metric Space has extra conditions, Saadati et. al [7] modified the idea of intuitionistic fuzzy metric spaces and presented the new notion of intuitionistic fuzzy metric spaces with the help of the notion of continuous t representable.

Aamri and Moutawakil [1] generalized the notion of non compatible mapping in metric space by E. A. property. E. A like property in fuzzy metric space was defined by Kamal Wadhawa. et al. [8]

Role of E. A. property in proving common fixed point theorems can be concluded by following,

1. It buys containment of ranges without any continuity requirements.
2. It minimizes the commutatively conditions of the maps to the commutatively at their points of coincidence.
3. It allows replacing the completeness requirement of the space with a more natural condition of closeness of the range.

Of course, if two mappings satisfy E. A. like property then they satisfies E. A. property also, but, onthe other hand, E. A. like property relaxes the condition of containment of ranges and closeness of the ranges to prove common fixed point theorems, which are necessary with E. A. property.

2 PRELIMINARIES

Definition 2. 1- L Fuzzy Set [114]

Let $L^* = (L, \leq_L)$ be a complete lattice, and U a non-empty set called a universe. An L^*-fuzzy set A on U is defined as a mapping A: $U \rightarrow L$. For each u in U, A(u) represents the degree (in L) to which u satisfies A.

In L fuzzy sets the requirement that the membership grades must be represented by numbers in the unit interval [0, 1] is relaxed and they are represented by symbols of an arbitrary set L i. e. at least partial ordered.

Definition 2. 2- Intuitionistic Fuzzy Set [10]

An Intuitionistic Fuzzy Set $A_{\zeta,\eta}$ on a universe U is an object $A_{\zeta,\eta} = \{(\zeta_A(u), \eta_A(u)) : u \in U\}$, where, for all $u \in U$, $\zeta_A(u) \in [0, 1]$ and $\eta_A(u) \in [0, 1]$ are called the membership degree and the non membership degree, respectively, of u in $A_{\zeta,\eta}$ and furthermore satisfy $\zeta_A(u) + \eta_A(u) \leq 1$

Classically, a triangular norm $*=T$ on $([0, 1], \leq)$ is defined as an increasing, Commutative, associative mapping T: $[0, 1]^2 \rightarrow [0, 1]$ satisfying $T(1, x) = x$, for all $x \in [0, 1]$.

A triangular co norm $\Diamond = S$ is defined as an increasing, commutative, associative mapping *S:* $[0, 1]^2 \rightarrow [0, 1]$ satisfying $S(0, x) = 0 \Diamond x = x$, for all $x \in [0, 1]$.

These definitions can be straight forwardly extended to any lattice $L = (L, \leq_L)$. Define first $0_L = \inf L$ and $1_L = \sup L$.

Definition 2. 3 –T norm [127]

A triangular norm (t-norm) on L is a mapping T: $L^2 \rightarrow L$ satisfying the following conditions:

1. For all $x \in L$, $T(x, 1_L) = x$; (Boundary Condition)
2. For all $(x, y) \in L^2$, $T(x, y) = T(y, x))$; (Commutativity)
3. For all $x, y, z \in L^3$, $T(x, T(y, z)) = T(T(x, y), z))$;) (Associativity)
4. For all x, x', y, y') $\in L^4$, $x \leq_L x'$ and $y \leq_L y'$ $(T(x, y) \leq_L T(x', y'))$. (Monotonicity)

A t-norm T on L* is called t-representabe if and only if there exist a t-norm $*$ and a t-co norm $\Diamond$ on [0, 1] such that, for all $x = (x_1, x_2)$, $y = (y_1, y_2) \in L^*$, $T(x, y) = ((x_1 * y_1), S(x_2 \Diamond y_2))$.

