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Error Performance Analysis for Equal Gain Combining with Gaussian Noise and Co-channel Interference in Nakagami-m Fading Channel

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Abstract

The performance analysis of noncoherent M-ary Frequency Shift Keying (NMFSK) in noise and co-channel interference limited environment in mobile communication system is considered. The closed form expression for Symbol Error Probability (SEP) is derived employing equal gain combining for Nakagami fading channel. The effect of Symbol Interference Ratio (SIR) on SEP in presence of signal to noise ratio (SNR) is studied. The analysis is restricted to the case of equal interferer powers.

Introduction

The performance of mobile radio system is affected by noise, multipath fading and the co-channel interface [1]. Several techniques like diversity scheme, additive arrays, equalization etc. can be utilized to combat the effect of miultipath fading and to reduce the co-channel interference. In general space diversity is useful in mitigating the fading effect and Co-Channel Interference (CCI) by weighting and combining the received signals from all antenna branches [2]. There are three combining techniques namely, Maximum Ratio Combiner (MRC), Selective Combining (SC) and Equal Gain Combiner (EGC) for space diversity [3]. In MRC technique two receivers are employed for two branch diversity and the receiver circuit is very complicated .At the same time, although h in EGC there is only one dB degradation as compared to MRC but the circuit is relatively simple. The outage probability of cellular system for EGC with CCI in Nakagami fading channel was studied by Abu-Dayya and Beauliu [4] with equal interferer powers. The same study was performed by author [5] using mean signal using mean signal power to mean interferer powers.

EGC with co-channel interference for Rayleigh fading channel was studied by

Soug et. al. [6] for both equal and unequal power distributor, Chayan and Aalo [7] and Winter [8] have discussed the effect of CCI with MRC and OS combing technique.

The detection of Binary Phase Shift Keying (BPSK) in the presence of interference and noise has been presented by various authors [9-12].

In this paper, we have derived the closed form expression using characteristics function for probability of error for noncoherent M-ary Frequency Shift Keying (NMFSK) with Post Detection EGC for correlated Nakagami fading channel in presence of noise and interference. Here, we have presented the effect of Signal to Noise Ratio (SNR) and Signal to Interference Ratio (SIR) both on the probability of error. The organization of paper is as follows. Section IInd contains the correlated Nakagami fading model and their characteristics function. Symbol Error Probability (SEP) expression have been derived in section III. Numerical results have been presented in section IV. Section V, contains the concluding remarks of paper.

System Model

Let us assume S(t) be a complex base band information bearing signal with average energy 2Es. The received signal at kth diversity branch is given as

$$\gamma_k(t) = \mathrm{Re}\left\{\left[C_{sk}s(t) + \sum_{i=1}^{N} C_{ik}S_i(t) + n_k(t)\right] \times e^{j2\pi f_c t}\right\} \tag{1}$$

Where k= 1, 2, …….L; $0 \le t \le T_s$

i=1, 2,………N; T_s is symbol interval $S_i(t)$ is the ith complex interfering signal. L is the number of channels. N is the number of interfering signals.

C_{sk} is the kth channel vector of desired signal and it is defined as

$$C_{sk} = \alpha_{sk} e^{-j\varphi_{sk}} \tag{2}$$

Where α_{sk} is the random magnitude of fade $C_{i,k}$ is the kth channel vector of ith interfering signal and it is expressed as

$$C_{ik} = \alpha_{ik} e^{-\varphi_{ik}} \tag{3}$$

Where α_{ik} is the amplitude and φ_{ik} is the phase of interfering user channel

We assume C_i and C_s are mutually independent. $n_k(t)$ represents the AWGN which is complex valued with zero mean and N_0 variance. All the $\{n_k(t)\}$ are assumed to be independent. They are also independent of channel gain vectors.

We assume α_{sk}'s and α_{ik}'s are correlated variables.

Nakagami probability density for fading amplitudes of desired and interfering signal can be written as

$$f_{\alpha_{sk}}(r)=\frac{2}{\Gamma(m_{sk})}\left(\frac{m_{sk}}{\Omega_{sk}}\right)^{m_{sk}} r^{2m_{sk}-1} e^{\left(\frac{-m_{sk}r^2}{\Omega_{sk}}\right)} \tag{4}$$

and

$$f_{\alpha_{ik}}(r)=\frac{2}{\Gamma(m_{ik})}\left(\frac{m_{ik}}{\Omega_{ik}}\right)^{m_{ik}} r^{2m_{ik}-1} e^{\left(\frac{-m_{ik}r^2}{\Omega_{ik}}\right)} \tag{5}$$

