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Properties of T – Anti Fuzzy Ideal of ℓ – Rings

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Abstract

In this paper, we made an attempt to study the properties of $T-$anti fuzzy ideal of $\ell-$ring and we introduce some definitions and theorems in join, union, join of a family and the union of a family of $T-$anti fuzzy ideal of $\ell-$ring.

Keywords: Fuzzy subset, $T-$fuzzy ideal, $T-$anti fuzzy ideal, join of $T-$anti fuzzy ideal, union of $T-$anti fuzzy ideal, join of a family $T-$anti fuzzy ideal and the union of a family of $T-$anti fuzzy ideal.

INTRODUCTION

The concept of fuzzy sets was initiated by L.A.Zadeh [9] in 1965. After the introduction of fuzzy sets several researchers explored on the generalization of the concept of fuzzy sets. In this paper we define, characterize and study the $T-$anti fuzzy right and left ideals. Z. D. Wang introduced the basic concepts of TL-ideals. We introduced $T-$anti fuzzy right ideals of $\ell-$ ring. We compare fuzzy ideal introduced by Liu to $T-$anti fuzzy ideals. We have shown that ring is regular if and only if union of any $T-$anti fuzzy right ideal with $T-$anti fuzzy left ideal is equal to its product. We discuss some of its properties. We have shown that the join of $T-$anti fuzzy ideal of $\ell-$ring, union of $T-$anti fuzzy ideal of $\ell-$ring, join of a family $T-$anti fuzzy ideal of $\ell-$ring and the union of a family of $T-$anti fuzzy ideal of $\ell-$ring.

Definition: 1

A nonempty set R together with two binary operation "+" and " . " is called a ring if

(i). $(R, +)$ is an abelian group,

(ii). $(R, .)$ is a semigroup

(iii). $x(y+z) = xy + xz$; $(x+y)z = xz + yz$, for all x, y, z in R

Definition: 2

A non-empty set R is called lattice ordered ring or $\ell-$ring if it has four binary operations "$+$", "$\cdot$", $\vee$, $\wedge$ defined on it and satisfy the following

(i) $(R, +, \cdot)$ is a ring

(ii) $(R, \vee, \wedge)$ is a lattice

(iii) $x+(y \vee z) = (x+y) \vee (x+z)$; $x+(y \wedge z) = (x+y) \wedge (x+z)$

$(y \vee z) + x = (y+x) \vee (z+x)$; $(y \wedge z) + x = (y+x) \wedge (z+x)$

(iv) $x \cdot (y \vee z) = (xy) \vee (xz)$; $x \cdot (y \wedge z) = (xy) \wedge (xz)$

$(y \vee z) \cdot x = (yx) \vee (zx)$; $(y \wedge z) \cdot x = (yx) \wedge (zx)$,

for all x, y, z in R and $x \geq 0$

Example: 1

$(Z, +, \cdot, \vee, \wedge)$ is a $\ell-$ring, where Z is the set of all integers.

Example: 2

$(nZ, +, \cdot, \vee, \wedge)$ is a $\ell-$ring, where Z is the set of all integers and $n \in Z$

Definition: 3

A mapping $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called a triangular norm [$t-$norm] if and only if it satisfies the following conditions:

(i). $T(x, 1) = T(1, x) = x$, for all $x \in [0, 1]$

(ii). if $x \geq x^*$, $y \geq y^*$ then $T(x, y) \geq T(x^*, y^*)$

(iii). $T(x, y) = T(y, x)$, for all x, y $\in [0, 1]$.

(iv). $T(x, T(\mathrm{y}, \mathrm{z})) = T(T(\mathrm{x}, \mathrm{y}), \mathrm{z})$.

Definition: 4

A mapping from a nonempty set X to $[0, 1]$ $\mu : X \rightarrow [0, 1]$ is called a fuzzy subset of X .

