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Estimation of Pulse Repetition Frequency for Ionospheric Communication

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Abstract

Interest in the study of propagation of pulsed signals through ionosphere leads to investigate the pulse repetition frequency (PRF) for high frequency (HF) radio link. Properties of propagating signal through ionosphere and the ionosphere itself, which is a dispersive medium, play a vital role on the performance of various existing and under-developed communication systems. Very long distance communication can be achieved by propagation of high frequency waves through ionosphere. But this channel imposes some severe restrictions on signal communication. For this reason, it becomes very difficult to achieve high PRF for communication through ionosphere. This paper intends to estimate theoretically the PRF for ionospheric communication. Applying ray theory approximation, the expressions for received signal after reflection from ionosphere have been derived. Those expressions are then applied to find out the shape of trains of pulses considering parabolic electron density profile. The results indicate that signal overlapping increases with increase in frequency but decreases with increase in angle of incidence. These results are analyzed to estimate the frequency of operation for transmitted signal which shows that, to obtain a high pulse repetition frequency, threshold level at the receiver performs a crucial role. Higher the angle of incidence, higher will be the frequency of operation. A comparison with other ionospheric electron density profile explains the reason to assume parabolic profile for the study.

Keywords: Electromagnetic waves, Ionosphere, Radio wave propagation.

Introduction
There has been increasing interest in the study of propagation of pulsed signals through ionosphere, which is considered to be a dispersive medium. The performance of various existing and under-developed communication systems depends on the properties of propagating signal and the ionosphere. Liu, Wernik and Yeh [4] computed the signal intensities of a pulse train after reflection from ionosphere. Yeh [5] and Xu [6] searched for the mean arrival time and mean pulse width of a signal propagating through dispersive medium. But in the above papers [4] – [6], no work has been found regarding the maximum rate at which signals can be transmitted for faithful reception after reflection from ionosphere. Perrine and others [7] have tried to present a high data rate communication system through the ionospheric channel, but theoretical analysis and predictions in this respect were not given in their work. Hence, this interest can be extended to search for high pulse repetition frequency (PRF) for communication through ionosphere.

High frequency waves, when propagated through ionosphere, can achieve very long distance transmissions. But this multimode channel strongly degrades transmissions mainly by fading and frequency selectivity. For this reason, it becomes very difficult to achieve high PRF for communication through ionosphere. In this paper, we investigated mathematically, the overlapping of consecutive pulses at reception when a pulse train is being transmitted through ionosphere. This overlapping leads to inter symbol interference (ISI) which imposes restriction to high PRF for communication. We here considered a simple propagation problem through the ionosphere, which is characterized by its electron density profile. The result indicates that, assuming a parabolic electron density profile, we can attain a pulse frequency of hundreds of KHz and can achieve long transmission distance with low acceptable overlapping.

In section 2, a brief overview of the theoretical formulation of signal propagation through the ionosphere has been given. The detail mathematical derivations have been given in [1] & [2]. Considering parabolic electron density profile, the shape of the received pulse train has been discussed in Section 3. A comparative study of frequency of operation for faithful reception of pulse train has been discussed in Section 4 with three different ionospheric electron density profiles. Conclusions are given in Section 5.

Theoretical Formulations
For the theoretical formulation, we consider a simple communication problem, i.e. we consider a signal $m(t)$ transmitted obliquely from the transmitter located at an earth station. The signal after reflection from the ionosphere comes back to earth and is received by a receiver located on earth at a distance D from the transmitter. D is known as the transmission distance. To simplify our calculation we make following assumptions:

a. The coordinate system is considered to be Cartesian with coordinates x & y horizontal and z vertically upward with direction cosines I_x, I_y and I_z respectively.

b. The electron concentration N varies vertically upward.
c. The incident wave is a plane wave traveling obliquely upward.
d. The transmitter location is considered as origin, i.e. $(x, y, z) = (0,0,0)$.
e. Ray treatment has been used for formulation.

The detail calculations were given in articles [1] & [2]. Here we restrict ourselves mainly on the solutions found in the above articles.

For isotropic medium, the expression for phase path is:

$$P = I_x x + I_y y + \int_0^z q dz \tag{1}$$

where, q is the booker quartic.

