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Homomorphism on T – Anti-Fuzzy Ideals of $\ell-$Ring

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Abstract

In this paper, we made an attempt to study the properties of $T-$fuzzy ideal of a $\ell-$ring and we introduce some theorems an homomorphic image, an epimorphic pre-image of a $T-$anti-fuzzy ideal of a $\ell-$ring.

Keywords: Fuzzy subset, $T-$fuzzy ideal, homomorphism of $T-$fuzzy ideal, homomorphism of $T-$anti-fuzzy ideal, homomorphic image of $T-$anti-fuzzy ideal, homomorphic pre-image $T-$anti-fuzzy ideal.

INTRODUCTION

In 1965, Zadeh introduced the concept of fuzzy sets. In 1967, Rosenfeld defined the idea of fuzzy subgroups and gave some of its properties. Biswas introduced the notion of anti fuzzy subgroups. In 1982, Wang-jin Liu introduced the concept of fuzzy ring and fuzzy ideal and discussed the operations on fuzzy ideals. Fuzzy subnear-rings are introduced by Abou-Zaid. In W. A. Dudek and Y. B. Jun introduced fuzzy subgroups over a $t-$norm. Many researchers are engaged in extending the concepts. Chandrasekhara Rao. K and V. Swaminathan [3] defined the anti-homomorphisms in near rings. In the year 1998, Sung M.H. et al. [12] proved the same result using the level fuzzy subsets and obtained some properties based on near-ring homomorphism. Homomorphic images and pre images of anti fuzzy ideals are investigated by K.H. Kim et al. [9].

In this paper we define, homomorphism and study the $\ell-$ring homomorphism. We introduced homomorphism in $T-$anti-fuzzy ideals of $\ell-$ ring. We discuss some of its properties. We have shown that homomorphism, homomorphic image of $T-$anti-fuzzy ideal, homomorphic pre-image $T-$anti-fuzzy ideal .

Definition: 1

A nonempty set R together with two binary operation "$+$" and "$.$" is called a ring if

(i). $(R, +)$ is an abelian group,

(ii). $(R, .)$ is a semigroup

(iii). $x(y+z) = xy+xz$; $(x+y)z = xz+yz$, for all x, y, z in R

Definition: 2

A non-empty set R is called lattice ordered ring or $\ell-$ring if it has four binary operations "$+$", "$\cdot$", $\vee$, $\wedge$ defined on it and satisfy the following

(i) $(R, +, \cdot)$ is a ring

(ii) $(R, \vee, \wedge)$ is a lattice

(iii) $x+(y \vee z) = (x+y) \vee (x+z)$; $x+(y \wedge z) = (x+y) \wedge (x+z)$

$(y \vee z) + x = (y+x) \vee (z+x)$; $(y \wedge z) + x = (y+x) \wedge (z+x)$

(iv) $x \cdot (y \vee z) = (xy) \vee (xz)$; $x \cdot (y \wedge z) = (xy) \wedge (xz)$

$(y \vee z) \cdot x = (yx) \vee (zx)$; $(y \wedge z) \cdot x = (yx) \wedge (zx)$,

for all x, y, z in R and $x \geq 0$

Example: 1

$(Z, +, \cdot, \vee, \wedge)$ is a $\ell-$ring, where Z is the set of all integers.

Example: 2

$(nZ, +, \cdot, \vee, \wedge)$ is a $\ell-$ring, where Z is the set of all integers and $n \in Z$

Definition: 3

A mapping $T : [0, 1] \times [0, 1] \to [0, 1]$ is called a triangular norm [$t-$norm] if and only if it satisfies the following conditions:

(i). $T(x, 1) = T(1, x) = x$, for all $x \in [0, 1]$

(ii). if $x \geq x^*$, $y \geq y^*$ then $T(x, y) \geq T(x^*, y^*)$

(iii). $T(x, y) = T(y, x)$, for all x, y $\in [0, 1]$.

(iv). $T(x, T(\text{y}, \text{z})) = T(T(\text{x}, \text{y}), \text{z})$.