A negation on L is any decreasing mapping N: L→ L satisfying $N(0_L) = 1_L$ and $N(1_L) = 0_L$. If N (N(x)) = x, for all x ∈ L, then N is called an involutive negation. If, for all x ∈ [0, 1], $N_{s(x)} = 1 - x$, we say that N_s is the standard negation on ([0, 1], ≤).

Definition 2. 4 L Fuzzy Metric Space [114]

The 3-tuple (X, M, T) is said to be an L-fuzzy metric space if X is an arbitrary (non-empty) set, T is a continuous t–norm on L and M is an L-fuzzy set on $X^2 \times (0, +\infty)$ satisfying the following conditions for every x, y, z in X and t, s in (0, +∞):

1. $M(x, y, t) >_L 0_L$;
2. $M(x, y, t) = 1_L$ for all t > 0 if and only if x = y;
3. M(x, y, t) =M(y, x, t);
4. $T(M(x, y, t), M(y, z, s)) \leq_L M(x, z, t + s)$;
5. $M(x, y, \cdot): (0, \infty) \rightarrow L$ is continuous.

Definition 2. 5 Modified Intuitionistic Fuzzy Metric Space [118]

The 3-tuple $(X, M_{M, N}, T)$ is said to be an Modified Intuitionistic Fuzzy Metric Space if X is an arbitrary (non-empty) set, *T* is a continuous t representable and $M_{M, N}$ is a mapping $X^2 \times (0, \infty) \rightarrow L^*$ satisfying the following conditions for every x, y ∈ X and t, s > 0:

1. $M_{M, N}(x, y, t) >_{L*} 0_{L*}$,
2. $M_{M, N}(x, y, t) = 1_{L*}$ if and only if x = y,
3. $M_{M, N}(x, y, t) = M_{M, N}(y, x, t)$,
4. $M_{M, N}(x, y, t + s) \geq_{L*} T(M_{M, N}(x, z, t), M_{M, N}(z, y, s))$,
5. $M_{M, N}(x, y, .) : (0, \infty,) \rightarrow L^*$ is continuous.

Where M, N are fuzzy sets from $X^2 \times (0, \infty)$ to [0, 1] such that M(x, y, t) + N(x, y, t) ≤ 1 for all x, y ∈ X and t > 0.

In this case $M_{M, N}$ is called an Intuitionistic Fuzzy Metric. Here, $M_{M, N}(x, y, t) = (M(x, y, t), N(x, y, t))$.

Remark 2. 1[118]

In an Intuitionistic Fuzzy Metric Space $(X, M_{M, N}, T)$, M(x, y,.) is non-decreasing and N(x, y,.) is non-increasing for all x, y ∈ X. Hence $(X, M_{M, N}, T)$ is non-decreasing function for all x, y ∈X.

Example 2. 1 [16]

Let (X, d) be a metric space. Denote $T(a, b) = (a_1 b_1, \min\{a_2 + b_2, 1\})$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2) \in L^*$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_{M, N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \left(\frac{t}{t+d(x,y)}, \frac{d(x,y)}{t+d(x,y)}\right)$$

Then $(X, M_{M, N}, T)$ is a modified intuitionistic fuzzy metric space.

Definition. 2. 6 – Cauchy Sequence [114]
Let (X, M, T) be an L-fuzzy metric space. Then a sequence $\{x_n\}$ in X is called a Cauchy sequence if for each $\epsilon \in L \setminus \{0_L\}$ and t > 0, there exist $n_0 \in N$ such that for all $m \geq n \geq n_0 (n \geq m \geq n_0)$, $M(x_n, x_m, t) >_L N(\epsilon)$

Definition 2. 7- Convergent Sequence [114]
Let (X, M, T) be an L-fuzzy metric space. Then a sequence $\{x_n\}$ in X is said to be converged to x in X (denote by $x_n \to x$) if $M(x_n, x, t) = M(x, x_n, t) \to 1_L$ whenever $n \to \infty$ for each t > 0.

Definition 2. 8- Complete Space [114]
A L-fuzzy metric space is said to be complete if and only if every Cauchy Sequence is convergent.