If the signal is incident obliquely making an angle θ with the vertical, we can write the booker quartic as:

$$q^2 = \mu^2 - \sin^2\theta \tag{2}$$

where, μ is the refractive index of the medium. Also the ray tracing equations can be written as:

$$x = \sin\theta \int_0^z \frac{dz}{q} \qquad \& \qquad y = 0 \tag{3}$$

Since the signal comes back to earth after reflection from ionosphere at $z = z_r$, the transmission distance can be written as:

$$D = \sin\theta \int_\Omega \frac{dz}{q} \tag{4}$$

where, Ω is the contour for complex z which starts and ends at $z = 0$ circumventing the reflection point z_r.

Now let the signal $m(t)$ is modulated by a carrier $\cos(2\pi f_0 t)$ such that the modulated signal is of the form:

$$S(t) = \mathrm{Re}\left[m(t)\exp\left(2\pi i f_0 t\right)\right] \tag{5}$$

If the ionosphere is characterized by a transfer function $g(t)$, then the received signal will be:

$$S_r(t) = S(t) \otimes g(t) \tag{6}$$

or

$$S_r(t) = \int_{-\infty}^{\infty} S(t-\tau)\, g(t)\, dt \tag{7}$$

Now transferring the equation (7) in frequency domain, extracting the carrier term and introducing a new measure of time $\tau = t - P^{/}/c$, where $P^{/}$ is group path and is given by $P^{/} = \frac{d}{df}(fP)$, we find the expression for received signal as:

$$m_r(\tau) = \int_{-\infty}^{\infty} M(\phi) G(\phi) \exp(2\pi i \phi t) d\phi \tag{8}$$

where, $\phi = (f - f_0)$, $M(\phi)$ & $G(\phi)$ are Fourier transform pairs for $m(t)$ & $g(t)$ respectively. For low value of ϕ, the expression for $g(t)$ can be approximated as:

$$g(t) = \frac{1}{t_1} \exp\left(-\frac{1}{4}\pi i + \frac{\pi i t^2}{t_1^2} \right) \tag{9}$$

here, $t_1 = f_1^{-1} = \left(P_1^{/}/c \right)^{1/2}$, and $P_1^{/} = \frac{dP^{/}}{df}$. The method to find $g(t)$ has been given in [2] & [3]. Using the expression (9), we can find out an explicit expression for received signal after reflection from the ionosphere. The expression for $P_1^{/}$ is:

$$P_1^{/} = \frac{\cos^2 \theta P^{/} \frac{\partial P^{/}}{\partial f}}{P^{/} - f \sin^3 \theta \frac{\partial P^{/}}{\partial f}} \tag{10}$$

(Quoted from (45) of [2]).

Shape of a Pulse Train

In this section, we applied the above theory for a rectangular pulse train transmitted obliquely towards ionosphere and received at a distance D from the transmission point after reflection from the ionosphere. We considered a parabolic electron density profile for the ionosphere which is in the form:

$$N = N_m \left[1 - \left(\frac{h - h_m}{a} \right)^2 \right] \tag{11}$$

where N_m is the maximum electron density at height h_m and a is the half thickness of the parabola. Using the above profile, the expression for transmission distance D is found out as:

$$D = 2h_0 \tan\theta + \frac{af}{f_p}\sin\theta \ln\left[\frac{f_p + f\cos\theta}{f_p - f\cos\theta}\right] \tag{12}$$

where h_0 is the height of the lower level of the ionosphere from earth, f_p is the plasma frequency for the ionosphere, θ is the angle of incidence.

Equations (10) and (12) are used to calculate t_1. Now considering (9) as the ionospheric transfer function we use equation (8) to find out the expression for received pulse train. The simulation results are shown in figures 1 and 2 for two consecutive square pulses. The result will be similar for a large pulse train.

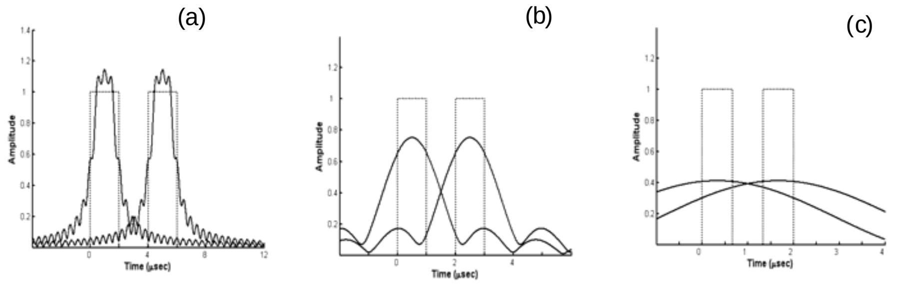

Figure 1: Received pulse shape for three different frequency of pulse train incident on ionosphere at a fixed angle of incidence of 30^0. (a) for f = 0.25 MHz, (b) for f = 0.5 MHz, (c) for f = 0.75 MHz.