Definition: 4

An anti- fuzzy subset μ_A of a ring R is called $T-$anti-fuzzy right (resp. left) ideal if

(i) $\mu_A(x-y) \leq T(\mu_A(x), \mu_A(y)) = \max\{\mu_A(x), \mu_A(y)\}$

(ii) $\mu_A(xy) \leq \{\mu_A(x)\}$ (resp. left $\mu_A(xy) \leq \{\mu_A(y)\}$), for all x, y in R

Definition: 5

An anti- fuzzy subset μ_A of a lattice ordered ring (or $\ell-$ring) R is called an anti-fuzzy sub $\ell-$ring of R, if the following conditions are satisfied

(i) $\mu_A(x \vee y) \leq \max\{\mu_A(x), \mu_A(y)\}$

(ii) $\mu_A(x \wedge y) \leq \max\{\mu_A(x), \mu_A(y)\}$

(iii) $\mu_A(x-y) \leq \max\{\mu_A(x), \mu_A(y)\}$

(iv) $\mu_A(xy) \leq \max\{\mu_A(x), \mu_A(y)\}$, for all x, y in R

Definition: 6

An anti-fuzzy subset μ_A of a $\ell-$ring R is called a $T-$anti-fuzzy ideal, if the following conditions are satisfied,

(i) $\mu_A(x-y) \le T(\mu_A(x), \mu_A(y))$

(ii) $\mu_A(xy) \le \mu_A(x);\ \mu_A(xy) \le \mu_A(y)$

(iii) $\mu_A(x \vee y) \le T(\mu_A(x), \mu_A(y))$

(iv) $\mu_A(x \wedge y) \le T(\mu_A(x), \mu_A(y))$, for all $x, y \in R$

Example: 3

Now $(R=\{a, b, c\}, +, \cdot, \vee, \wedge)$ is a $\ell-$ring. The operations $+, \cdot, \vee$ and $\wedge$ defined by the following tables

Consider the anti-fuzzy subset μ_A of the $\ell-$ring R

$$\mu_A(x) = \begin{cases} 0.3 & if\ x=a \\ 0.5 & if\ x=b \\ 0.7 & if\ x=c \end{cases}$$

Then μ_A is a $T-$anti-fuzzy ideal of $\ell-$ring R

Definition: 7

Let R_1 and R_2 be two $\ell-$rings. Then the function $f : R_1 \to R_2$ is called a $\ell-$ring homomorphism if satisfies the following conditions

(i) $f(x+y) = f(x) + f(y)$

(ii) $f(xy) = f(x)\ f(y)$

(iii) $f(x \vee y) = f(x) \vee f(y)$

(iv) $f(x \wedge y) = f(x) \wedge f(y)$, for all x, y in R

Example: 4

Let $R = \{m + n\sqrt{2}\ \ for\,all\ m, n \in Z\}$

R is a $\ell-$ring under usual addition and multiplication

Define $f : R \to R$ by

$f\left(m+n\sqrt{2}\right)=m-n\sqrt{2}$ is $\ell-$ring homomorphism,

where Z is set of all integer

Definition: 8

Let R_1 and R_2 be two $\ell-$rings. A mapping $f:R_1\to R_2$ is called a $\ell-$ring isomorphism if

(i) f is one-to-one

(ii) f is onto , for all x,y in R

Definition: 9

An anti-fuzzy set μ_A of a $\ell-$ring R has the **inf** property if for any subset N of R, there exists a $a_0\in N$ such that $\mu_A\left(a_0\right) = \inf\limits_{a\in N}\mu_A\left(a\right)$

Definition: 10

Let M and N be any two sets and let $f:M\to N$ be any function. An anti-fuzzy subset μ_A of a M is called $f-$f-invariant if $f\left(x\right)= f\left(y\right)$implies $\mu_A\left(x\right)=\mu_A\left(y\right)$, for all $x,\ y\in M$

Definition: 11

Let R be a $\ell-$ring. Let μ_A be an anti-fuzzy set of a $T-$anti-fuzzy ideals of a $\ell-$ring R and f be a function defined on R, then the anti-fuzzy set μ_A^f in $f\left(R\right)$ is defined by $\mu_A^f\left(y\right) = \inf\limits_{n\in f^{-1}\left(y\right)}\mu_A\left(x\right)$, for all $y\in f\left(R\right)$and is called the image of μ under f

Definition: 12

Let R be a $\ell-$ring. Let μ_A be an anti-fuzzy set of a $T-$anti-fuzzy ideals of a $\ell-$ring R and f be a function defined on R, if v is a fuzzy set in $f\left(R\right)$, then

$\mu_A = v \circ f$ in R is defined by $\mu_A(x) = v(f(x))$, for all $x \in R$ and is called the pre-image of v under f

Theorem: 1

An onto homomorphism image of a $T-$anti-fuzzy ideal of a $\ell-$ring R with **inf** property is a $T-$anti-fuzzy ideal of a $\ell-$ring R.