Proposition: 1
Every $t-norm\ T$, has a useful property
(i) $T(\alpha, \beta) \le \min\{\alpha, \beta\}$, and
(ii) $T(\alpha, 0) = 0$, for all $\alpha, \beta \in [0,1]$

Definition: 5
Let μ_A and λ_A be an anti fuzzy subsets of a set X. An anti fuzzy subset $\mu_A \cup \lambda_A$ is defined as $(\mu_A \cup \lambda_A)(x) = \max\{\mu_A(x), \lambda_A(x)\}$

Example: 3
Let $\mu_A = \{\langle a, 0.4\rangle, \langle b, 0.7\rangle, \langle c, 0.3\rangle\}$ and
$\lambda_A = \{\langle a, 0.5\rangle, \langle b, 0.3\rangle, \langle c, 0.43\rangle\}$ be an anti fuzzy subsets of $X = \{a, b, c\}$
The union of two anti fuzzy subsets of μ_A and λ_A is
$\mu_A \cup \lambda_A = \{\langle a, 0.5\rangle, \langle b, 0.7\rangle, \langle c, 0.43\rangle\}$

Definition: 6
Let μ_A and λ_A be the fuzzy subsets of a set X. An anti-fuzzy subset $\mu_A \vee \lambda_A$ is defined as $(\mu_A \vee \lambda_A)(x) = T(\mu_A(x), \lambda_A(x))$

Definition: 7
An anti fuzzy subset μ_A of a ring R is called $T-$anti fuzzy left (resp. right) ideal if
(i) $\mu_A(x-y) \le T(\mu_A(x), \mu_A(y)) = \max\{\mu_A(x), \mu_A(y)\}$
(ii) $\mu_A(xy) \le \{\mu_A(x)\}$ (resp. left $\mu_A(xy) \le \{\mu_A(y)\}$), for all x, y in R

Theorem: 1
Every anti-fuzzy right ideal of a ring R is an $T-$anti fuzzy right ideal.
Proof:
Let μ_A be an anti fuzzy right ideal of R.
Then $\mu_A(x-y) \le T(\mu_A(x), \mu_A(y))$ and $\mu_A(xy) \le \{\mu_A(x)\}$, for all $x, y \in R$.
Hence μ_A is an $T-$anti fuzzy ideal.

Definition: 8
An anti fuzzy subset μ_A of a lattice ordered ring (or $\ell-$ring) R is called an anti fuzzy sub $\ell-$ring of R, if the following conditions are satisfied
(i) $\mu_A(x \vee y) \le \max\{\mu_A(x), \mu_A(y)\}$
(ii) $\mu_A(x \wedge y) \le \max\{\mu_A(x), \mu_A(y)\}$
(iii) $\mu_A(x-y) \le \max\{\mu_A(x), \mu_A(y)\}$

(iv) $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$, for all x, y in R

Example: 4
Consider an ant-fuzzy subset μ_1 of the $\ell-$ring $(Z, +, \cdot, \vee, \wedge)$

$\mu_1(x) = \begin{cases} 0.4 & if \ x \in \langle 2 \rangle \\ 0.7 & Z - \langle 2 \rangle \end{cases}$ Then μ_1 is an anti-fuzzy $\ell-$sub ring

Definition: 9
An anti fuzzy subset μ_A of an $\ell-$ring R is called an anti fuzzy $\ell-$ring ideal (or) fuzzy $\ell-$ideal of R, if for all x, y in R the following conditions are satisfied

(i) $\mu_A(x \vee y) \leq \max\{\mu_A(x), \mu_A(y)\}$

(ii) $\mu_A(x \wedge y) \leq \min\{\mu_A(x), \mu_A(y)\}$

(iii) $\mu_A(x - y) \leq \max\{\mu_A(x), \mu_A(y)\}$

(iv) $\mu_A(xy) \leq \min\{\mu_A(x), \mu_A(y)\}$

Definition: 10
An anti fuzzy subset μ_A of a ring R is called an $T-$anti fuzzy ideal, if the following conditions are satisfied,

(i) $\mu_A(x - y) \leq T(\mu_A(x), \mu_A(y))$

(ii) $\mu_A(xy) \leq \mu_A(x); \ \mu_A(xy) \leq \mu_A(y)$, for all $x, y \in R$

Definition: 11
An anti fuzzy subset μ_A of a $\ell-$ring R is called an $T-$anti fuzzy ideal, if the following conditions are satisfied,

