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Reliability of Computer Systems with Minimal Repairs

Dr. R. Chinnaiyan

Professor, Department of Computer Applications,
New Horizon College of Engineering, Bangalore

Dr. V. Ilango

Professor, Department of Computer Applications,
New Horizon College of Engineering, Bangalore

Abstract

This paper discusses the reliability of computer systems and its performance. The computer system's performance policy undergoes minimal repairs on failures between replacements is performed with respect to Preventative Maintenance. The Minimal repairs follow Non-Homogeneous Poisson Process. The Operational characteristics and Cost Benefit analysis are obtained. The inspection policy minimizing the expected cost per unit of time for an infinite time span is also discussed

Keywords: Computer Systems, Non-Homogeneous Poisson Process, Minimal Repairs, Preventative Maintenance, Reliability

1. Introduction

Computer systems can undergo minor failures as well as catastrophic ones. When the former take place in a particular computer system, it returns to the right operating state with slight repairs and at low costs, even if the failure cause the mechanism to stop working. Consider for instance the case of a computer that presents some difficulties to work due to lack of connection between hardware devices, or a battery that do not properly supplies power via SMPS. The problem is easily solved after testing the power connections or the battery level. The catastrophic failures are those that cause the computer system to stop working properly but major repairs and high costs are required. Often, a replacement of the whole hardware unit or a perfect repair that restore the computer system to an as-good as-new condition is to be carried out. For example, failures due to viruses that make the user to install a new hard disk and, in general, unavoidable over-loads, warming environments, etc may have serious consequences.

As many users of modern technology know, complex computer systems subject to both types of failures are very common to find in practice and maintenance policies should deal with them. This is the case of computer systems that can be affected by inoffensive spy programmes as well as by dangerous viruses.

1. 1 Reliability Model for Computer Systems

A reliability model for computer systems represents a clear picture of the computer's functional interdependencies providing a means to trade-off design alternatives and to identify areas for design improvement of a computer. The reliability models are also helpful in:

(i). Identifying of critical items and single points of failure of a computer
(ii). Allocating reliability goals to portions of the design of a computer
(iii). Providing a framework for comparing estimated reliability of a computer
(iv). Trading-off alternative fault tolerance approaches for a computer

Reliability models are derived from, and traceable to, functional requirements of computer system. They represent the required modes of operation, the duty cycles, and are consistent with a specified definition of what constitutes a computer system failure. There has been continuing interest in the policies for computer systems that are subject to stochastic failures, as the uncalled for failures may prove to be costly and dangerous. Barlow and Prochan [2] first considered a policy called an age replacement policy in which system is replaced at age t or at the time of failure whichever occurs first. But for the complex and expensive systems, it is not advisable to replace the entire system just because of the failure of one component The minimal repair model introduced by Barlow and Hunter [3] has been extended in later works that propose maintenance policies according to the state of the system. Block *et al* [4] present an interesting survey concerning maintenance policies with time dependent costs and probabilities. The text due to Ascher and Feingold [1] constitutes a general framework for repairable systems and deals, in particular, with the minimal repair model and the underlying theory on non-homogeneous Poisson processes. In this paper a policy for the computer system is considered that undergoes minimal repairs on failures between replacements. Minimal repairs follow Non-Homogeneous Poisson Process (NHPP).

2. Model Assumptions

(a) Computer System is replaced at the time of Preventive Maintenance (PM) and it undergoes minimal repairs on failures between replacements
(b) Hazard rate of the Computer System is continuous, increasing and is not disturbed by minimal repairs
(c) All maintenance events time are negligible
(d) All failure events are (statistically independent) s-independent
(e) Planning time horizon is infinite

2. 1 Notations

$F(t)$	:	the failure time cumulative distribution function (cdf) of the computer system with probability density function (pdf)
$H(t)$	:	recurrence time cdf from state 0
$q(t)$	:	hazard rate of the random life of the computer system
$Q(t)$	:	cumulative hazard rate
$A(t)$	:	cdf time to PM defined as: $A(t) = 1, \text{ if } t \geq T\ 0, \text{ if } t < T$
$Q_{ij}(t)$	:	the probability that after transiting from state into the state j, the in unit amount of time ≤ t.
$q_{ij}(t)$	:	pdf of $Q_{ij}(t)$
$M_{ij}(t)$	:	the expected number of visits to state j during the interval (0, t], given that the computer system starts from the state i.
*	:	the Laplace Stieltjes convolution $A(t) * B(t) = {}_0\int^t A(u)\, dB(t-u)$
$f(s)$	=	$LS[f(t)] = {}_0\int^\infty e^{-st}\, dF(t)$
$M(t)$	**:**	the expected number of failures during the time interval [0, T)
M	:	the expected number of failures per unit time in the steady state
M_j	:	the expected number of visits per unit time from the state 0 to the state j in the steady state

3. Derivation of *Qij (t)*

The following transition probabilities have been obtained from makov renewal processes MRP $Q_{01}(t) = {}_0\int^1 \sum P_j(x)\, d A(x) = A(t)$
With $Q_{10}(t) = 1$

Where $P_j(x) = [Q_j(x)] / j!\ e^{-Q(x)}$, $j = 0, 1, 2..$, denote the probability of j failures in the interval [0, x].

