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Influence of Variable Viscosity and Thermal Conductivity of Micropolar Flow in a Porous Channel with High Mass Transfer in Presence of Magnetic Field

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Abstract

The effect of variable viscosity and thermal conductivity of two dimensional flow of a micropolar fluid in a porous channel with high mass transfer in presence of magnetic field is investigated when the viscosity and thermal conductivity are assumed as the inverse linear function of temperature. The flow is driven by suction or injection at the channel walls and the the similarity transformations are applied to reduce the system of partial differential equations and their boundary conditions describing this problem, into a ordinary differential equations and then solved numerically using Range-Kutta shooting technique. The effects of the different parameters such as velocity distribution, micro-rotation distribution, temperature distribution Prandtl number, magnetic parameter etc. on flow and heat transfer has been studied numerically. The graphs are plotted for velocity distribution, temperature distribution and microrotation distribution for various values of non-dimensional parameters. It is found that the effects of the parameters giving variable property of viscosity and thermal conductivity are significant.

Key words:-Viscosity, Thermal conductivity, Porous channel, Suction or injection.

INTRODUCTION:-

The theory of micropolar fluids was originally formulated by Eringen [3]. In essence, the theory introduces new material parameters, an additional independent vector field-the microrotation-and new constitutive equations which must be solved simultaneously with the usual equations for Newtonian flow. The desire to model the non-Newtonian flow of fluid containing rotating micro-constituents prporous media, turbulent shear flows, and flowing capillaries and microchannels by Lukaszewiez [6].

We analyze the effect of the variable viscosity and the variable thermal conductivity on self-similar boundary layer flow of a micropolar fluid in a porous channel, where the flow is driven by uniform mass transfer through the channel walls. The corresponding Newtonian fluid model was first studied by Berman [1], who described an exact solution of the Navier-Stokes equations by assuming a self-similar solution and reducing the governing partial differential equations to a nonlinear ordinary differential equation of fourth order. The solution is of potential value in understanding more realistic flow in channels and pipes, and study of Berman’s exact solution and generalizations of it have attracted numerous studies subsequently, for example Yuan [8], Zaturska et. al. [9], Desseaux [2].

Through the viscosity and thermal conductivity are assumed as constant properties but in actual these are temperature dependent (Schlichiting [7], Eckert[4]). Therefore, in this paper we consider the effect of variable viscosity and variable thermal conductivity on steady incompressible laminar flow of a micropolar fluid in a porous channel with high mass transfer due to suction or injection. By means of similarity transformation the governing equations are reduced to boundary value problem of nonlinear coupled ordinary differential equations and solved numerically and results are shown graphically. The effects of the different parameters such as velocity distribution, micro-rotation distribution, temperature distribution Prandtl number, magnetic parameter etc. on flow and heat transfer has been studied numerically. The missing values of the velocity, angular velocity and thermal conductivity are tabulated for a wide range of material parameters of the fluid.

FORMULATION OF THE PROBLEM:-

We consider steady, incompressible, laminar flow of a micropolar fluid along a two-dimensional channel with porous walls through which fluid is uniformly injected or removed with speed q. Using Cartesian coordinate, the channel walls are parallel to the x-axis and located at y = ± h, where 2h is the channel width. The governing equations under the assumptions become

Mass equation : $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-p_x+\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+\kappa\frac{\partial^2 u}{\partial y^2}+\kappa\frac{\partial N}{\partial y}-\sigma_0\beta_0{}^2u \qquad (2)$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-p_y+\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)+\kappa\frac{\partial^2 v}{\partial y^2}-\kappa\frac{\partial N}{\partial x}-\sigma_0\beta_0{}^2u\,, \qquad (3)$$

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=-\kappa\left(2N+\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial y}\left(\nu_s\frac{\partial N}{\partial y}\right) \qquad (4)$$

$$\rho C_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\frac{\partial}{\partial x}\left(\lambda\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right)+(\mu+\kappa)\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right]+\sigma_0\beta_0^2(u^2+v^2) \qquad (5)$$

where μ and ν_s are respectively the viscosity and microrotation (or spin-gradient) viscosity, j is the micro-inertia density, ρ is the density of the fluid, κ is the microrotation coupling coefficient (or coefficient of gyro viscosity or vortex viscosity) p is the pressure, C_p is the specific heat at constant pressure and λ is the thermal conductivity.