Definition 2. 9 Sequentially Compact L Fuzzy Metric Space [114]
Let (X, M, T) be an L-fuzzy metric space is called Sequentially Compact Metric Space compact if every sequence in X has a convergent subsequence in it.

Definition 2. 10 –Weakly Compatible mapping [81]
Let A and S be mappings from an L-fuzzy metric space (X, M, T) into itself. Then the mappings are said to be weak compatible if they commute at their coincidence point, that is, Ax = Sx implies that ASx = SAx.

Example 2. 2:
Let (X, d) be a usual metric space where X=[0, 2]. The continuous t-norm $*$ is defined as $a * b = a\,b$ for all $a, b \in [0, 1]$ and let M be Fuzzy Sets on $X^2 \times (0, \infty)$ defined as :
$M(x, y, t) = \frac{t}{t+d(x,y)}$

Let A and S be two self mapping defined as-
$A(x)=\begin{cases} 1-x, 0 \leq x < 1 \\ 2, 1 \leq x \leq 2 \end{cases}$ and Sx = x

Clearly $x=\frac{1}{2}$ and x=2 are the coincidence point of {A, S}

Since at $x = \frac{1}{2}$ we have

$A(\frac{1}{2}) = 1 - \frac{1}{2} = \frac{1}{2}$ and $S(\frac{1}{2}) = \frac{1}{2}$. Thus $A(\frac{1}{2}) = S(\frac{1}{2})$.

Also

$A[S(\frac{1}{2})] = A[\frac{1}{2}] = 1 - \frac{1}{2} = \frac{1}{2}$

$S[A(\frac{1}{2})\ ] = S(1 - \frac{1}{2}) = S(\frac{1}{2}) = \frac{1}{2}$

Thus $A(\frac{1}{2}) = S(\frac{1}{2}) = \frac{1}{2}$ implies that $A[S(\frac{1}{2})] = S[A(\frac{1}{2})] = \frac{1}{2}$

At x = 2 we have

A(2) = 2 and S(2) = 2. Thus A (2) = S (2).

Also

A[S(2)] = A[2] =2 and S[A(2)] =S(2) =2

Thus A(2) =S(2) = 2 implies that A[S(2)] = S[A(2)] = 2

Hence the pair (A, S) is weakly compatible map.

Definition 2. 11 – A Class of Implicit Function [3]
Let Ψ be the set of all continuous functions $F(t_1, t_2, \ldots, t_5) : L^{*5} \to L^*$, satisfying the following conditions (for all u, v, $1 \in L^*$, $u = (u_1, u_2)$, $v = (v_1, v_2)$ and $1 = 1_{L^*} = (1, 0)$):
(F1) for all u, $v >_{L^*} 0_{L^*}$, $F(u, 1, 1, u, 1) \geq_{L^*} 0_{L^*}$ or $F(u, u, 1, 1, u) \geq_{L^*} 0_{L^*}$ or $F(u, 1, u, 1, u) \geq_{L^*} 0_{L^*}$, implies that $u \geq_{L^*} 1_{L^*}$ (F2) $F(u, v, u, v, 1) \geq_{L^*} 0_{L^*}$ implies that $u \geq_{L^*} v$
Then $F \in \Psi$

Definition 2. 12 – EA like Property [7]
Let A and B be two self-maps of a fuzzy metric space (X, M, T). We say that Aand B satisfy the E. A. Like Property if there exists a sequence $\{x_n\}$ such that, $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Bx_n z$ for some $z \in A(X)$ or $z \in B(X)$, i. e., $z \in A(X) \cup B(X)$.

Definition 2. 13 –Common EA like Property [7]
Let A, B, S and T be self maps of a fuzzy metric space (X, M, T), then the pairs (A, S) and (B, T) said to satisfy common E. A. Like property if there exists two sequences $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Sx_n = z$ where $z \in S(X) \cap T(X)$ or $z \in A(X) \cap B(X)$.

