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Design and Development of New Parametric Wavelet for Image Denoising

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Abstract

A lot of research work in the image processing has been carried out in the last few years using wavelets. But image denoising has remained a fundamental problem in the field of image processing. The problem of estimating an image that is corrupted by Additive White Gaussian Noise has been of interest for practical and theoretical reasons. Earlier several wavelet systems were designed for achieving good results in denoising the images using various non-linear thresholding techniques in wavelet domain such as hard and soft thresholding , wavelet shrinkages such as Visu shrink, Sure shrink, Bayes shrink etc. The main aim of this paper is to construct a new wavelet structure which could provide better results than compared to already available scalar wavelet [7, 9, 10, 11].

Keywords: Wavelets, Parameterization , Subdivision scheme, Wave manager, Denoising, Thresholding, Wavelet Shrinkage, Orthogonality, Symmetry.

Introduction

Image denoising is used to produce good estimates of original image from noisy observations. Wavelet Transform due to its excellent localization property, has rapidly became an indispensable signal and image processing tool for a variety of applications, including compression and denoising .Wavelet denoising attempts to remove the noise present in signal or image while preserving signal characteristics, regardless of frequency content making use of wavelet thresholding (first proposed by Donoho). In spite of many methods available for the design of new wavelet, we have here concentrated on 'Parametric Wavelet Design through Orthogonality condition and Parameterization'. Then we have developed appropriate coding to test the performance of this new wavelet system and compare it with available wavelet system.

In section 2, we have presented parameterization method to derive the LPF coefficients and HPF coefficients. In section 3, we presented the subdivision scheme to visualize the new wavelet shape and procedure for adding new wavelet to wavelet toolbox using Wave manager "wavemngr" command. In section 4, comparison between new wavelet system and available wavelet system for different transforms is made and finally the conclusion and Future Scope is drawn.

Parametric Wavelet Design through Orthogonality condition and parameterization : [1, 4, 8]

The Orthogonal wavelet system must satisfy certain conditions like,

Normality

$$\int \boldsymbol{\Phi}(\mathbf{t})\mathbf{dt} = \mathbf{1} \rightarrow \sum_{\mathbf{k}} \mathbf{h}(\mathbf{k}) = \sqrt{\mathbf{2}} \quad (1)$$

Orthogonality of integer translates of Φ(t)

$$\int \boldsymbol{\Phi}(\mathbf{t})\, \boldsymbol{\Phi}(\mathbf{t}-\mathbf{k})\, \mathbf{dt} = \boldsymbol{\delta}\mathbf{k},\mathbf{0} \rightarrow \sum_{k} \mathbf{h}(\mathbf{k})\, \mathbf{h}(\mathbf{k}-\mathbf{2n}) = \boldsymbol{\delta}\mathbf{n},\mathbf{0} \quad (2)$$

Where n = 0, 1, 2, ... (N/2) - 1

For N=4 and n=1, h(0) h(2) + h(1) h(3) = 0

Orthogonality of Φ(t) & Ψ (t)

$$\int \boldsymbol{\Phi}(\mathbf{t})\, \boldsymbol{\Psi}\,(\mathbf{t}-\mathbf{k})\mathbf{dt} = \mathbf{0} \rightarrow \mathbf{g}(\mathbf{n}) = (-\mathbf{1})^{\,\boldsymbol{n}}\, \mathbf{h}(\mathbf{N}-\mathbf{1}-\mathbf{n}) \quad (3)$$

First vanishing moment of Ψ (t)

$$\int \boldsymbol{\Psi}\,(\mathbf{t})\, \mathbf{dt} = \mathbf{0} \rightarrow \sum_{\mathbf{n}} \mathbf{g}(\mathbf{n}) = \mathbf{0} \rightarrow \sum \sum_{\mathbf{n}} (-\mathbf{1})^{\mathbf{n}}\, \mathbf{h}(\mathbf{N}-\mathbf{1}-\mathbf{n}) = \mathbf{0}$$

$$\sum_{n-even} \boldsymbol{h}(\boldsymbol{n}) = \sum_{n-odd} \boldsymbol{h}(\boldsymbol{n}) \quad (4)$$

$$\sum_{n} \boldsymbol{g}(\boldsymbol{n}) = \mathbf{0} \rightarrow \mathbf{h}(\mathbf{0}) + \mathbf{h}(\mathbf{2}) = \mathbf{h}(\mathbf{1}) + \mathbf{h}(\mathbf{3}) \quad (5)$$

