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Properties Of Level Subsets Of An Intuitionistic Fuzzy $\ell-$Subsemiring Of A $\ell-$Semiring

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Abstract

In this paper, we made an attempt to study the properties of level subsets of an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring and we introduce some theorems on this.

Key Words: Fuzzy subset, intuitionistic fuzzy subset, intuitionistic fuzzy $\ell-$subsemiring, level $\ell-$subsemiring and strongest intuitionistic fuzzy relation.

Introduction

The concept of fuzzy sets was initiated by L.A.Zadeh [7] in 1965. After the introduction of fuzzy sets several researchers explored on the generalization of the concept of fuzzy sets. K.T.Atanassov introduced [2] intuitionistic fuzzy subset in 1983. George Gargor named new sets as the intuitionistic fuzzy subset. In this paper to introduced the concept the properties of level subsets of an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring and established some results on these.

Definition: 1.1

Let R be a $\ell-$semiring. An intuitionistic fuzzy subset A of R is said to be an intuitionistic fuzzy $\ell-$subsemiring of R if it satisfies the following conditions:

(i) $\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$ (ii) $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$

(iii) $\mu_A(x\vee y) \geq \min\{\mu_A(x), \mu_A(y)\}$ (iv) $\mu_A(x\wedge y) \geq \min\{\mu_A(x), \mu_A(y)\}$

(v) $\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$ (vi) $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$

(vii) $\nu_A(x\vee y) \leq \max\{\nu_A(x), \nu_A(y)\}$ (viii) $\nu_A(x\wedge y) \leq \max\{\nu_A(x), \nu_A(y)\}$

for all $x,\ y \in R$

Definition: 1.2
Let A be an intuitionistic fuzzy subset in a set S, the strongest intuitionistic fuzzy relation on S, that is a intuitionistic fuzzy relation on A is V given by $\mu_V(x,y) = \min\{\mu_A(x), \mu_A(y)\}$ and $\nu_V(x,y) = \max\{\nu_A(x), \nu_A(y)\}$, for all $x, y \in S$.

Definition: 1.3
Let A be an intuitionistic fuzzy subset of X. For α, β in $[0,1]$ the level subset of A is the $A_{(\alpha,\beta)} = \{x \in X / \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta\}$.

Theorem: 1.1
Let A be an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring R. Then for α and β in $[0,1]$, $A_{(\alpha,\beta)}$ is a $\ell-$subsemiring of R.

Proof:
Given A is an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring R. Take $x, y \in A_{(\alpha,\beta)}$
Then $\mu_A(x) \geq \alpha$, $\nu_A(x) \leq \beta$ and $\mu_A(y) \geq \alpha$, $\nu_A(y) \leq \beta$. Now,
$\mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \min\{\alpha, \alpha\} = \alpha \Rightarrow \mu_A(x+y) \geq \alpha$,
$\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \min\{\alpha, \alpha\} = \alpha \Rightarrow \mu_A(xy) \geq \alpha$,
$\mu_A(x \vee y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \min\{\alpha, \alpha\} = \alpha \Rightarrow \mu_A(x \vee y) \geq \alpha$,
$\mu_A(x \wedge y) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \min\{\alpha, \alpha\} = \alpha \Rightarrow \mu_A(x \wedge y) \geq \alpha$,
$\nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \max\{\beta, \beta\} = \beta \Rightarrow \nu_A(x+y) \leq \beta$,
$\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \max\{\beta, \beta\} = \beta \Rightarrow \nu_A(xy) \leq \beta$,
$\nu_A(x \vee y) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \max\{\beta, \beta\} = \beta \Rightarrow \nu_A(x \vee y) \leq \beta$,
$\nu_A(x \wedge y) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \max\{\beta, \beta\} = \beta \Rightarrow \nu_A(x \wedge y) \leq \beta$,
$\forall x, y \in A_{(\alpha,\beta)}$

Therefore $\mu_A(x+y) \geq \alpha$, $\mu_A(xy) \geq \alpha$, $\mu_A(x \vee y) \geq \alpha$ and $\mu_A(x \wedge y) \geq \alpha$ and $\nu_A(x+y) \leq \beta$, $\nu_A(xy) \leq \beta$, $\nu_A(x \vee y) \leq \beta$ and $\nu_A(x \wedge y) \leq \beta \Rightarrow x+y$, xy, $x \vee y$ and $x \wedge y$ are in $A_{(\alpha,\beta)}$. Hence $A_{(\alpha,\beta)}$ is a $\ell-$subsemiring R.