Example 2. 3:
Let X=[0, 1] and $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$ then (X, M, T) is a L fuzzy metric space where $T(a, b) = \min\{a, b\}$.

Define the self mappings A, B, S and T as – $A(x) = \left(\frac{x}{2} - \frac{1}{4}\right)$, $B(x) = \frac{x}{2}$, $S(x) = \left(x - \frac{1}{2}\right)$, $T(x) = x^2$

Define the sequences $\{x_n\}$ and $\{y_n\}$ where $x_n = \left(\frac{1}{2} + \frac{1}{n}\right)$ and $y_n = \frac{1}{n}$.

We have $A(X) = [-\frac{1}{4}, \frac{1}{4}]$, $B(X) = [0, \frac{1}{2}]$, $S(X) = [-\frac{1}{2}, \frac{1}{2}]$ and $T(x)=[0, 1]$

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} \left(\frac{\left(\frac{1}{2}+\frac{1}{n}\right)}{2} - \frac{1}{4}\right) = 0 \in S(X)$$

$$\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} \left(\left(\frac{1}{2}+\frac{1}{n}\right) - \frac{1}{2}\right) = 0 \in A(X)$$

$$\lim_{n\to\infty} Ty_n = \lim_{n\to\infty} \left(\frac{1}{n}\right)^2 = 0 \in B(X)$$

$$\lim_{n\to\infty} By_n = \lim_{n\to\infty} \left(\frac{\frac{1}{n}}{2}\right) = 0 \in T(X)$$

Thus $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} By_n = \lim_{n\to\infty} Ty_n = \lim_{n\to\infty} Sx_n = 0$
Where $0 \in S(X) \cap T(X)$ or $z \in A(X) \cap B(X)$.
Hence the pairs (A, S) and (B, T) satisfies common E. A. Like property.

3. Main Result

Suman Jain, Bhawna Mundra and Sangita Aske [4] proved the following result –

Theorem:

Let A, B and T be self mappings of a complete fuzzy metric space (X, M, $*$) satisfying:

1. $A(X) \subset T(X)$, $B(X) \subset T(X)$,
2. The pairs (A, T) and (B, T) are weakly compatible,
3. T(X) is complete,
4. For some $F \in \Phi$, there exists $k \in (0, 1)$ such that $\forall$ x, y $\in$ X, $\forall$ t > 0,

$$F\begin{Bmatrix} M(A^2x, B^2y\, kt), M(T^2x, T^2y, t), M(A^2x, T^2x, t), \\ M(B^2y, T^2y, kt), M\,(T^2y, A^2x, t) \end{Bmatrix} \geq 0.$$

Then A, B and T have unique common fixed point in X.

We prove the following result -

Theorem 3. 1

Let A, B, S and T be self mapping of an Modified Intuitionistic Fuzzy Metric Space (X, M_{MN}, T) where T is continuous t norm such that

1. The pair (A, S) and (B, T) share Common EA Like property and
2. The pair (B, T) and (A, S) are weakly compatible.
3. AS=SA and BT=TB
4. For some $F \in \Phi$ there exists a constant $k \in (0, 1)$ such that for all

x, y $\in$ X, for some t > 0 we have

$$F\begin{Bmatrix} M_{MN}(A^2x, B^2y\, k\, t), M_{MN}(S^2x, A^2x, kt), M_{MN}(T^2y, S^2x, kt) \\ M_{MN}(B^2y, T^2y, k\, t), M_{MN}(T^2y, A^2x, kt) \end{Bmatrix} \geq_L 0_L. \qquad (3.\ 1.\ a)$$

Then A, B, S and T have a unique common fixed point in X.