From Eq.(1) &Eq. (5), for 4-tap wavelet system, we have,

$$h(0) + h(2) = h(1) + h(3) = 1/\sqrt{2}$$

In general,

$$\sum_{n-even} \boldsymbol{h}(\boldsymbol{n}) = \sum_{n-odd} \boldsymbol{h}(\boldsymbol{n}) = \mathbf{1}/\sqrt{\mathbf{2}} \quad (6)$$

From Eq. (6)

$$[\sum_{n-even} \boldsymbol{h(n)}]^2 + [\sum_{n-odd} \boldsymbol{h(n)}]^2 = \mathbf{1/2} + \mathbf{1/2} = \mathbf{1} \tag{7}$$

From above fact, for a 4-tap wavelet system
we set,

$$h(0) + h(2) = \text{Cos}\,(\Theta_1 + \Theta_2)$$

and

$$h(1) + h(3) = \text{Sin}\,(\Theta_1 + \Theta_2)$$

where

$$\Theta_1 + \Theta_2 = \Pi/4$$
$$h(0) + h(2) = \text{Cos}\,(\Theta_1 + \Theta_2) = \text{Cos}\Theta_1\,\text{Cos}\Theta_2 - \text{Sin}\Theta_1\,\text{Sin}\Theta_2$$
$$h(1) + h(3) = \text{Sin}\,(\Theta_1 + \Theta_2) = \text{Sin}\Theta_1\,\text{Cos}\Theta_2 + \text{Cos}\Theta_1\,\text{Sin}\Theta_2$$

which implies,

$$\left.\begin{aligned} h(0) &= \text{Cos}\Theta_1\,\text{Cos}\Theta_2 \\ h(2) &= -\,\text{Sin}\Theta_1\,\text{Sin}\Theta_2 \\ h(1) &= \text{Sin}\Theta_1\,\text{Cos}\Theta_2 \\ h(3) &= \text{Cos}\Theta_1\,\text{Sin}\Theta_2 \end{aligned}\right\} \tag{8}$$

This approach specifies that, by choosing a value of Θ_1 randomly

$$\Theta_2 = (\Pi/4) - \Theta_1$$

Generalization of idea,

$$\sum_{n-even} \boldsymbol{h(n)} = \textbf{Cos}\,(\boldsymbol{\Theta_1} + \boldsymbol{\Theta_2} + \ldots\ldots\ldots\ldots\ldots + \boldsymbol{\Theta_{N/2}}) = \mathbf{1/\sqrt{2}} \tag{9}$$
$$\sum_{n-odd} \boldsymbol{h(n)} = \textbf{Cos}\,(\boldsymbol{\Theta_1} + \boldsymbol{\Theta_2} + \ldots\ldots\ldots\ldots\ldots + \boldsymbol{\Theta_{N/2}}) = \mathbf{1/\sqrt{2}} \tag{10}$$

and

$$\sum_{i=1}^{N/2} \boldsymbol{\Theta i} = \Pi/\mathbf{4} \tag{11}$$

From above concept of finding scaling function coefficient, we came to know that for various Θ_1 and Θ_2 values in Eq.(8) such that they satisfy Eq. (11), we can obtain a set of scaling function coefficients.

We have gone for various sets of scaling function coefficients for 4-tap system satisfying the above discussed conditions. Out of various set of scaling function coefficients, we have found, that the results obtained for scaling function coefficients for $\Theta_1 = \Pi/2$, $\Theta_2 = -\Pi/4$ .The set of scaling function coefficients obtained are quite good for image denoising application.Values for h_0, h_1, h_2, h_3 (scaling function coefficients) are 4.3298e-017 ≈ 0, 0.7071, 0.7071, -4.3298e-017 ≈ 0 respectively.

Plotting the new Scaling and Wavelet functions [1]

Once the set of new scaling and wavelet function coefficients (easily obtained by reversing the scaling function coefficients and assigning negative unity value at the alternate position) are obtained, then by using various methods, we can plot the shape of our new wavelet system. Here we make use of Subdivision Scheme to obtain scaling and wavelet functions.

