International Journal of Computational Science and Mathematics.
ISSN 0974-3189 Volume 8, Number 1 (2016), pp. 1-7
© International Research Publication House
http://www.irphouse.com

Split block subdivision domination in fuzzy graphs

C.|V.R.Harinarayanan[1] and S.Geetha [2]

[1]*Government Arts college, Paramakudi Tamilnadu, India*
[2]*Kings College of Engineering, Punalkulam, Thanjavur Tamilnadu, India*

Abstract

Let G be a fuzzy graph. B(G) is a fuzzy block graph of G. SB[G] is a subdivision fuzzy block graph of B[G]. A dominating set D of V[SB(G)] is a split dominating set in SB[G], if the induced subgraph $V[SB(G)] - D$ is disconnected in [SB(G)]. The split domination number of [SB(G)] is denoted by $\gamma_{ssb}(G)$ which is the minimum cardinality of a split dominating set in [SB(G)]. In this paper bounds on γ_{ssb} were obtained in terms of vertices, blocks, and othe domination parameters of G.

Keywords Fuzzy block graph, subdivision fuzzy block graph, split domination number.

1.Introduction

Let V be a finite non empty set and E be the collection of all two element subsets of V.

A fuzzy graph $G = (\sigma, \mu)$ is a set with a pair of relations $\sigma : V \to [0,1]$ *and* $\mu : V \times V \to [0,1]$ *such that* $\mu(u, v) \le \sigma(u) \wedge \sigma(v)$ for all $u, v \in V$.

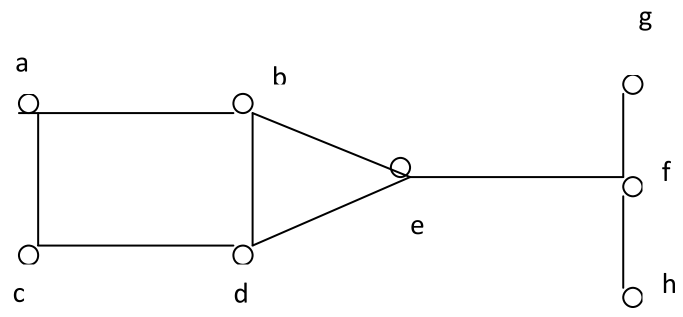
A non empty set $D \subseteq V$ of a fuzzy graph $G = (\sigma, \mu)$ is a dominating set of G if every vertex in V-D is adjacent to some vertex in D.The domination number $\gamma(G)$ is the minimum cardinality taken over all the minimal dominating sets of G.

The order of a fuzzy graph G is O (G)= $\sum_{u \in V} \sigma(u)$

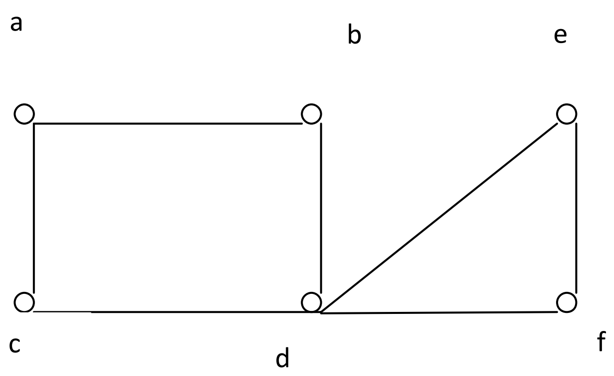
The size of a fuzzy graph G is S (G)= $\sum_{uv \in E} \mu(uv)$

A dominating set of a fuzzy graph G is a split (non split) dominating set if the induced