Proof:

Since the pair (A, S) and (B, T) satisfy Common EA Like property therefore there exists a sequence $\{x_n\}$ and $\{y_n\}$ in X such that $\lim_{n\to\infty} A^2y_n = \lim_{n\to\infty} T^2x_n = \lim_{n\to\infty} B^2x_n = \lim_{n\to\infty} S^2y_n = z$ where $z \in S(X) \cap T(X)$ or $z \in A(X) \cap B(X)$.

Suppose that $z \in S(X) \cap T(X)$, now we have $\lim_{n\to\infty} A^2x_n = z \in S(X)$

Then $z = S^2u$ for some $u \in X$………… (. 3. 1. b)

We claim that $A^2u = S^2u$.

$$F\begin{Bmatrix} M_{MN}(A^2u, B^2y_n, k\, t), M_{MN}(S^2u, A^2u, kt), \\ M_{MN}(T^2y_n, S^2u, kt) \\ M_{MN}(B^2y_n, T^2y_n, k\, t), M_{MN}(T^2y_n, A^2u, kt) \end{Bmatrix} \geq_L 0_L.$$

Taking limit $n\to\infty$ we have

$$F\begin{Bmatrix} M_{MN}(A^2u, z, k\, t), M_{MN}(z, A^2u, kt), M_{MN}(z, z, kt), \\ M_{MN}(z, z, k\, t), M_{MN}(z, A^2u, kt) \end{Bmatrix} \geq_L 0_L.$$

$$F\{M_{MN}(A^2u, z, k\, t), M_{MN}(z, A^2u, kt), 1_L, 1_L, M_{MN}(z, A^2u, kt)\} \geq_L 0_L.$$

$M_{MN}(z, A^2u, kt) \geq_L 1_L$.

Hence $z = A^2u$.

From (3. 1. b) we have $z = A^2u = S^2u$.

Again $\lim_{n\to\infty} B^2y_n = z \in T(X)$ then $z = T^2v$ for some $v \in X$.

We claim that $B^2v=T^2v$.

$$F\left\{\begin{matrix} M_{MN}(A^2x_n, B^2v\, k\, t), M_{MN}(S^2x_n, A^2x_n, kt), \\ M_{MN}(T^2v, S^2x_n, kt) \\ M_{MN}(B^2v, T^2v, k\, t), M_{MN}(T^2v, A^2x_n, kt) \end{matrix}\right\} \geq_L 0_L$$

Taking limit $n\to\infty$ we have

$$F\left\{\begin{matrix} M_{MN}(z, B^2v\, k\, t), M_{MN}(z, z, kt), M_{MN}(z, z, kt) \\ M_{MN}(B^2v, z, k\, t), M_{MN}(z, z, kt) \end{matrix}\right\} \geq_L 0_L$$

$F\{M_{MN}(z, B^2v\, k\, t), 1_L, 1_L, M_{MN}(B^2v, z, k\, t), 1_L\} \geq_L 0_L$

$M_{MN}(B^2v, z, k\, t) \geq 1_L$.

Thus $B^2v=z$.

Hence $B^2v=T^2v=z$

Since u is the coincidence point of the pair (A, S) therefore

$A^2S^2u = S^2A^2u$

Hence $A^2z = S^2z$.

since v is the coincidence point of the pair {B, T} therefore

$B^2T^2v = T^2B^2v$

Hence $B^2z = T^2z$.

Next we claim that $A^2z=z$

$$F\left\{\begin{matrix} M_{MN}(A^2z, B^2y_n, k\, t), M_{MN}(S^2z, A^2z, kt), \\ M_{MN}(T^2y_n, S^2z, kt) \\ M_{MN}(B^2y_n, T^2y_n, k\, t), M_{MN}(T^2y_n, A^2z, kt) \end{matrix}\right\} \geq_L 0_L.$$

Taking limit $n\to\infty$ we have

$$F\left\{\begin{matrix} M_{MN}(A^2z, z, k\, t), M_{MN}(z, A^2z, kt), M_{MN}(z, z, kt) \\ M_{MN}(z, z, k\, t), M_{MN}(z, A^2z, kt) \end{matrix}\right\} \geq_L 0_L.$$

$F\{M_{MN}(A^2z, z, k\, t), M_{MN}(z, A^2z, kt), 1_L, 1_L, M_{MN}(z, A^2z, kt)\} \geq_L 0_L$.