Theorem: 1.2
Let A be an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring R. Then two

level $\ell-$subsemirings $A_{(\alpha_1,\beta_1)}$, $A_{(\alpha_2,\beta_2)}$ and α_1, α_2 in $[0,1]$, β_1, β_2 in $[0,1]$ with $\alpha_2 < \alpha_1$ and $\beta_1 < \beta_2$ of A are equal if and only if there is no x in R such that $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < v_A(x) < \beta_2$.

Proof:
Assume that $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$. Suppose there exists $x \in R$ such that $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < v_A(x) < \beta_2$. Then $A_{(\alpha_1,\beta_1)} \subseteq A_{(\alpha_2,\beta_2)}$
$\Rightarrow$ $x \in A_{(\alpha_2,\beta_2)}$, but $x \notin A_{(\alpha_1,\beta_1)}$ $\Rightarrow$ a contradiction to $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$
Therefore there is no $x \in R$ such that $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < v_A(x) < \beta_2$.
Conversely if there is no $x \in R$ such that $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < v_A(x) < \beta_2$
Then $A_{(\alpha_1,\beta_1)} = A_{(\alpha_2,\beta_2)}$

Theorem: 1.3
Let R be a $\ell-$semiring and A be an intuitionistic fuzzy subset of R such that $A_{(\alpha,\, \beta)}$ be a $\ell-$subsemiring of R. If α and β in $[0,1]$ then A is an intuitionistic fuzzy $\ell-$subsemiring of R.

Proof:
Let R be a $\ell-$subsemiring. For x and y in R.
Let $\mu_A(x) = \alpha_1$, $\mu_A(y) = \alpha_2$, $v_A(x) = \beta_1$ and $v_A(x) = \beta_2$
Case (i) : If $\alpha_1 < \alpha_2$ and $\beta_1 > \beta_2$ then $x, y \in A_{(\alpha_1,\beta_1)}$.
As $A_{(\alpha_1,\beta_1)}$ is a $\ell-$subsemiring of R, $x+y$, xy, $x \vee y$ and $x \wedge y$ in $A_{(\alpha_1,\beta_1)}$

$\mu_A(x+y) \geq \alpha_1 = \min\{\alpha_1,\ \alpha_2\} \Rightarrow \mu_A(x+y) \geq \min\{\mu_A(x),\ \mu_A(y)\}$,

$\mu_A(xy) \geq \alpha_1 = \min\{\alpha_1,\ \alpha_2\} \Rightarrow \mu_A(xy) \geq \min\{\mu_A(x),\ \mu_A(y)\}$,

$\mu_A(x \vee y) \geq \alpha_1 = \min\{\alpha_1,\ \alpha_2\} \Rightarrow \mu_A(x \vee y) \geq \min\{\mu_A(x),\ \mu_A(y)\}$,

$\mu_A(x \wedge y) \geq \alpha_1 = \min\{\alpha_1,\ \alpha_2\} \Rightarrow \mu_A(x \wedge y) \geq \min\{\mu_A(x),\ \mu_A(y)\}$,

$v_A(x+y) \leq \beta_1 = \max\{\beta_1,\ \beta_2\} \Rightarrow v_A(x+y) \leq \max\{v_A(x),\ v_A(y)\}$,

$v_A(xy) \leq \beta_1 = \max\{\beta_1,\ \beta_2\} \Rightarrow v_A(xy) \leq \max\{v_A(x),\ v_A(y)\}$,

$v_A(x \vee y) \leq \beta_1 = \max\{\beta_1,\ \beta_2\} \Rightarrow v_A(x \vee y) \leq \max\{v_A(x),\ v_A(y)\}$,

$v_A(x \wedge y) \leq \beta_1 = \max\{\beta_1,\ \beta_2\} \Rightarrow v_A(x \wedge y) \leq \max\{v_A(x),\ v_A(y)\}$,

$\forall\ x, y \in R$

Case (ii) : If $\alpha_1 < \alpha_2$ and $\beta_1 < \beta_2$ then x and y in $A_{(\alpha_1,\beta_2)}$

As $A_{(\alpha_1,\beta_2)}$ is a $\ell-$subsemiring of R, $x+y$, xy, $x \vee y$ and $x \wedge y$ in $A_{(\alpha_1,\beta_2)}$

$\mu_A(x+y) \geq \alpha_1 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(xy) \geq \alpha_1 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(x \vee y) \geq \alpha_1 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(x \vee y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(x \wedge y) \geq \alpha_1 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(x \wedge y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\nu_A(x+y) \leq \beta_2 = \max\{\beta_1, \beta_2\} \Rightarrow \nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$,