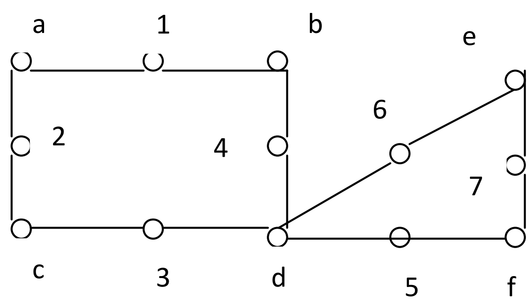
subgraph $\langle V-D\rangle$ is disconnected (connected).
The split (non split) domination number $\gamma_s(G)[\gamma_{ns}(G)]$ is the minimum cardinality of a split(non split) dominating set.
Two nodes that are joined by a path are said to be connected.
A vertex v of a fuzzy graph G is called a cut vertex if removing it from G increases the number of components of G.
The vertex cover in a fuzzy graph G is a set of vertices that covers all the edges of G
The vertex covering number $\alpha_0(G)$ is a minimum cardinality of of a vertex cover in G
An edge cover of G is the set of edges that covers all the vertices. The edge covering number $\alpha_1(G)$ of G is the minimum cardinality of an edge cover.
The edge independence number $\beta_1(G)$ of G is the minimum cardinality of an independent set of edges.
A dominating set D of a Fuzzy graph B(G) is a split block dominating set if the induced subgraph $\langle V[B(G)]-D\rangle$ is disconnected. The split block domination number $\gamma_{sb}(G)$is the minimum cardinality of split block dominating set.
A dominating set D of G is a cototal dominating set if the induced subgraph $\langle V-D\rangle$ has no isolated vertices. The co total domination number $\gamma_{\cot}(G)$ is the minimum minimum cardinality of a co total dominating set. The split dominating set of SB(G) is denoted by $\gamma_{ssb}(G)$
The subdivision fuzzy graph S(G) of a fuzzy graph G is the fuzzy graph obtained from G by subdividing each edge of a fuzzy graph G.



Example:
σ (a)=0.5 σ (b)=0.4 σ (c)=0.6 σ (d)=0.3
σ (e)=0.2 σ (f)=0.1 σ (g)=0.3 σ (h)=0.2



σ(a)=0.6 σ(b)=0.2 σ(c)=0.3 σ(d)=0.2
σ(e)=0.3 σ(f)=0.2



σ(a)=0.6 σ(b)=0.2 σ(c)=0.3 σ(d)=0.2
σ(e)=0.3 σ(f)=0.2
σ(1)=0.4 σ(2)=0.4 σ(3)=0.2 σ(4)=0.1
σ(5)=0.1 σ(6)=0.2 σ(7)=0.2

Theorem:I

A split dominating set D of G is minimal for each vertex $v \in D,$ one of the following conditions holds:

i) There exists a vertex $u \in V - D,\ \textit{such that}\ \ N(u) \cap D = \{v\}$

ii) v is an isolated vertex in $\langle D \rangle$

iii) $\langle (V - D) \cup \{v\} \rangle$ is connected

Theorem:II

For any fuzzy graph G, $\gamma_{sb}(G) \leq \dfrac{p.\Delta(G)}{1 + \Delta(G)}$

2.Main Results

Theorem2.1:

Let G bae fuzzy graph G with n blocks and $n \geq 2$, then $\gamma_{ssb}(G) \leq n-1$

Proof:

For any fuzzy graph G with n=1 block, a split domination does not exists. Hence we need $n \geq 2$ blocks. Let $S = \{B_1, B_2, ...B_n\}$ be the number of blocks of G and M $= \{b_1, b_2, ...b_n\}$ be the vertices in B(G) with corresponding to blocks of S. V $= \{v_1, v_2, ...v_n\}$ denote the set of vertices in [SB(G)]. Let $V_1 = \{v_1, v_2, ...v_i\}, 1 \leq i \leq n$, $V_1 \subset V$ be a set of cut vertices.Again consider a subset V_1^1 of V such that $\forall v_i \in N(v) \cap N(V_1^1)$ *and* $V_1 = V - V_1^1$.

Let $V_2 = \{v_1, v_2, ...v_s\}, 1 \leq s \leq n$, $\forall v_s \in V$ which are not cut vertices such that $N(V_1) \cap N(V_2) = \Phi$, then $\{V_1 \cup V_2\}$ is a dominating set. Clearly V[SB(G)].- $\{V_1 \cup V_2\}$ =H is a disconnected fuzzy graph. Then $(V_1 \cup V_2)$ is a γ_{ssb} set of G. Hence $|V_1 \cup V_2| = \gamma_{ssb}(G) \Rightarrow \gamma_{ssb} \leq n-1$

Theorem 2.2:

Let G be a fuzzy graph G with $n \geq 2$ blocks then $\gamma_{ssb}(G) \leq \left\lfloor \dfrac{p\Delta(G)}{1+\Delta(G)} \right\rfloor$

Proof:

Consider fuzzy graphs with $n \geq 2$ blocks. If n=1, split dominating set does not exists.. Let $S = \{B_1, B_2, ...B_n\}$ be the number of blocks of G and M$= \{b_1, b_2, ...b_n\}$ be the vertices in B(G) with corresponding to blocks of S. V$= \{v_1, v_2, ...v_n\}$ denote the set of vertices in [SB(G)].Let D be a split dominating set of [SB(G)].