The appropriate physical boundary conditions are

$$u(x,\ \pm h)=0,\quad \mathrm{v}(x,\ \pm h)=\pm q,\quad \mathrm{N}(x,\ \pm h)=-s\left.\frac{\partial u}{\partial y}\right|_{(x,\ \pm h)},\quad T_w=T_0+Bx^2\Big|_{(x,\ \pm h)} \tag{6}$$

and assuming that that the flow is symmetric about y = 0,

$$\frac{\partial u}{\partial y}(x,0)=v(x,0)=0,\qquad T=T_0(x,0), \tag{7}$$

where q > 0 correspondence to suction, q < 0 to injection, and s is a boundary parameter that is used to model the extent to which microelements are free to rotate in the vicinity of the channel walls. For example, the value s = 0 corresponds to the case where microelements close to a wall are unable to rotate, whereas the value s = ½ corresponds to the case where the microrotation is equal to the fluid vorticity at the boundary (Lukaszewiez [5]).

To simplify the governing equations, we generalize Berman's similarity solutions to include micropolar effects by assuming a stream function and microrotation of the form

$$\Psi=-qxf(\eta),\qquad N=\frac{qx}{h^2}g(\eta), \tag{8}$$

where

$$\eta=\frac{y}{h},\quad u=\frac{\partial\Psi}{\partial y}=-\frac{qx}{h}f'(\eta),\quad v=-\frac{\partial\Psi}{\partial x}=qf(\eta) \tag{9}$$

and the temperature is taken in the form as

$$\theta(\eta)=\frac{T(\eta)-T_0}{T_w-T_0}. \tag{10}$$

In addition we introduce the dimensionless micropolar parameters, non-zore cross-flow Reynolds number, Prandtl number and Eckert number respectively as

$$\mathrm{Re}=\frac{qh}{v},\quad P_r=\frac{\rho v C_p}{\lambda_\infty}\quad \textit{and}\quad \mathrm{E_c}=\frac{q^2}{C_p(T_w-T_\infty)}, \tag{11}$$

where Re > 0 corresponds to suction, and Re < 0 to injection..

The fluid viscosity is assumed to be inverse linear function of temperature (Lai and Kulacki [6] as

$$\frac{1}{\mu}=\frac{1}{\mu_\infty}\left[1+\alpha(T-T_\infty)\right],\quad \frac{1}{\mu}=a(T-T_r),\quad a=\frac{\alpha}{\mu_\infty}\ \text{ and }\ T_r=T_\infty-\frac{1}{\alpha}, \tag{12}$$

where a and T_r are constants and their values depends on the reference state and the thermal property of the fluid. In general, a > 0 for liquids and a < 0 for gases. T_r is transformed reference temperature related to viscosity parameter. α is constant based on thermal property and μ_∞ is the viscosity at $T=T_\infty$. Similarly, consider the variation of thermal conductivity as,

$$\frac{1}{\lambda}=\frac{1}{\lambda_\infty}\left[1+\xi\left(T-T_\infty\right)\right],\quad \frac{1}{\lambda}=b\left(T-T_k\right),\quad \mathrm{b}=\frac{\xi}{\lambda_\infty}\quad \text{and}\quad T_k=T_\infty-\frac{1}{\xi}\ , \tag{13}$$

where b and T_k are constants and their values depends on the reference state and the thermal property of the fluid. ξ is constant based on thermal property and λ_∞ is the viscosity at T=T_{∞}..

Using equations (8)and (9), it can be easily verified that the continuity equation is satisfied automatically and using equations (8)-(13) in the equations (2), (4) and (5) become,

$$\mathrm{Re}\left(\frac{\theta_v-\theta}{\theta_v}\right)(f'^2-ff'')=M\left(\frac{\theta_v-\theta}{\theta_v}\right)f'+\left(1+K\frac{\theta_v-\theta}{\theta_v}\right)f'''+\frac{\theta'}{\theta_v-\theta}f''+K\left(\frac{\theta_v-\theta}{\theta_v}\right)g' \tag{14}$$

$$G_1(fg'-f'g)=-K(f''-2g)+G_2g'' \tag{15}$$

$$\theta''=\mathrm{Pr}\left[f\theta'-E_c\left(f'^2+f''^2\right)\right]\frac{\theta_c-\theta}{\theta_c}-M\frac{\theta_c-\theta}{\theta_c}\left(f'^2+f^2\right)-\frac{\theta_c}{\theta_c-\theta}\theta'^{\,2} \tag{16}$$

The transformed boundary conditions are

$f^1(x,\pm h)=0,\ f(x,\pm h)=1,\ g(x,\pm h)=0,\ \theta(x,\pm h)=\dfrac{Bx^2}{T_w-T_0}$