Proof:

Let R and S be a $\ell-$rings

Let $f : R \to S$ be an epimorphism and μ_A be a $T-$ anti-fuzzy ideal of a $\ell-$ ring R with **inf** property.

Let $x, y \in S$

Let $x_0 \in f^{-1}(x)$, $y_0 \in f^{-1}(y)$ and $z_0 \in f^{-1}(z)$ be such that

$\mu_A(x_0) = \inf\limits_{n \in f^{-1}(x)} \mu_A(n)$,

$\mu_A(y_0) = \inf\limits_{n \in f^{-1}(y)} \mu_A(n)$ and

$\mu_A(z_0) = \inf\limits_{n \in f^{-1}(z)} \in \mu_A(n)$ respectively, then

We have

(i) $$\begin{aligned} \mu_A^f(x-y) &= \inf_{z \in f^{-1}(x-y)} \mu_A(z) \\ &\leq \mu_A(x_0 - y_0) \\ &\leq \max(\mu_A(x_0),\ \mu_A(y_0)) \\ &= T(\mu_A(x_0),\ \mu_A(y_0)) \\ &\leq T\left(\inf_{n \in f^{-1}(x)} \mu_A(n),\ \inf_{n \in f^{-1}(y)} \mu_A(n) \right) \\ &= T(\mu_A^f(x),\ \mu_A^f(y)) \end{aligned}$$

Therefore $\mu_A^f(x-y) \leq T(\mu_A^f(x),\ \mu_A^f(y))$, for all $x, y \in S$

(ii) Let μ_A be a $T-$fuzzy ideals of R and let $x, y \in R$

Since $\mu_A(xy) \leq \mu_A(x)$ and $\lambda_A(xy) \leq \lambda_A(y)$

Also we have

$$\begin{aligned}
\mu_A^f(xy) &= \inf_{z \in f^{-1}(xy)} \mu_A(z) \\
&\leq \mu_A(x_0 y_0) \\
&\leq \mu_A(x_0) \\
&\leq \inf_{n \in f^{-1}(x)} \mu_A(n) \\
&= \mu_A^f(x)
\end{aligned}$$

Therefore $\mu_A^f(xy) \leq \mu_A^f(x)$, for all $x, y \in S$

(iii)
$$\begin{aligned}
\mu_A^f(x \vee y) &= \inf_{z \in f^{-1}(x \vee y)} \mu_A(z) \\
&\leq \mu_A(x_0 \vee y_0) \\
&\leq \max\big(\mu_A(x_0),\ \mu_A(y_0)\big) \\
&= T\big(\mu_A(x_0),\ \mu_A(y_0)\big) \\
&\leq T\left(\inf_{n \in f^{-1}(x)} \mu_A(n),\ \inf_{n \in f^{-1}(y)} \mu_A(n) \right) \\
&= T\left(\mu_A^f(x),\ \mu_A^f(y) \right)
\end{aligned}$$

Therefore $\mu_A^f(x \vee y) \leq T\left(\mu_A^f(x),\ \mu_A^f(y) \right)$, for all $x, y \in S$

(iv)
$$\begin{aligned}
\mu_A^f(x \wedge y) &= \inf_{z \in f^{-1}(x \wedge y)} \mu_A(z) \\
&\leq \mu_A(x_0 \wedge y_0) \\
&\leq \max\big(\mu_A(x_0),\ \mu_A(y_0)\big) \\
&= T\big(\mu_A(x_0),\ \mu_A(y_0)\big)
\end{aligned}$$

$$\leq T\left(\inf_{n\in f^{-1}(x)} \mu_A(n), \quad \inf_{n\in f^{-1}(y)} \mu_A(n) \right)$$

$$= T\left(\mu_A^f(x), \ \mu_A^f(y) \right)$$

Therefore $\mu_A^f(x \wedge y) \leq T\left(\mu_A^f(x), \ \mu_A^f(y) \right)$, for all $x, y \in S$

Thus an onto homomorphic image of a $T-$anti-fuzzy ideal of a $\ell-$ ring R with **inf** property is a $T-$anti-fuzzy ideal of a $\ell-$ring R .