By theorem, each vertex $v \in D$, there exists a vertex $u \in V[SB(G)] - D$ is a split dominating set in [SB(G)]. Thus $\gamma(G) \leq V[SB(G)] - D$. This implies $\gamma(G) \leq p - \gamma_{ssb}$

For any fuzzy graph G, $\gamma_s(G) \leq \dfrac{p.\Delta(G)}{1+\Delta(G)}$

By using the above theorem II, we have $\gamma_{ssb}(G) \leq \left\lfloor \dfrac{p\Delta(G)}{1+\Delta(G)} \right\rfloor$

Theorem2.3:

For any fuzzy graph G with $n \geq 2$ blocks, then $\gamma_{ssb}(G) \geq \alpha_0[B(G)]$, *where* α_0 is vertex covering number of B(G).

Proof:

We consider only fuzzy graphs for which $n \neq 1$. Let $S = \{B_1, B_2, ...B_n\}$ be the number of blocks of G and $M=\{b_1,b_2,.....b_n\}$ be the vertices in B(G) with corresponding to blocks of S.Let $V=\{v_1,v_2,.....v_n\}$ denote the set of vertices in [SB(G)] such that $M\subset V$

Again D== $\{v_1,v_2,.....v_i\}$, $1\le i\le n$, $D\subset V$ such that $N(v_i)\cap N(v_j)=v_k$,v_i ,$v_j\in D$

Hence $\langle V[SB(G)]-D\rangle$ is disconnected, which gives $|V[SB(G)]-D|=\gamma_{ssb}(G)$

Now $M_1=\{b_1,b_2,,...............b_i\}$, $1\le i\le n$ *and* $M_1\subset M$ and each edge in B(G) is adjacent to atleast one vertex in M_1. *Clearly* $|M_1|=\alpha_0[B(G)]$. *Hence* $|V[SB(G)]-D|\ge|M_1|$ *which gives* $\gamma_{ssb}(G)\ge\alpha_0[B(G)]$

Theorem 2.4:

For any connected fuzzy graph G with $n\ge 2$ blocks and N end blocks, then $\gamma_{ssb}(G)\le\gamma(G)+N$

Proof:

Suppose fuzzy graph G is a block.Then by definition split domination does not exists. Now assume G is a fuzzy graph with at least 2 blocks. $n\ne 1$. Let $S=\{B_1,B_2,.....B_n\}$ be the number of blocks of G and $M=\{b_1,b_2,.....b_n\}$ be the vertices in B(G) with corresponding to blocks of S.Let $V=\{v_1,v_2,.....v_n\}$ denote the set of vertices in [SB(G)]. Suppose D is a γ_s −set [SB(G)] of G, whose vertices is $V=\{v_1,v_2,.....v_i\}$

Note that atleast one $v_i\in S$. Moreover, any component of V-S is of size atleast two.Thus D is minimal which gives $|D|=\gamma_{ssb}(G)$. Again $S_1=\{u_1,u_2,.....u_n\}$ be the vertices in G and $D_1=\{u_1,u_2,.....u_i\}$,$1\le i\le n$, $D_1\subset S_1$. Every vertex of S_1-D_1 is adjacent to atleast one vertex of D_1

Suppose there exists a vertex $v\in D_1$ such that every vertex of D_1-V_1 is not adjacent to atleast one vertex $u\in[S_1-\{D_1-v\}]$. Thus $|S_1-D_1|=\gamma(G)$. Hence $|D|\le|S_1-D_1|+N$ which gives $\gamma_{ssb}(G)\le\gamma(G)+N$

Theorem2.5:

For any connected fuzzy graph G with $n\ge 2$ blocks then $\gamma_{ssb}(G)\ge\beta_0(G)-1$, where $\beta_0(G)$ is the independent number of G.

Proof:

By the definition of split domination, $n\ne 1$. Let $S=\{B_1,B_2,.....B_n\}$ be the number of blocks of G and $M=\{b_1,b_2,.....b_n\}$ be the vertices in B(G) with corresponding to blocks of S. Let $V=\{v_1,v_2,.....v_n\}$ denote the set of vertices in [SB(G)] such that