$\nu_A(xy) \leq \beta_2 = \max\{\beta_1, \beta_2\} \Rightarrow \nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$,

$\nu_A(x \vee y) \leq \beta_2 = \max\{\beta_1, \beta_2\} \Rightarrow \nu_A(x \vee y) \leq \max\{\nu_A(x), \nu_A(y)\}$,

$\nu_A(x \wedge y) \leq \beta_2 = \max\{\beta_1, \beta_2\} \Rightarrow \nu_A(x \wedge y) \leq \max\{\nu_A(x), \nu_A(y)\}$,

$\forall\ x, y \in R$

Case (iii) : If $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$ then x and y in $A_{(\alpha_2,\beta_1)}$

As $A_{(\alpha_2,\beta_1)}$ is a $\ell-$subsemiring of R, $x+y$, xy, $x \vee y$ and $x \wedge y$ in $A_{(\alpha_2,\beta_1)}$

$\mu_A(x+y) \geq \alpha_2 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(xy) \geq \alpha_2 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(x \vee y) \geq \alpha_2 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(x \vee y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(x \wedge y) \geq \alpha_2 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(x \wedge y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\nu_A(x+y) \leq \beta_1 = \max\{\beta_1, \beta_2\} \Rightarrow \nu_A(x+y) \leq \max\{\nu_A(x), \nu_A(y)\}$,

$\nu_A(xy) \leq \beta_1 = \max\{\beta_1, \beta_2\} \Rightarrow \nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\}$,

$\nu_A(x \vee y) \leq \beta_1 = \max\{\beta_1, \beta_2\} \Rightarrow \nu_A(x \vee y) \leq \max\{\nu_A(x), \nu_A(y)\}$,

$\nu_A(x \wedge y) \leq \beta_1 = \max\{\beta_1, \beta_2\} \Rightarrow \nu_A(x \wedge y) \leq \max\{\nu_A(x), \nu_A(y)\}$,

$\forall\ x, y \in R$

Case (iv) : If $\alpha_1 > \alpha_2$ and $\beta_1 < \beta_2$ then x and y in $A_{(\alpha_2,\beta_2)}$

As $A_{(\alpha_2,\beta_2)}$ is a $\ell-$subsemiring of R, $x+y$, xy, $x \vee y$ and $x \wedge y$ in $A_{(\alpha_2,\beta_2)}$

$\mu_A(x+y) \geq \alpha_2 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(x+y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(xy) \geq \alpha_2 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(x \vee y) \geq \alpha_2 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(x \vee y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$\mu_A(x \wedge y) \geq \alpha_2 = \min\{\alpha_1, \alpha_2\} \Rightarrow \mu_A(x \wedge y) \geq \min\{\mu_A(x), \mu_A(y)\}$,

$v_A(x+y) \leq \beta_2 = \max\{\beta_1, \beta_2\} \Rightarrow v_A(x+y) \leq \max\{v_A(x), v_A(y)\}$,

$v_A(xy) \leq \beta_2 = \max\{\beta_1, \beta_2\} \Rightarrow v_A(xy) \leq \max\{v_A(x), v_A(y)\}$,

$v_A(x \vee y) \leq \beta_2 = \max\{\beta_1, \beta_2\} \Rightarrow v_A(x \vee y) \leq \max\{v_A(x), v_A(y)\}$,

$v_A(x \wedge y) \leq \beta_2 = \max\{\beta_1, \beta_2\} \Rightarrow v_A(x \wedge y) \leq \max\{v_A(x), v_A(y)\}$,

$\forall\ x, y \in R$

Case (v) : If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then it is trivial.

In all the cases, A is an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring R.

Theorem: 1.4

Let A be an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring R. If any two level $\ell-$subsemirings of A belongs to R then their intersection is also level $\ell-$subsemirings of A in R.

Proof:

Given A is an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring R.