Theorem: 2

An epimorphic pre-image of a $T-$anti-fuzzy ideal of a $\ell-$ring is a $T-$anti-fuzzy ideal of a $\ell-$ring R.

Proof:

Let R and S be a $\ell-$rings

Let $f : R \to S$ be an epimorphism

Let v be a $T-$anti-fuzzy ideal of a $\ell-$ring S and μ_A be the pre-image of v under f for any $x, y, z \in R$.

(i) we have $\mu_A(x-y) = (v \circ f)(x-y)$

$$= v\left(f(x-y)\right)$$

$$= v\left(f(x) - f(y)\right)$$

$$\leq T\left(v\left(f(x)\right), \ v\left(f(y)\right)\right)$$

$$\leq T\left((v \circ f)(x), \ (v \circ f)(y)\right)$$

$$= T\left(\mu_A(x), \ \mu_A(y)\right)$$

Therefore $\mu_A(x-y) \leq T\left(\mu_A(x), \ \mu_A(y)\right)$, for all $x, y \in R$

(ii) Since $\mu_A(xy) \leq \mu_A(x)$ and $\lambda_A(xy) \leq \lambda_A(y)$

$$\mu_A(xy) = (v \circ f)(xy)$$

$$= v\left(f(xy)\right)$$

$$= v\left(f(x)f(y)\right)$$
$$\le T\left(v\left(f(x)\right)\right)$$
$$\le T\left((v\circ f)(x)\right)$$
$$= (v\circ f)(x)$$
$$= \mu_A(x)$$

Therefore $\mu_A(xy) \le \mu_A(x)$, for all $x, y\in R$

(iii) we have $\mu_A(x\vee y) = (v\circ f)(x\vee y)$

$$= v\left(f(x\vee y)\right)$$
$$= v\left(f(x)\vee f(y)\right)$$
$$\le T\left(v\left(f(x)\right),\ v\left(f(y)\right)\right)$$
$$\le T\left((v\circ f)(x),\ (v\circ f)(y)\right)$$
$$= T\left(\mu_A(x),\ \mu_A(y)\right)$$

Therefore $\mu_A(x\vee y) \le T\left(\mu_A(x),\ \mu_A(y)\right)$, for all $x, y\in R$

(iv) we have $\mu_A(x\wedge y) = (v\circ f)(x\wedge y)$

$$= v\left(f(x\wedge y)\right)$$
$$= v\left(f(x)\wedge f(y)\right)$$
$$\le T\left(v\left(f(x)\right),\ v\left(f(y)\right)\right)$$
$$\le S\ \ T\left((v\circ f)(x),\ (v\circ f)(y)\right)$$
$$= T\left(\mu_A(x),\ \mu_A(y)\right)$$

Therefore $\mu_A(x\wedge y) \le T\left(\mu_A(x),\ \mu_A(y)\right)$, for all $x, y\in R$

Thus an epimorphic pre-image of a $T-$anti-fuzzy ideal of a $\ell-$ ring is a $T-$anti-fuzzy ideal of a $\ell-$ring R.

Proposition: 1

Let R and S be $\ell-$rings R and let $f:R\to S$ be a homomorphism. Let μ_A be $f-$invariant anti-fuzzy ideal of a $\ell-$ring R. If $x=f(a)$, then $f(a)(x)=\mu_A(a)$, for all $a\in R$.

Theorem: 3

Let $f:R\to S$ be an epimorphism of $\ell-$rings R and S. If μ_A is $f-$invariant anti-fuzzy ideal of a $\ell-$ring R, then $f(\mu_A)$ is a $T-$anti-fuzzy ideal of S.