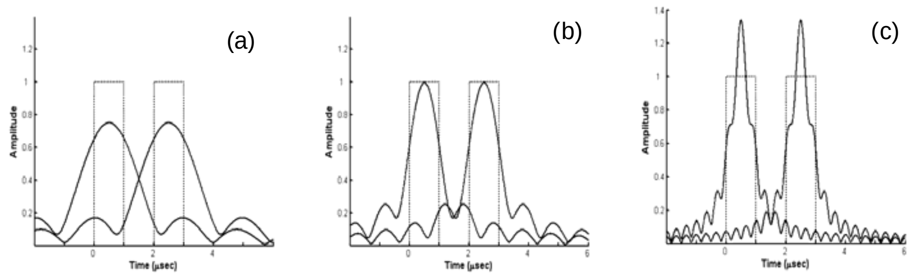

Figure 2: Received pulse shape for three different angles of incidence for pulse train of frequency 0.5 MHz incident on ionosphere. (a) for $\theta = 30^0$, (b) for $\theta = 45^0$, (c) for $\theta = 60^0$.

Figure 1 depicts the received pulse shape for three different frequency of pulse train incident on ionosphere at a fixed angle of incidence of 30^0. Here we choose h_0 = 80 km, a = 10 km, and f_p = 5 MHz. The frequency of pulse train for figure 1-a is 0.25 MHz, for figure 1-b is 0.5 MHz, and for figure 1-c is 0.75 MHz. The result indicates that the received pulses overlap with each other and the amount of overlapping increases as we increase the frequency of the pulse. The implication of the study of amount of overlapping is to estimate the inter symbol interference (ISI) and frequency of operation. From the outcome of the simulation, it is clear that for fixed angle of incidence, increasing the frequency of operation increases the ISI. Hence, in this case, to receive pulses with low ISI, the pulse repetition frequency becomes low. As an example, for 0.25 MHz, the overlapping of two consecutive pulses is ~ 25%.

The same type of simulation is carried out to obtain figure 2, where the frequency of operation is kept constant at 0.5 MHz and the angle of incidence is increased from 30^0 for figure 2-a, in step of 15^0 up to 60^0 for figure 2-c. This result explains just the opposite phenomena, i.e., the amount of overlapping decreases with increase in angle of incidence.

Frequency of Operation

From the above discussion, we are now able to estimate the frequency of operation of a signal for communication on earth through ionosphere. The extensive study of the pulse shape for the case of parabolic electron density profile of the ionosphere reveals the characteristics that indicates the change of overlapping region with frequency for fixed angle of incidence (as shown in figure 3-a) and also change of overlapping region with angle of incidence for fixed frequency (as shown in figure 3-b). The frequency of operation may be extended above 1 MHz if we transmit signal at a large grazing angle. As discussed in previous section, ISI incorporates a severe limitation for communication with high PRF, such as signal degradation, lost of information, etc. Hence, following the figure 3, if threshold level for detection of two pulses is chosen to such a value that 30% overlapping is acceptable, the frequency of operation becomes restricted up to nearly 400 KHz with an incidence angle of ~ 60^0. Thus to obtain a high pulse repetition frequency, threshold level at the receiver performs a crucial role. Higher the angle of incidence, higher will be the frequency of operation.

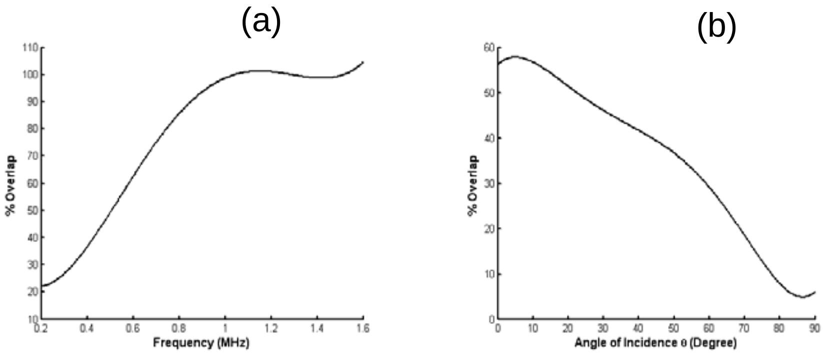

Figure 3: Change of overlapping region with (a) frequency, and with (b) angle of incidence for parabolic electron density profile.