$M_{MN}(z, A^2z, kt) \geq_L 1_L$.

Hence $z= A^2z$.

By similar argument we have $z= B^2z$

Hence $T^2z = A^2z = B^2z = S^2z =z$

Thus z is a common fixed point A^2, B^2, S^2, T^2.

Let w be any other fixed point of A^2, B^2, S^2, T^2.

$$F\left\{\begin{matrix} M_{MN}(A^2z, B^2w\, k\, t), M_{MN}(S^2z, A^2z, kt), M_{MN}(T^2w, S^2z, kt) \\ M_{MN}(B^2w, T^2w, k\, t), M_{MN}(T^2w, A^2z, kt) \end{matrix}\right\} \geq_L 0_L$$

Taking limit $n\to\infty$ we have

$$F\left\{\begin{matrix} M_{MN}(z, w\, k\, t), M_{MN}(z, z, kt), M_{MN}(w, z, kt) \\ M_{MN}(w, w, k\, t), M_{MN}(w, z, kt) \end{matrix}\right\} \geq_L 0_L$$

$F\{M_{MN}(z, w\, k\, t), 1_L, M_{MN}(w, z, kt), 1_L, M_{MN}(w, z, kt)\} \geq_L 0_L$

$M_{MN}(w, z, kt)\} \geq_L 1_L$.

Hence $z= w$.

Thus z is a unique common fixed point A, B, S, and T

Since (A, S) and (B, T) commutes therefore (A^2, S^2) and (B^2, T^2) commutes.

Now $Az=A(A^2z)= A^2(Az)$ and $Az =A(S^2z)= S^2(Az)$

Thus Az is the Common fixed point of A^2 and S^2.

$Bz=B(B^2z)= B^2(Bz)$ and $Bz=B(T^2z)= T^2(Bz)$

Thus Bz is the Common fixed point of B^2 and T^2.

We claim that Az= Bz. $F\begin{Bmatrix} M_{MN}(A^2Az, B^2Bz, k\,t), M_{MN}(S^2Az, A^2Az, kt), \\ M_{MN}(T^2Bz, S^2Az, kt) \\ M_{MN}(B^2Bz, T^2Bz, k\,t), M_{MN}(T^2Bz, A^2Az, kt) \end{Bmatrix} \geq_L 0_L$

Taking limit $n \to \infty$ we have

$$F\begin{Bmatrix} M_{MN}(Az, Bz, k\,t), M_{MN}(Az, Az, kt), M_{MN}(Bz, Az, kt), \\ M_{MN}(Bz, Bz, k\,t), M_{MN}(Bz, Az, kt) \end{Bmatrix} \geq_L 0_L$$

$F\{M_{MN}(Az, Bz, k\,t), 1_L, M_{MN}(Bz, Az, kt), 1_{L,}, M_{MN}(Bz, Az, kt)\} \geq_L 0_L$

$M_{MN}(Bz, Az, kt)\} \geq_L 1_L$.

Hence Az= Bz.

Now $Sz=S(S^2z)= S^2(Sz)$ and $Sz=S(A^2z)= A^2(Sz)$

Thus Sz is the Common fixed point of A^2 and S^2.

$Tz=T(T^2z)= T^2(Tz)$ and $Tz=T(B^2z)= B^2(Tz)$

Thus Tz is the Common fixed point of B^2 and T^2.