Proof:

Let $a,b,c\in S$

then there exists $x,y,z\in R$ such that $f(x)=a$, $f(y)=b$ and $f(z)=c$

Suppose μ_A is $f-$invariant anti-fuzzy ideal of a $\ell-$ring R,

then by Proposition 1

(i) we have $f(\mu_A)(a-b) = f(\mu_A)\big(f(x)-f(y)\big)$

$$= f(\mu_A)\big(f(x-y)\big)$$
$$= \mu_A(x-y)$$
$$\le T\big(\mu_A(x),\ \mu_A(y)\big)$$
$$= T\big(f(\mu_A)(a),\ f(\mu_A)(b)\big)$$

Therefore $f(\mu_A)(a-b) \le T\big(f(\mu_A)(a),\ f(\mu_A)(b)\big)$, for all $a,b\in S$ and $x,y\in R$

(ii) Since $\mu_A(xy) \le \mu_A(x)$ and $\lambda_A(xy) \le \lambda_A(y)$

We have $f(\mu_A)(ab) = f(\mu_A)\big(f(x)f(y)\big)$

$$= f(\mu_A)\big(f(xy)\big)$$
$$= \mu_A(xy)$$
$$\le \mu_A(x)$$
$$= f(\mu_A)(a)$$

Therefore $f(\mu_A)(ab) \leq f(\mu_A)(a)$, for all $a,b \in S$, $x,y \in R$

(iii) we have $f(\mu_A)(a \vee b) = f(\mu_A)(f(x) \vee f(y))$

$$= f(\mu_A)(f(x \vee y))$$

$$= \mu_A(x \vee y)$$

$$\leq T(\mu_A(x),\ \mu_A(y))$$

$$= T(f(\mu_A)(a),\ f(\mu_A)(b))$$

Therefore $f(\mu_A)(a \vee b) \leq T(f(\mu_A)(a),\ f(\mu_A)(b))$, for all $a,b \in S$ and $x,y \in R$

(iv) we have $f(\mu_A)(a \wedge b) = f(\mu_A)(f(x) \wedge f(y))$

$$= f(\mu_A)(f(x \wedge y))$$

$$= \mu_A(x \wedge y)$$

$$\leq T(\mu_A(x),\ \mu_A(y))$$

$$= T(f(\mu_A)(a),\ f(\mu_A)(b))$$

Therefore $f(\mu_A)(a \wedge b) \leq T(f(\mu_A)(a),\ f(\mu_A)(b))$, for all $a,b \in S$ and $x,y \in R$

Hence $f(\mu_A)$ is a $T-$anti-fuzzy ideal of a $\ell-$ring S.

Theorem: 4

Let $f: R_1 \to R_2$ be an onto homomorphism of a $\ell-$rings. If μ_A is $T-$anti-fuzzy ideal of R_1, then $f(\mu_A)$ is a $T-$anti-fuzzy ideal of R_2.

Proof:

Let μ_A be a $T-$anti-fuzzy ideal of a $\ell-$ring R_1

Let $\mu_1 = f^{-1}(y_1)$ and $\mu_2 = f^{-1}(y_2)$, where $y_1, y_2 \in R_2$ are non-empty subsets of R_2

Similarly $\mu_3 = f^{-1}(y_1 - y_2)$

Consider the set $\mu_1 - \mu_2 = \{a_1 - a_2 \ / \ a_1 \in \mu_1, \ a_2 \in \mu_2\}$

If $x \in \mu_1 - \mu_2$, then $x = x_1 - x_2$, for some $x_1 \in \mu_1, \ x_2 \in \mu_2$ and so,

$$f(x) = f(x_1 - x_2)$$

$$= f(x_1) - f(x_2)$$

$$= y_1 - y_2$$

$$\Rightarrow \qquad x \in f^{-1}(y_1 - y_2) = \mu_3$$

Thus $\mu_1 - \mu_2 \subseteq \mu_3$

that is $\{x \ / \ x \in f^{-1}(y_1 - y_2)\} \supseteq \{x_1 - x_2 \ / \ x_1 \in f^{-1}(y_1), \ x_2 \in f^{-1}(y_2)\}$