The above discussed method to find out the overlapping region has been applied to two other electron density profiles, first is the sech^2 profile, and second is exponential profile. The outcomes of the simulations are shown in figure 4 and 5. From both the figures we find that the characteristics are similar to that of parabolic profile discussed above but, frequency of operation is much lower compared to the case of parabolic profile.

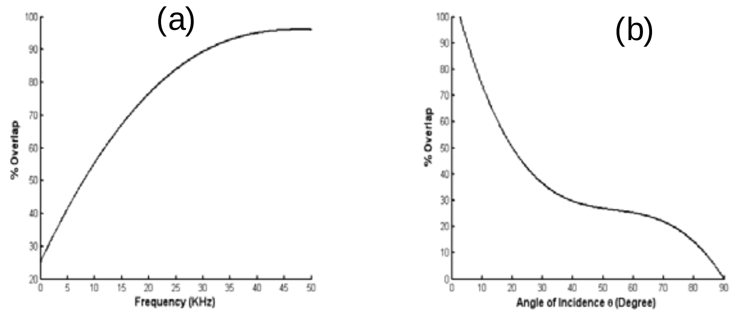

Figure 4: Change of overlapping region with (a) frequency and with (b) angle of incidence for $sech^2$ electron density profile.

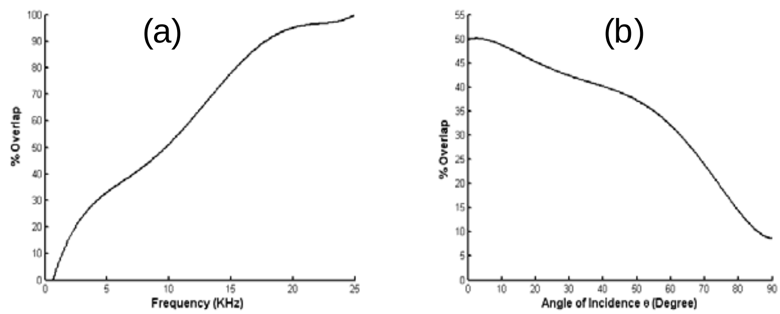

Figure 5: Change of overlapping region with (a) frequency, and with (b) angle of incidence for exponential electron density profile.

The difference in result is due to the fact that, for both $sech^2$ and exponential profile, the electron concentration is assumed to vary from ground level, i.e. there are some ionization effect (though very low) below the ionosphere. The theory discussed in section 2 is based on ray theory. For parabolic profile it is assumed that the region below the ionosphere is free of electrons so that electromagnetic wave will propagate with out any change of properties up to the ionosphere. For the case of $sech^2$ and exponential profile, this assumption will not be valid. Due to this reason, the frequency of operation and hence calculation of pulse repetition frequency is very different and very low compared to the case of parabolic profile. From the IRI model data sheet for the ionospheric electron concentration it is evident that the ratio of electron concentration to the maximum concentration (N / N_m) is negligible below the height of the ionosphere. In that context, parabolic profile assumption is more accurate for determining the ionosphere characteristics.

Conclusion

In this paper, the propagation of pulsed signals through ionosphere has been discussed to estimate the pulse repetition frequency for communication in a HF radio link. Under the assumption of ray theory approach, expressions for received signal after reflection from ionosphere have been derived. Applying those expressions and considering parabolic electron density profile for ionosphere, the shape of trains of pulses at reception has been found out. These results are used to estimate the frequency of operation for good reception with low ISI. It has been discussed that, to obtain a high pulse repetition frequency, threshold level at the receiver performs a crucial role. The variation of overlapping region with frequency and with angle of incidence together reveals the fact that higher the angle of incidence, higher will be the frequency of operation. Hence, it is interpreted that higher PRF can be achieved with transmission at high grazing angle. Also, a comparative study has been given for different other accepted ionospheric electron density profile, and it is found that parabolic profile is more accurate assumption for ionospheric communication study. The results of our study in this paper have important implications in ionospheric communication study. The study in this paper is based on ray theory approach. A wave theory approach can also be used in similar way to estimate more accurately the pulse repetition frequency for communication through ionosphere.

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