(i) $\mu_A(x - y) \leq T(\mu_A(x), \mu_A(y))$

(ii) $\mu_A(xy) \leq \mu_A(x); \ \mu_A(xy) \leq \mu_A(y)$

(iii) $\mu_A(x \vee y) \leq T(\mu_A(x), \mu_A(y))$

(iv) $\mu_A(x \wedge y) \leq T(\mu_A(x), \mu_A(y))$, for all $x, y \in R$

Example: 5
Now $(R=\{a, b, c\}, +, \cdot, \vee, \wedge)$ is a $\ell-$ring. The operations $+, \cdot, \vee$ and $\wedge$ defined by the following tables
Consider an anti-fuzzy subset μ_A of the $\ell-$ring R

$$\mu_A(x) = \begin{cases} 0.2 & if \ x = a \\ 0.5 & if \ x = b \\ 0.8 & if \ x = c \end{cases}$$

Then μ_A is an $T-$anti fuzzy ideal of $\ell-$ring R

Theorem: 2
If μ_A and λ_A are $T-$anti fuzzy ideals of a $\ell-$ring R, then $\mu_A \vee \lambda_A$ is an $T-$anti fuzzy ideal of a $\ell-$ring R.
Proof:
Given μ_A and λ_A are $T-$anti fuzzy ideals of a $\ell-$ring R, Let $x, y \in R$

(i) $(\mu_A \vee \lambda_A)(x-y) = T\left(\mu_A(x-y), \lambda_A(x-y)\right)$

$\leq T\left(T\left(\mu_A(x), \mu_A(y)\right), T\left(\lambda_A(x), \lambda_A(y)\right)\right)$, (by definition)

$= T\left(T\left(T\left(\mu_A(x), \mu_A(y)\right), \lambda_A(x)\right), \lambda_A(y)\right)$

$= T\left(T\left(T\left(\mu_A(x), \lambda_A(x)\right), \mu_A(y)\right), \lambda_A(y)\right)$

$= T\left(T\left(\mu_A(x), \lambda_A(x)\right), T\left(\mu_A(y), \lambda_A(y)\right)\right)$

$= T\left((\mu_A \vee \lambda_A)(x), (\mu_A \vee \lambda_A)(y)\right)$

Therefore $(\mu_A \vee \lambda_A)(x-y) \leq T\left((\mu_A \vee \lambda_A)(x), (\mu_A \vee \lambda_A)(y)\right)$, for all $x, y \in R$

(ii) Since $\mu_A(xy) \leq \mu_A(x)$ and $\lambda_A(xy) \leq \lambda_A(x)$

Now $(\mu_A \vee \lambda_A)(xy) \leq T\left(\mu_A(xy), \lambda_A(xy)\right)$, (by definition)

$\leq T\left(\mu_A(x), \lambda_A(x)\right)$

$\leq (\mu_A \vee \lambda_A)(x)$

Therefore $(\mu_A \vee \lambda_A)(xy) \leq (\mu_A \vee \lambda_A)(x)$, for all $x, y \in R$

(iii) $(\mu_A \vee \lambda_A)(x \vee y) = T\left(\mu_A(x \vee y), \lambda_A(x \vee y)\right)$

$\leq T\left(T\left(\mu_A(x), \mu_A(y)\right), T\left(\lambda_A(x), \lambda_A(y)\right)\right)$, (by definition)

$= T\left(T\left(T\left(\mu_A(x), \mu_A(y)\right), \lambda_A(x)\right), \lambda_A(y)\right)$

$= T\left(T\left(T\left(\mu_A(x), \lambda_A(x)\right), \mu_A(y)\right), \lambda_A(y)\right)$

$= T\left(T\left(\mu_A(x), \lambda_A(x)\right), T\left(\mu_A(y), \lambda_A(y)\right)\right)$

$= T\left((\mu_A \vee \lambda_A)(x), (\mu_A \vee \lambda_A)(y)\right)$

Therefore $(\mu_A \vee \lambda_A)(x \vee y) \leq T\left((\mu_A \vee \lambda_A)(x), (\mu_A \vee \lambda_A)(y)\right)$, for all $x, y \in R$