Take α_1, α_2 in $[0, 1]$ and β_1, β_2 in $[0, 1]$

Case (i) : If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 > v_A(x) > \beta_2$, then $A_{(\alpha_2,\beta_2)} \subseteq A_{(\alpha_1,\beta_1)}$

$\Rightarrow A_{(\alpha_1,\beta_1)} \cap A_{(\alpha_2,\beta_2)} = A_{(\alpha_2,\beta_2)}$, where $A_{(\alpha_2,\beta_2)}$ is a level $\ell-$subsemiring of A

Case (ii) : If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < v_A(x) < \beta_2$, then $A_{(\alpha_1,\beta_1)} \subseteq A_{(\alpha_2,\beta_2)}$

$\Rightarrow A_{(\alpha_1,\beta_1)} \cap A_{(\alpha_2,\beta_2)} = A_{(\alpha_1,\beta_1)}$, where $A_{(\alpha_1,\beta_1)}$ is a level $\ell-$subsemiring of A

Case (iii) : If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 < v_A(x) < \beta_2$, then $A_{(\alpha_2,\beta_1)} \subseteq A_{(\alpha_1,\beta_2)}$

$\Rightarrow A_{(\alpha_2,\beta_1)} \cap A_{(\alpha_1,\beta_2)} = A_{(\alpha_2,\beta_1)}$, where $A_{(\alpha_2,\beta_1)}$ is a level $\ell-$subsemiring of A

Case (iv) : If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < v_A(x) < \beta_2$, then $A_{(\alpha_1,\beta_2)} \subseteq A_{(\alpha_2,\beta_1)}$

$\Rightarrow A_{(\alpha_1,\beta_2)} \cap A_{(\alpha_2,\beta_1)} = A_{(\alpha_1,\beta_2)}$, where $A_{(\alpha_1,\beta_2)}$ is a level $\ell-$subsemiring of A

Case (v) : If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha_1,\beta_1)} = A_{(\alpha_1,\beta_1)}$

Therefore in all the cases, the intersection of any two level $\ell-$subsemiring is a level $\ell-$subsemiring of A.

Corollary: 1.1

Let A be an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring R. If α_i and β_i in $[0, 1]$ and $A_{(\alpha_i, \beta_j)}$; $i,\ j \in I$ is a collection of level $\ell-$subsemirings of A, then their intersection is also a level $\ell-$subsemirings of A in R.

Theorem: 1.5

Let A be an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring R. If any two level $\ell-$subsemirings of A belongs to R then their union is also level $\ell-$subsemiring of A in R.

Proof:

Given A is an intuitionistic fuzzy $\ell-$subsemiring of a $\ell-$semiring R

Let α_1, α_2 in $[0, 1]$ and $\beta_1,\ \beta_2$ in $[0, 1]$

Case (i): If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 > v_A(x) > \beta_2$, then $A_{(\alpha_2, \beta_2)} \subseteq A_{(\alpha_1, \beta_1)}$

$\Rightarrow\ A_{(\alpha_1, \beta_1)} \cup A_{(\alpha_2, \beta_2)} = A_{(\alpha_1, \beta_1)}$, where $A_{(\alpha_1, \beta_1)}$ is a level $\ell-$subsemiring of A

Case (ii) : If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 < v_A(x) < \beta_2$, then $A_{(\alpha_1, \beta_1)} \subseteq A_{(\alpha_2, \beta_2)}$

$\Rightarrow\ A_{(\alpha_1, \beta_1)} \cup A_{(\alpha_2, \beta_2)} = A_{(\alpha_2, \beta_2)}$, where $A_{(\alpha_2, \beta_2)}$ is a level $\ell-$subsemiring of A

Case (iii) : If $\alpha_1 < \mu_A(x) < \alpha_2$ and $\beta_1 < v_A(x) < \beta_2$, then $A_{(\alpha_2, \beta_1)} \subseteq A_{(\alpha_1, \beta_2)}$

$\Rightarrow\ A_{(\alpha_2, \beta_1)} \cup A_{(\alpha_1, \beta_2)} = A_{(\alpha_1, \beta_2)}$, where $A_{(\alpha_1, \beta_2)}$ is a level $\ell-$subsemiring of A

Case (iv) : If $\alpha_1 > \mu_A(x) > \alpha_2$ and $\beta_1 > v_A(x) > \beta_2$, then $A_{(\alpha_1, \beta_2)} \subseteq A_{(\alpha_2, \beta_1)}$

$\Rightarrow\ A_{(\alpha_1, \beta_2)} \cup A_{(\alpha_2, \beta_1)} = A_{(\alpha_2, \beta_1)}$, where $A_{(\alpha_2, \beta_1)}$ is a level $\ell-$subsemiring of A

Case (v) : If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, then $A_{(\alpha_1, \beta_1)} = A_{(\alpha_2, \beta_2)}$

Therefore in all the cases, the union of any two level $\ell-$subsemiring is also a level $\ell-$subsemiring of A.

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