Subdivision Scheme

Here we generate Φ(t) & Ψ(t) from the LPF coefficients and HPF coefficients obtained from above mentioned procedure. We assume a pulse of unit strength is given to system as input. After upsampling, convolution process is carried out to desired number of times. The final output is equally spaced samples of Φ(t), in range [0, N-1] for N taps. Multiplication factor √2 can be absorbed in filter itself.

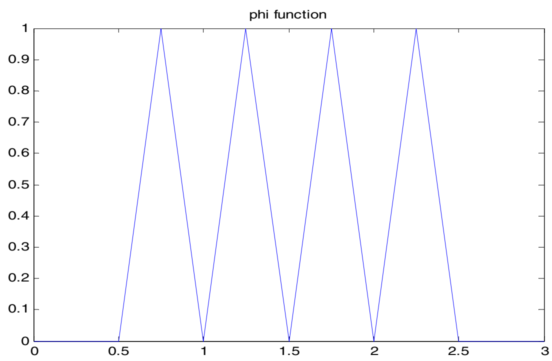


Figure 2(a): Scaling function.

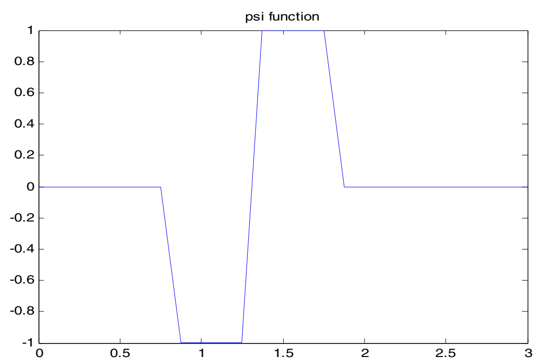


Figure 2(b): Wavelet function.

Adding the obtained Wavelet to the MATLAB Using the Wave manager "wavemngr" function [2][5]

By using the wavemngr function, we can add new wavelets to the existing ones to implement our favorite wavelet or try out one of our own design. The wavemngr command permits us to add new wavelets and wavelet families to the predefined ones. However, before we can use the wavemngr command to add a new wavelet, we must

Choose the Wavelet Family Full Name

The full name of the wavelet family, fn, must be a string. Predefined wavelet family names are Haar, Daubechies, Symlets, Coiflets, Gaussian, Mexican_hat, Morlet etc.

Choose the Wavelet Family Short Name
The short name of the wavelet family, fsn, must be a string of four characters or less. Predefined wavelet family short names are haar, db, sym, coif, gaus, mexh, morl, etc.

Determine the Wavelet Type: We distinguish five types of wavelets:
i) Orthogonal wavelets with FIR filters:
These wavelets can be defined through the scaling filter w. Predefined families of such wavelets include Haar, Daubechies, Coiflets etc.

ii) Biorthogonal wavelets with FIR filters :
These wavelets can be defined through the two scaling filters wr and wd, for reconstruction and decomposition respectively.

iii) Orthogonal wavelets without FIR filter, but with scale function:
These wavelets can be defined through the definition of the wavelet function and the scaling function. The Meyer wavelet family is a predefined family of this type.

iv) Wavelets without FIR filter and without scale function:
These wavelets can be defined through the definition of the wavelet function. Examples include Morlet and Mexican_hat.

v) Complex wavelets without FIR filter and without scale function:
These wavelets can be defined through the definition of the wavelet function. Examples include Complex Gaussian and Shannon. The new wavelet system designed here comes under wavelet type 1.

Define the Orders of Wavelets Within the Given Family :
If a family contains many wavelets, the short name and the order are appended to form the wavelet name. Argument nums is a string containing the orders separated with blanks. This argument is not used for wavelet families that only have a single wavelet (Haar, Meyer, and Morlet for example). For example, for the first Daubechies wavelets, fsn = 'db', nums = '1 2 3' , yield the three wavelets db1, db2, and db3.

Build a MAT-File or M-File
The wavemngr command requires a file argument, which is a string containing a MAT-file or M-file name. If a family contains many wavelets, an M-file must be defined and must be of a specific form that depends on the wavelet type.If a family contains a single wavelet, then a MAT-file can be defined for wavelets of type 1. Examples of such M-files for predefined wavelets are dbwavf.m for Daubechies, coifwavf.m for coiflets, and symwavf.m for symlets. Here we have made use of any type 1 wavelet family like dbwavf.m available at
C:\programfiles\MATLAB\R2007a\Toolbox\wavelet\wavelet:

Just replace values of coefficients for 4-tap in db family with our new scaling coefficients.