RESULTS AND DICUSSION:-

The equations (14)-(16) together with the boundary conditions (6) are solved for various condition of the Parameters involved in the equations using an algorithms based on the shooting method and presented results for the distribution of dimensionless velocity distribution, dimensionless micro-rotation distribution and temperature distribution with the variation of different parameters. Solution have been also been found for different values of Coupling Constant Parameter (K), Prandtl number (Pr), Eckert number (Ec), Magnetic parameter(M). The variation in velocity distribution, micro-rotation distribution and temperature distribution are illustrated in figures (1 – 4). From the equation (9) it is found that the velocity 'u' is dependent on $f'(\eta)$. The figures (1) and (4) display the variation in velocity (u) distribution with the variation of viscosity parameter θv and magnetic parameter M. The figure (1) indicate that velocity increases with the increase of θr i.e.,-θr decreases, while the velocity decreases with the increase of magnetic parameter M. Figure(2) indicate that lower values of-θc ,higher the temperature, while from figure (3), it is seen that the microrotation increases with the increases of the parameter G.

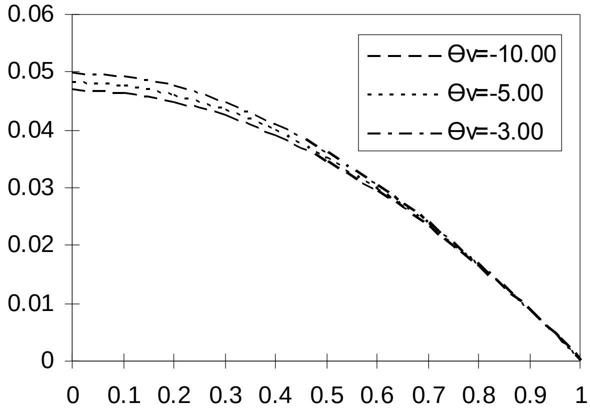

Fig 1: Velocity distribution f' with the variation of viscosity parameter θ_v .

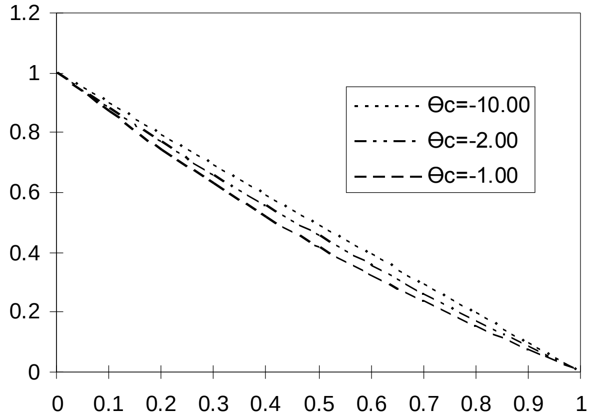

Fig 2: Temparature Distribution (θ) with the variation of thermal conductivity parameter θ_c .

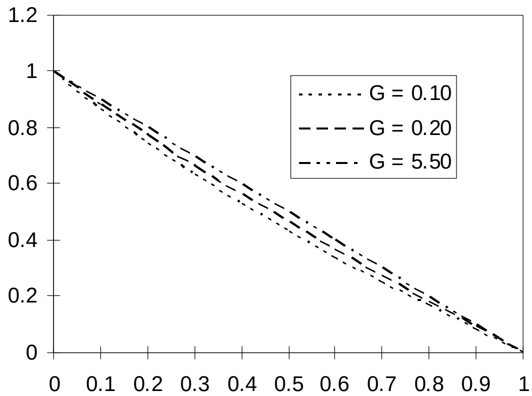

Fig-3 : Microrotation Distribution(g) with the variation of Parameter G.

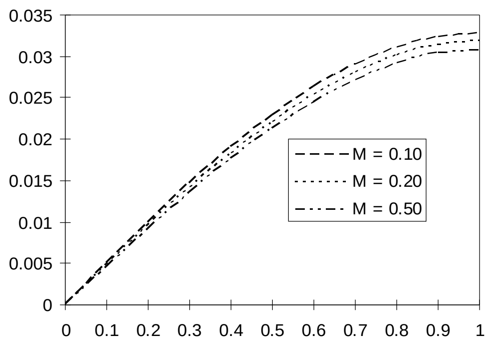

Fig 1: Velocity distribution with the variation of magnetic parameter M .

CONCLUSION:-

In this study, the effect of variable viscosity and thermal conductivity on flow and heat transfer for micropolar flow in a porous channel with high mass transfer through the channel walls in presence of magnetic field is examined. The results presented demonstrate clearly that the viscosity and thermal conductivity parameters have a substantial effect on velocity, temperature and micro-rotation distribution within the boundary layer. The effects of magnetic Parameter (M), Prandtl number Pr, Eckert number Ec is quite significant. Thus the assumption on constant properties may cause a significant error in the flow problems and in the prediction of skin friction while designing fluid machinery.

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