(iv) $(\mu_A \vee \lambda_A)(x \wedge y) = T\left(\mu_A(x \wedge y), \lambda_A(x \wedge y)\right)$

$\leq T\left(T\left(\mu_A(x), \mu_A(y)\right), T\left(\lambda_A(x), \lambda_A(y)\right)\right)$, (by definition)

$= T\left(T\left(T\left(\mu_A(x), \mu_A(y)\right), \lambda_A(x)\right), \lambda_A(y)\right)$

$= T\left(T\left(T\left(\mu_A(x), \lambda_A(x)\right), \mu_A(y)\right), \lambda_A(y)\right)$

$= T\left(T\left(\mu_A(x), \lambda_A(x)\right), T\left(\mu_A(y), \lambda_A(y)\right)\right)$

$= T\left((\mu_A \vee \lambda_A)(x), (\mu_A \vee \lambda_A)(y)\right)$

Therefore $(\mu_A \vee \lambda_A)(x \wedge y) \leq T\left((\mu_A \vee \lambda_A)(x), (\mu_A \vee \lambda_A)(y)\right)$, for all $x, y \in R$

Thus $\mu_A \vee \lambda_A$, is an $T-$anti fuzzy right ideal of a $\ell-$ring R.

Theorem: 3

If μ_A and λ_A are $T-$anti fuzzy ideals of a $\ell-$ring R, then $\mu_A \cup \lambda_A$, is an $T-$anti fuzzy ideal of a $\ell-$ring R.

Proof:

Let μ_A and λ_A are $T-$anti fuzzy ideals of a $\ell-$ring R,

Let $x, y \in R$

(i) $(\mu_A \cup \lambda_A)(x-y) = \max\{\mu_A(x-y), \lambda_A(x-y)\}$

$\leq \max\{\max\{\mu_A(x), \mu_A(y)\}, \max\{\lambda_A(x), \lambda_A(y)\}\}$

$= \max\{\max\{\max\{\mu_A(x), \mu_A(y)\}, \lambda_A(x)\}, \lambda_A(y)\}$

$= \max\{\max\{\max\{\mu_A(x), \lambda_A(x)\}, \mu_A(y)\}, \lambda_A(y)\}$

$= \max\{\max\{\mu_A(x), \lambda_A(x)\}, \max\{\mu_A(y), \lambda_A(y)\}\}$

$= \max\{(\mu_A \cup \lambda_A)(x), (\mu_A \cup \lambda_A)(y)\}$

Therefore $(\mu_A \cup \lambda_A)(x-y) \leq \max\{(\mu_A \cup \lambda_A)(x), (\mu_A \cup \lambda_A)(y)\}$, for all $x, y \in R$

(ii) Since $\mu_A(xy) \leq \mu_A(x)$ and $\lambda_A(xy) \leq \lambda_A(x)$

Now $(\mu_A \cup \lambda_A)(xy) \leq \max\{\mu_A(xy), \lambda_A(xy)\}$

$\leq \max\{\mu_A(x), \lambda_A(x)\}$

$\leq (\mu_A \cup \lambda_A)(x)$

Therefore $(\mu_A \cup \lambda_A)(xy) \leq (\mu_A \cup \lambda_A)(x)$, for all $x, y \in R$

(iii) $(\mu_A \cup \lambda_A)(x \vee y) = \max\{\mu_A(x \vee y), \lambda_A(x \vee y)\}$

$\leq \max\{\max\{\mu_A(x), \mu_A(y)\}, \max\{\lambda_A(x), \lambda_A(y)\}\}$

$= \max\{\max\{\max\{\mu_A(x), \mu_A(y)\}, \lambda_A(x)\}, \lambda_A(y)\}$

$= \max\{\max\{\max\{\mu_A(x), \lambda_A(x)\}, \mu_A(y)\}, \lambda_A(y)\}$

$= \max\{\max\{\mu_A(x), \lambda_A(x)\}, \max\{\mu_A(y), \lambda_A(y)\}\}$

$= \max\{(\mu_A \cup \lambda_A)(x), (\mu_A \cup \lambda_A)(y)\}$

Therefore $(\mu_A \cup \lambda_A)(x \vee y) \leq \max\{(\mu_A \cup \lambda_A)(x), (\mu_A \cup \lambda_A)(y)\}$, for all $x, y \in R$