We claim that Sz= Tz.

$$F\begin{Bmatrix} M_{MN}(A^2Sz, B^2Tz\, k\,t), M_{MN}(S^2Sz, A^2Sz, kt), \\ M_{MN}(T^2Tz, S^2Sz, kt) \\ M_{MN}(B^2Tz, T^2Tz, k\,t), M_{MN}(T^2Tz, A^2Sz, kt) \end{Bmatrix} \geq_L 0_L$$

Taking limit $n \to \infty$ we have

$$F\begin{Bmatrix} M_{MN}(Sz, Tz, k\,t), M_{MN}(Sz, Sz, kt), M_{MN}(Tz, Sz, kt), \\ M_{MN}(Tz, Tz, k\,t), M_{MN}(Tz, Sz, kt) \end{Bmatrix} \geq_L 0_L$$

$F\{M_{MN}(Sz, Tz, k\,t), 1_L, M_{MN}(Tz, Sz, kt), 1_{L,}, M_{MN}(Tz, Sz, kt)\} \geq_L 0_L$

$M_{MN}(Tz, Sz, kt)\} \geq_L 1_L$.

Hence Sz= Tz.

Therefore Az=Bz, Sz=Tz is the Common Fixed Point of A^2, B^2, S^2, T^2.

But z is the Unique Common Fixed Point of A^2, B^2, S^2, T^2.

Therefore z= Az=Bz=Sz=Tz.

Hence z is the Unique Common Fixed Point of A, B, S and T.

Corollary 3. 1

Let B, S and T be self mapping of an Modified Intuitionistic Fuzzy Metric Space (X, M_{MN}, T) where T is continuous t norm such that

The pair (T, S) and (B, T) share Common EA Like property and

The pair (B, T) and (T, S) are weakly compatible.

TS=ST and BT=TB

For some $F \in \Phi$ there exists a constant $k \in (0, 1)$ such that for all $x, y \in X$, for some $t > 0$ we have

$$F\begin{Bmatrix} M_{MN}(T^2x, B^2y\, k\,t), M_{MN}(S^2x, T^2x, kt), M_{MN}(T^2y, S^2x, kt) \\ M_{MN}(B^2y, T^2y, k\,t), M_{MN}(T^2y, T^2x, kt) \end{Bmatrix} \geq_L 0_L. \qquad (3.\ 1.\ a)$$

Then B, S and T have a unique common fixed point in X.

Proof – Replace A= T in the above result.

References –

[1] Aamri, M. and Moutawakil, D. E., 2002 "Some new common fixed point theorems under strict contractive conditions", *J. Math. Appl*. 270, pp. 181-188.

[2] Atanassov. K, 1986, "Intuitionistic Fuzzy Sets", Fuzzy Sets and System, 20, pp 87-96.

[3] George, A. and Veeramani, P., 1994, "On Some Result in Fuzzy Metric spaces", Fuzzy Sets and Systems 64, pp. 395-399.

[4] Jain, S., Mundra, B. and Aske, S., 2009, "Common fixed Point Theorem using implicit function", International Mahematical Forum, 4, No. 3, pp. 135-141.

[5] Park JH, 2004, "Intutionistic Fuzzy Metric Space", Chaos Solitons and Fractals, 22, pp. 1039-1046.

[6] Saadati, R. and Park, J. H., 2006, "On the Intuitionistic Fuzzy Topological Spaces", Chaos, Solitons Fractals, 27, pp. 331-344.

[7] Saadati, R., Sedghi, S., and Shobe, N., 2008, "Modified Intuitionistic fuzzy metric spaces and some fixed point theorems", Chaos, Solitons & Fractals, Volume 38, no. 1, pp. 36 – 47.

[8] Wadhwa, K., Dubey, H. and Jain, R., 2012, " Impact of "E. A. Like" Property on Common Fixed Point Theorems in Fuzzy Metric Spaces", Journal of Advanced Studies in Topology, Vol. 3, No. 1, pp. 52-59

[9] Zadeh L. A, 1965, "Fuzzy Sets", Information and Control, pp. 338-353.