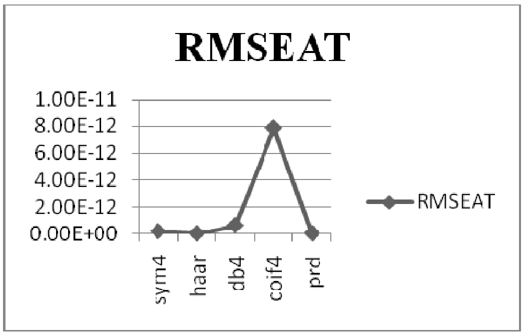
For our new wavelet family the syntax is , wavemngr('add','proposed','prd',1,' 2 ','prdwavf');

To see whether our family is added or not we can give the command as wavemngr ('read') in the command window. Once the new wavelet is added to the existing ones then ,we need to add the new set of scaling function coefficients in prdwavf.m file. Here we have created prdwavf.m by utilizing the dbwavf.m or lemwavf.m. After we create our own prdwavf file than we can use our own wavelet (prd) any where in the program for image denoising using many built in MATLAB function.

Comparison of performance of new wavelet system with other wavelet systems. [7]

Table 1: SWT ,Global Soft Threshold Level.

S.no	Wavelet type	RMSEAT	SNRAT
1	sym4	1.63E-13	255.7509
2	haar	8.52E-16	301.3885
3	db4	6.15E-13	244.2208
4	coif4	7.88E-12	222.0662
5	prd	8.40E-16	301.5133



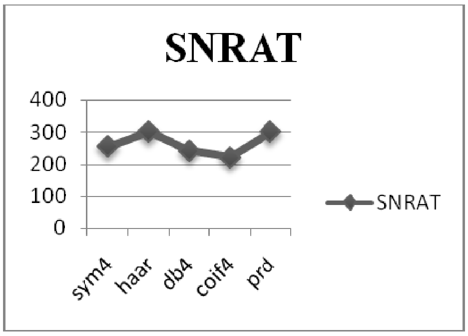
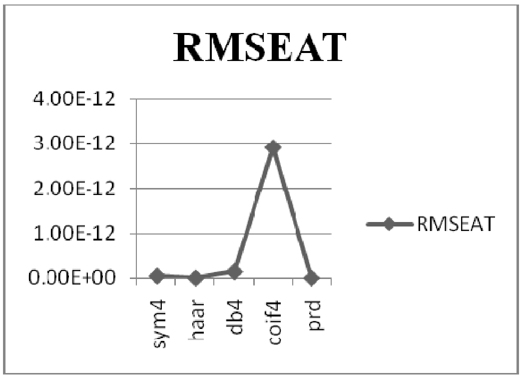


Figure 3: Grahical representation of SWT ,Global Soft Threshold Level.

Table 2: DWT, Global Soft Threshold Level 1.

S.no	Wavelet type	RMSEAT	SNRAT
1	sym4	4.48E-14	266.9819
2	haar	2.87E-16	310.8327
3	db4	1.39E-13	257.1591
4	coif4	2.91E-12	230.7168
5	prd	2.80E-16	311.0434



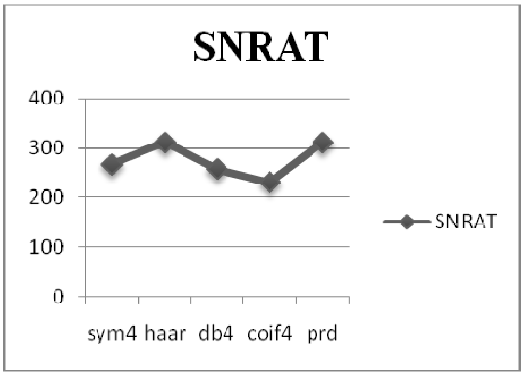
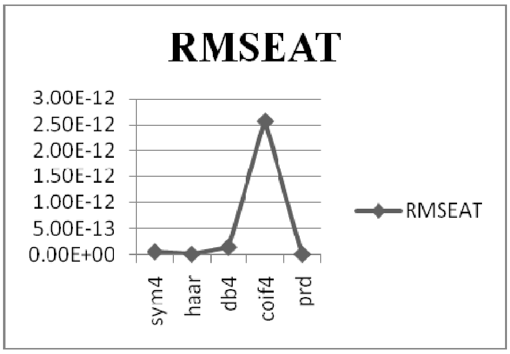


Figure 4: Grahical representation of DWT , Global Soft Threshold Level 1.