(iv) $(\mu_A \cup \lambda_A)(x \wedge y) = \max\{\mu_A(x \wedge y), \lambda_A(x \wedge y)\}$

$\leq \max\{\max\{\mu_A(x), \mu_A(y)\}, \max\{\lambda_A(x), \lambda_A(y)\}\}$

$= \max\{\max\{\max\{\mu_A(x), \mu_A(y)\}, \lambda_A(x)\}, \lambda_A(y)\}$

$= \max\{\max\{\max\{\mu_A(x), \lambda_A(x)\}, \mu_A(y)\}, \lambda_A(y)\}$

$= \max\{\max\{\mu_A(x), \lambda_A(x)\}, \max\{\mu_A(y), \lambda_A(y)\}\}$

$= \max\{(\mu_A \cup \lambda_A)(x), (\mu_A \cup \lambda_A)(y)\}$

Therefore $(\mu_A \cup \lambda_A)(x \wedge y) \leq \max\{(\mu_A \cup \lambda_A)(x), (\mu_A \cup \lambda_A)(y)\}$, for all $x, y \in R$

Thus $\mu_A \cup \lambda_A$, is an $T-$anti fuzzy ideal of a $\ell-$ring R.

Theorem: 4

The join of a family of an $T-$anti fuzzy ideal of $\ell-$ring R is an $T-$anti fuzzy ideal of a $\ell-$ring R.

Proof:

Let $\{u_\alpha : \alpha \in I\}$ be a family of $T-$anti fuzzy ideal of $\ell-$ring R

Let $A = \vee_{\alpha \in I} u_\alpha$ and Let x and y in R. Then

(i) $\mu_A(x-y) = T(\mu_A(x-y), \mu_A(x-y))$

$\leq T(T(\mu_A(x), \mu_A(y)), T(\mu_A(x), \mu_A(y)))$, (by definition)

$= T(T(\mu_A(x), \mu_A(y)))$

$= T(\mu_V(x), \mu_V(y))$

Therefore $\mu_A(x-y) \leq T(\mu_A(x), \mu_A(y))$, for all $x, y \in R$

(ii) Since $\mu_A(xy) \leq \mu_A(x)$ and $\mu_A(xy) \leq \mu_A(y)$

Now $\mu_A(xy) \leq T(\mu_A(xy), \mu_A(xy))$

$= T(\mu_A(x), \mu_A(x))$, (by definition)

$= \mu_A(x)$

Therefore $\mu_A(xy) \leq \mu_A(x)$, for all $x, y \in R$

(iii) $\mu_A(x \vee y) = T(\mu_A(x \vee y), \mu_A(x \vee y))$

$\leq T(T(\mu_A(x), \mu_A(y)), T(\mu_A(x), \mu_A(y)))$, (by definition)

$= T(T(\mu_A(x), \mu_A(y)))$

$= T\left(\mu_A(x), \mu_A(y)\right)$

Therefore $\mu_A(x \vee y) \leq T\left(\mu_A(x), \mu_A(y)\right)$, for all $x, y \in R$

(iv) $\mu_A(x \wedge y) = T\left(\mu_A(x \wedge y), \mu_A(x \wedge y)\right)$

$\leq T\left(T\left(\mu_A(x), \mu_A(y)\right), T\left(\mu_A(x), \mu_A(y)\right)\right)$, (by definition)

$= T\left(T\left(\mu_A(x), \mu_A(y)\right)\right)$

$= T\left(\mu_A(x), \mu_A(y)\right)$

Therefore $\mu_A(x \wedge y) \leq T\left(\mu_A(x), \mu_A(y)\right)$, for all $x, y \in R$

Thus the join of a family of an $T-$anti fuzzy ideal of $\ell-$ring R is an $T-$anti fuzzy ideal of a $\ell-$ring R.