4. Expected Number of Failures in the steady state

By renewal theoretic arguments, we have

$$M(t) = \sum_{J=0}^{\infty} j\, p_j(t)\, A(t) + \sum_{J=0}^{\infty} j\, P_j(x)\, d A(x) + Q_{01}(t) * Q_{10}(t) * M(t)$$

Taking LST and simplifying gives way

$$m(s) = {}_0\int^T e^{-st}\, dQ(t) / (1 - q_{01}(s) q_{10}(s))$$

Thus, the expected number of failures per unit amount of time in the steady state is given by,

$$M = \lim_{S \to 0} s\, m(s)$$

$$= Q(T) / T$$

5. Expected Number of Visits From State 0 To State 1
Similarly, by renewal theoretic arguments, we have
$M_{01}(t) = Q_{01}(t) * Q_{10}(t) * [1+ M_{01}(t)]+Q_{01}(t) * (1-Q_{10}(t))$

Taking LST and simplifying gives
$m_{01}(s) = e^{-sT} /(1-e^{-sT})$

Thus, the expected number of visits per unit amount of time from state 0 to state 1 in the steady state is given by,
$M_1 = \lim_{S\to 0} s\, m_{01}(s)$

$= 1 / T$

Thus, the expected cost rate is the steady state (infinite time horizon) is given by
$C (T) = (C_1 Q (T) +C_2) /T$

Where, C1 denotes the cost for each minimal repair and C2 (>C1) denotes the cost of replacement due to PM. This is the classical Minimal Repair Policy introduced by Barlow and Hunter[3]. To optimize the cost function C(T), consider
$C (T) = (C_1 Q (T) +C_2) / T$

Differentiating with respect to T, we get,
$C^1(T) = TC_1q (T) – [C_1Q (T) +C_2] / T^2$

For the minimum value of C(T), $C_1(T) = 0$, This gives
$C_1[Tq (T)-Q (T)] – C_2 = 0$
Or
$\int_0^T [q (T) – q (t)]\, dt = C_2 / C_1$
Let $L (T) = \int_0^T [q(T) – q(t)]\, dt$. Therefore
$L (0) = 0< C_2 / C_1$

Since, q(T) is an increasing function of T, one can easily see that L′ (T) >0, which implies that L(T) is an increasing function of T and
$L(\infty) = \int_0^{\infty} [q(\infty) – q(t)]dt > C_2 / C_1$

Thus there exists a unique T which minimizes C(T) such that
$Tq(T) – Q(T) = C_2 / C_1$

One may note here that
$C'(T) \le 0$ {if $0 < T \le T'$}
>0 {if $T' < T < \infty$}

6. Conclusion
The model discussed in this paper depicts a clear picture of the computer system’s functional interdependencies providing a means to trade-off design alternatives and to

identify areas for design improvement of the computer system. The models are also supportive in identifying the critical items and single points of failure, allocating reliability goals to portions of the design of the computer, providing a framework for comparing the estimated reliability for computer system goals and trading-off alternative fault tolerance approaches.

7. References

[1] Ascher. H, Feingold. H, Repairable Systems Reliability, Marcel Dekker, 1984

[2] Barlow, R. E., F. Proschan Mathematical Theory of Reliability. John Wiley and Sons, New York 1965.

[3] Barlow R. E and Hunter. L. C, Optimum preventive maintenance policies, *Operations Research,* **8**, (1960), 90-100.

[4] H. W. Block, W. S. Borges and T. H. Savits, Preventive Maintenance Policies, *Reliability and Quality Control*, A. P. Basu (Editor) Elsevier (1986), 101-106.

[5] J. D. Musa, A. Iannino, and K. Okumoto, Software Reliability, Measurement, Prediction and Application. McGraw-Hill, 1987.

[6] J. D. Musa, A. Iannino, and K. Okumoto, Software Reliability, Measurement, Prediction and Application. McGraw-Hill, 1987.

[7] R. Chillarege, I. S. Bhandari, J. K. Chaar, M. J. Halliday, D. S. Moebus, B. K. Ray, and M.-Y. Wong, Orthogonal Defect Classification—A Concept for In-Process Measurements, IEEE Transactions on Software Engineering SE-18(11), 94–956 (November 1992).

[8] R. H. Hou, S. Y. Kuo, and Y. P. Chang, "On a Unified Theory of Some Nonhomogenous Poisson Process Models for Software Reliability, " Proc. 1998 Int'l Conf. Software Eng., Education &Practice (SEEP '98), pp. 60-67, Jan. 1998.

[9] M. R. Lyu, Handbook of Software Reliability Engineering, McGraw-Hill IEEE Computer Society Press, New York, 1996.

[10] R. Chinnaiyan, S. Somasundaram, Evaluating the reliability of component-based software systems, International Journal of Quality &Reliability Management 27 (1) (2010) 78–88.

[11] R. Chinnaiyan & S.Somasundaram, "Reliability Estimation Model for Software Components Using CEP" International Journal of Mechanical and Industrial Engineering (IJMIE), , Volume-2, Issue-2, 2012.