Let $y_3 \in R_2$, then

(i) we have $f(\mu_A)(y_1 - y_2) = \inf\{\mu_A(x) \ / \ x \in f^{-1}(y_1 - y_2)\}$

$$\leq \inf\{\mu_A(x_1 - x_2) \ / \ x_1 \in f^{-1}(y_1), \ x_2 \in f^{-1}(y_2)\}$$

$$\leq \inf\{\max\{\mu_A(x_1), \ \mu_A(x_2)\} \ / \ x_1 \in f^{-1}(y_1), \ x_2 \in f^{-1}(y_2)\}$$

$$\leq \inf\{T(\mu_A(x_1), \ \mu_A(x_2)) \ / \ x_1 \in f^{-1}(y_1), \ x_2 \in f^{-1}(y_2)\}$$

$$= T(\inf\{\mu_A(x_1)\} \ / \ x_1 \in f^{-1}(y_1), \ \inf\{\mu_A(x_2)\} \ / \ x_2 \in f^{-1}(y_2))$$

$$= T(f(\mu_A)(y_1), \ f(\mu_A)(y_2))$$

Therefore $f(\mu_A)(y_1 - y_2) \leq T(f(\mu_A)(y_1), \ f(\mu_A)(y_2))$, for all $y_1, y_2 \in R_2$

(ii) We have $\mu_A(xy) \leq \mu_A(x)$ and $\lambda_A(xy) \leq \lambda_A(y)$

We have $f(\mu_A)(x_1 x_2) = \inf\{\mu_A(y) \ / \ y \in f^{-1}(x_1 x_2)\}$

$$\leq \inf\{\mu_A(x_1 x_2) \ / \ y_1 \in f^{-1}(x_1), \ y_2 \in f^{-1}(x_2)\}$$

$$\leq \inf\{\mu_A(x_2) \ / \ y_2 \in f^{-1}(x_2)\}$$

$$= f(\mu_A)(x_2)$$

Therefore $\quad f(\mu_A)(x_1 x_2) \le f(\mu_A)(x_2)$, for all $y_1, y_2 \in R_2$ and $\alpha \in \Gamma$

(iii) we have $f(\mu_A)(y_1 \vee y_2) = \inf\{\mu_A(x) \,/\, x \in f^{-1}(y_1 \vee y_2)\}$

$$\le \inf\{\mu_A(x_1 \vee x_2) \,/\, x_1 \in f^{-1}(y_1),\ x_2 \in f^{-1}(y_2)\}$$

$$\le \inf\{\max\{\mu_A(x_1),\ \mu_B(x_2)\} \,/\, x_1 \in f^{-1}(y_1),\ x_2 \in f^{-1}(y_2)\}$$

$$\le \inf\{T(\mu_A(x_1),\ \mu_A(x_2)) \,/\, x_1 \in f^{-1}(y_1),\ x_2 \in f^{-1}(y_2)\}$$

$$= T(\inf\{\mu_A(x_1)\} \,/\, x_1 \in f^{-1}(y_1),\ \inf\{\mu_A(x_2)\} \,/\, x_2 \in f^{-1}(y_2))$$

$$= T(f(\mu_A)(y_1),\ f(\mu_A)(y_2))$$

Therefore $\quad f(\mu_A)(y_1 \vee y_2) \le T(f(\mu_A)(y_1),\ f(\mu_A)(y_2))$, for all $y_1, y_2 \in R_2$

(iv) we have $f(\mu_A)(y_1 \wedge y_2) = \inf\{\mu_A(x) \,/\, x \in f^{-1}(y_1 \wedge y_2)\}$

$$\le \inf\{\mu_A(x_1 \wedge x_2) \,/\, x_1 \in f^{-1}(y_1),\ x_2 \in f^{-1}(y_2)\}$$

$$\le \inf\{\max\{\mu_A(x_1),\ \mu_A(x_2)\} \,/\, x_1 \in f^{-1}(y_1),\ x_2 \in f^{-1}(y_2)\}$$

$$\le \inf\{T(\mu_A(x_1),\ \mu_A(x_2)) \,/\, x_1 \in f^{-1}(y_1),\ x_2 \in f^{-1}(y_2)\}$$

$$= T(\inf\{\mu_A(x_1)\} \,/\, x_1 \in f^{-1}(y_1),\ \inf\{\mu_A(x_2)\} \,/\, x_2 \in f^{-1}(y_2))$$

$$= T(f(\mu_A)(y_1),\ f(\mu_A)(y_2))$$

Therefore $\quad f(\mu_A)(y_1 \wedge y_2) \le T(f(\mu_A)(y_1),\ f(\mu_A)(y_2))$, for all $y_1, y_2 \in R_2$

Hence $f(\mu_A)$ is a $T-$anti-fuzzy ideal of a $\ell -$ ring R_2.

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