Theorem: 5

The union of a family of an $T-$anti fuzzy ideal of $\ell-$ring R is an $T-$anti fuzzy ideal of a $\ell-$ring R.

Proof:

Let $\{U_\alpha : \alpha \in I\}$ be a family of $T-$fuzzy ideal of R and let $A = \bigcup_{\alpha \in I} U_\alpha$.

Let x and y in R

Then

(i) $\mu_A(x-y) = \max\left\{\mu_A(x-y), \mu_A(x-y)\right\}$

$\leq \max\left\{\max\{\mu_A(x), \mu_A(y)\}, \max\{\mu_A(x), \mu_A(y)\}\right\}$

$= \max\left\{\max\left(\mu_A(x), \mu_A(y)\right)\right\}$

$= \max\{\mu_A(x), \mu_A(y)\}$

Therefore $\mu_A(x-y) \leq \max\{\mu_A(x), \mu_A(y)\}$, for all $x, y \in R$

(ii). Since $\mu_A(xy) \leq \mu_A(x)$ and $\mu_A(xy) \leq \mu_A(y)$

Now $\mu_A(xy) \leq \max\{\mu_A(xy), \mu_A(xy)\}$

$\leq \max\{\mu_A(x), \mu_A(x)\}$

$\leq \mu_A(x)$

Therefore $\mu_A(xy) \leq \mu_A(x)$, for all $x, y \in R$

(iii) $\mu_A(x \vee y) = \max\left\{\mu_A(x \vee y), \mu_A(x \vee y)\right\}$

$\leq \max\left\{\max\{\mu_A(x), \mu_A(y)\}, \max\{\mu_A(x), \mu_A(y)\}\right\}$

$= \max\left\{\max\{\mu_A(x), \mu_A(y)\}\right\}$

$= \max\left(\mu_A(x), \mu_A(y)\right)$

Therefore $\mu_A(x \vee y) \leq \max\{\mu_A(x), \mu_A(y)\}$, for all $x, y \in R$

(iv) $\mu_A(x \wedge y) = \max\{\mu_A(x \wedge y), \mu_A(x \wedge y)\}$

$\leq \max\{\max\{\mu_A(x), \mu_A(y)\}, \max\{\mu_A(x), \mu_A(y)\}\}$

$= \max\{\max\{\mu_A(x), \mu_A(y)\}\}$

$= \min\{\mu_A(x), \mu_A(y)\}$

Therefore $\mu_A(x \wedge y) \leq \max(\mu_A(x), \mu_A(y))$, for all $x, y \in R$

Thus union of a family of $T-$anti fuzzy ideal of $\ell-$ring R is an $T-$anti fuzzy ideal of a $\ell-$ring R.

REFERENCES

[1] **W. Liu.** Fuzzy invariant subgroups and fuzzy ideals. Fuzzy Sets and Systems, 59:205–210, 1993.

[2] **Y.-D. Yu and Z.-D. Wang.** TL-subrings and TL-ideals part1: basic concepts. Fuzzy Sets and Systems, 68:93–103, 1994.

[3] **M. T. Abu Osman,** On some product of fuzzy subgroups, Fuzzy Sets and Systems 24 (1987), 79-86.

[4] **W. E. Barnes,** On the Γ-rings of Nobusawa, Pacific J. Math. 18 (1966), 411-422.

[5] **W. E. Coppage and J. Luh,** Radicals of gamma-rings, J. Math. Soc. Japan 23 (1971), 40-52.

[6] **N. Nobusawa,** On a generalization of the ring theory, Osaka J. Math. 1 (1964), 81-89.

[7] **B. Schweizer and A. Sklar,** Statistical metric spaces, Pacific Journal of Mathematics. 10 (No. 1) (1963), 313-334.

[8] **Y. Yu, J. N. Mordeson and S. C. Cheng,** Elements of L-algebra, Lecture Notes in Fuzzy Math. and Computer Sciences, Creighton Univ., Omaha, Nebraska 68178, USA (1994).

[9] **L. A. Zadeh,** Fuzzy sets, Inform. and Control, 8 (1965), 338-353.