Table 3: WPT, Global Soft Threshold Level 1.

s.no	Wavelet type	RMSEAT	SNRAT
1	sym4	4.72E-14	266.516
2	haar	2.83E-16	310.9741
3	db4	1.33E-13	257.5237
4	coif4	2.57E-12	231.8053
5	prd	2.85E-16	310.9003



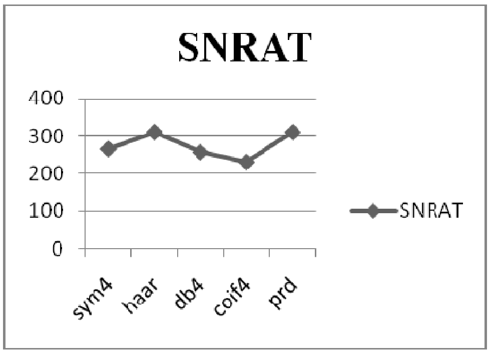


Figure 5: Grahical representation of WPT ,Global Soft Threshold Level 1.

Table 4: performance of proposed wavelet using various transforms.

S.no	Wavelet Transform	RMSEAT	SNRAT
1	SWT	8.40E-16	301.5133
2	DWT	2.80E-16	311.0434
3	WPT	2.85E-16	310.9003

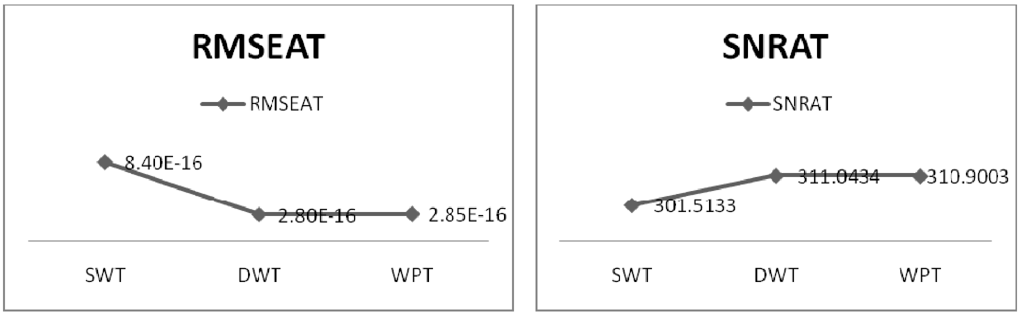


Figure 5: Grahical representation of WPT, Global Soft Threshold Level 1 performance of proposed wavelet for various transforms.

The Table 1 through Table 3 shows the result for Stationary Wavelet Transform, Discrete Wavelet Transform and Wavelet Packet Transform using various types of wavelets for global soft thresholding including newly proposed wavelet named as “prd”. [12, 13, 14,15,16]

Conclusion
The parametric wavelet system obtained above has some of the important properties like Orthogonality, Symmetry and Short support. In this paper we have concentrated on construction of a new wavelet system which was then utilised for image denoising process and the results obtained using the new wavelet system were better than compared to some already existing scalar wavelets and quite comparable to some pre existing wavelet system as shown above in the comparison tables 1 through 3 for SWT, DWT and WPT. Table 4 shows the comparison of SWT, DWT and WPT with Haar, Daubechies, Symlet, Coiflet, and Proposed(prd) wavelets. For various transformations the result obtained with proposed new wavelet system named as ‘prd’ are better with significant increase in SNR after thresholding and significant decrease in RMSE value, after thresholding than compared to the many members of Daubachies, Coiflet, Symlet, Haar.

Future Scope
None the less there is always room for improvement. Since wavelets are relatively a new subject of study (compared to FT, STFT), only a few construction methods for wavelets are available. Future construction methods may add even higher order of approximation while preserving the desirable features of the image. Moreover, there is a possibility that in future many more wavelet systems might be developed with coefficients, which could provide even better results in the field of image processing.

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