$M \subset V$. Let H$=\{v_1, v_2, ...v_n\}$ be the vertices in G. We have the following cases:
Case i) Suppose B(G) is a tree. Let $V_1^1 = \{v_1, v_2, ...v_n\}$ are cut vertices in [SB(G)]. Again $V_1^{"} = \{v_1, v_2, ...v_t\}$ $1 \le t \le s$ and $V_1^{"} \subset V_1^1$ *for all* $v_t \in V_1^{"}$. Then we consider V_2^1, V_3^1, V_4^1 *where* $V_1^{"} = \{v_1, v_2, ...v_t\} = V_2^1 \cup V_3^1 \cup V_4^1$ with the property that $N(v_i) \cap N(v_j) = \phi$, $\forall v_i \in V_2^1$ *and* $\forall v_j \in V_3^1$ *and* V_4^1 is a set of all end vertices in SB(G).
Again $\langle V[SB(G)] \rangle = J$ where every $v \in J$ *is an isolates*. *Thus* $|V_1^{"}| = \gamma_{ssb}(G)$
Case 2 Suppose B(G) is not a tree.

Subcase2.1
Assume B(G) is a block. Then in [SB(G)], V[SB(G)]=V[B(G)]+{k} $|K| = P_0$ is the number of isolates in V[SB(G)]-V[B(G)]. Hence $\|V[B(G)]\| \ge \beta_0 - 1$. This implies that $\gamma_{ssb}(G) \ge \beta_0(G) - 1$

Theorem 2.6:
For any fuzzy graph G with $n \ge 2$ blocks then $\gamma_{ssb}(G) + \gamma(G) \le p + 1$

Proof:
Suppose the fuzzy graph G has only one block, then split domination does not exists. Hence $n \ge 2$. Suppose $n \ne 1$. Let $S = \{B_1, B_2, ...B_n\}$ be the number of blocks of G and M$=\{b_1, b_2, ...b_n\}$ be the vertices in B(G) with corresponding to blocks of S.Let H $=\{v_1, v_2, ...v_n\}$ denote the set of vertices in G. Take V $=\{v_1, v_2, ...v_i\}$ $1 \le i \le n$ *such that* $J \subset H$ *and* $\forall v_i \in H - J$ is adjacent to one vertex of J. Hence $|J| = \gamma(G)$. Let V$=\{v_1, v_2, ...v_s\}$ denote the set of vertices in [SB (G)]. Now let $S_1 = \{B_i\}$ *where* $1 \le i \le n$, $S_1 \subset S$ *and* $\forall B_i \in S_i$ are non end blocks in G. Then we have $V_1 \subset V$ which corresponds to the elements of $S[S_1]$ such that V_1 forms a minimal dominating set of [SB(G)]. Since each element of V_1 is a cutvertex, then $|V_1| = \gamma_{ssb}(G)$. *Further* $V_1 \cup J \le p + 1$. This implies that $\gamma_{ssb}(G) + \gamma(G) \le p + 1$.

Theorem 2.7
For any nontrivial fuzzy tree with $n \ge 2$ blocks, $\gamma_{ssb}(G) \ge \gamma_{\cot}(G) - 1$

Proof:
Consider fuzzy graphs with $n \ne 1$. Let H$=\{v_1, v_2, ...v_p\}$. $H_1 = \{v_1, v_2, ...v_i\}, 1 \le i \le p$ be a subset of V(G)=H which are end vertices in G.Let T= $=\{v_1, v_2, ...v_j\} \subseteq V(G)$ *with* $1 \le j \le p$ such that

$\forall v_i \in J, N(v_i) \cap N(v_j) = \phi$ *and* $\langle V(G) - (H_1 \cup J) \rangle$ has no isolates, then $|H_1 \cup J| = \gamma_{\cot}(G.)$ Let V $= \{v_1, v_2, ...v_n\}$ be the vetices in [SB(G)]. Consider D $= \{v_1, v_2, ...v_t\} = V_1 \cup V_2 \cup V_3$ be the set of all vertices of [SB(G)] where $\forall v_s \in V_1$ *and* $v_t \in V_2$ with the property that $v_s \cap N(v_t) = \phi$,$\forall v_t \in V_3$ is, the set of all end vertices in [SB(G)]. The $\langle D \rangle$ is an isolates.$|D|$ gives minimum split dominating set in [SB(G)].

References :-

[1]. Q.M.Mahyoub and N.D.Soner The Split domination number of fuzzy graphs

[2]. M.H.Muddebihal, Shabbir Ahamed, P.Shekanna.Split Block domination in Graphs

[3]. K.Ameenal Bibi and R.Selvakumar The inverse split and non –split dominations in graphs

[4]. A.Somasundaram, S.Somasundaram, domination in fuzzy graphs - I, February - 1998.

[5]. John N.Mordeson, Premchand S.Nair Fuzzy graphs and fuzzy hyper graphs