Where m_{sk} and m_{ik} are fading parameters defined as

$$m_{sk} = \frac{\Omega_{sk}{}^2}{E\left[\left(\alpha_{sk}{}^2 - \Omega_{sk}{}^2\right)\right]} \tag{6}$$

$$m_{ik} = \frac{\Omega_{ik}{}^2}{E\left[\left(\alpha_{ik}{}^2 - \Omega_{ik}{}^2\right)\right]} \tag{7}$$

Variances $\Omega_{sk} = E\left[\alpha_{sk}{}^2\right]$

And

$\Omega_{sk} = E\left[\alpha_{ik}{}^2\right]$

So, instantaneous Signal to Interference Pulse Noise Ratio (SINR) defined as

$$SINR = \frac{s}{\sum_{i=1}^{N} I_i + N_0} \tag{8}$$

Let γ_{KIN} denote instantaneous SINR at the kth diversity branch

So

$$\gamma_{KIN} = \frac{E_s(\alpha_{sk})^2}{\sum_{i=1}^{N} E_i\left|\alpha_{ik}\right|^2 + N_0} \tag{9}$$

E_s and E_i are the energies of desired and interfering signals.
The average SINR of the kth branch is given as

$$\Gamma_{KIN} = E\left[\gamma_{KIN}\right] = \frac{E_s\Omega_{sk}}{\sum_{i=1}^{N} E_i\Omega_{ik} + N_0} \tag{10}$$

Now,

$$\gamma_{Tot} = \frac{1}{\sum_{k=1}^{L}\frac{1}{\gamma_{kI}} + \sum_{k=1}^{L}\frac{1}{\gamma_{kN}}} \tag{11}$$

Where γ_{kI} is instantaneous SIR and γ_{kN} is instantaneous SNR and they can be written as

$$\gamma_{kI} = \frac{\sum_{k=1}^{L}\left(E_s \alpha_{sk}{}^2\right)}{\sum_{k=1}^{L}\left(\sum_{i=1}^{N} E_i \alpha_{ik}{}^2\right)} \; ; \; \gamma_{kN} = \frac{\sum_{k=1}^{L} E_s \alpha_{sk}{}^2}{N_0}$$

γ_{kI} and γ_{kN} are independent

So,

$$\frac{1}{\gamma_{Tot}} = \sum_{k=1}^{L}\frac{1}{\gamma_{kI}} + \sum_{k=1}^{L}\frac{1}{\gamma_{kN}} \tag{12}$$

The average of $\gamma_{kI}{}^{-1}$ is given by

$$\Gamma_{kI} = E\left[\gamma_{kI}{}^{-1}\right] = \frac{\sum_{i=1}^{N} E_i \sum_{k=1}^{L} \frac{m_{kI}}{m_{ks}}}{E_s} \tag{13}$$

Where $\frac{E[\alpha_{ik}^2]}{E[\alpha_{sk}^2]} = \frac{m_{kI}}{m_{ks}}$

Here, we have assumed $m_{kI} = m_{ks}$

Similarly,

$$\Gamma_{kN} = E\left[\gamma_{kN}{}^{-1}\right] = \frac{N_0}{\sum_{k=1}^{L} E_s 2m_{ks}} \; . \tag{14}$$

For our analysis, we need the characteristics function of $\frac{1}{\gamma_{Tot}}$ Here,

$E\left[\gamma_{KIN}{}^{-1}\right] = \Gamma_{KIN}$

Following the method [13],

$$\gamma_{kI}{}^{-1} = \left(X_{kI}{}^{-1}\right)^T \left(X_{kI}{}^{-1}\right) \tag{15}$$

and $$\gamma_{kN}^{-1} = \left(X_{kN}^{-1}\right)^T \left(X_{kN}^{-1}\right) \tag{16}$$

Where $\left(X_{kI}^{-1}\right)$ is (2m_{ik} X 1) vector and $\left(X_{kN}^{-1}\right)$ is (2m_{sk} X 1) vector. $(\ )^T$ indicates the transpose of vector.

$$X_{kI}^{-1} = \left[X_{kI,1}^{-1}, X_{kI,2}^{-1}, \ldots\ldots. X_{kI,2m_k I}^{-1}\right] \tag{17}$$

and

$$X_{kN}^{-1} = \left[X_{kN,1}^{-1}, X_{kN,2}^{-1}, \ldots\ldots.. X_{kN,2m_{ks}}^{-1}\right] \tag{18}$$

k=1, 2,…….L

These vectors are independent and identically distributed Gaussian random variables with zero mean and variables $E\left[X_{kI}^{-1}\right]$ and $E\left[X_{kN}^{-1}\right]$

Here,

$$\gamma_{kN}^{-1} \stackrel{L}{=} \left(X_{kN}^{-1}\right)^T . \left(X_{kN}^{-1}\right) \tag{19}$$

where $\stackrel{L}{=}$ indicates equal in their respective distribution.

and

$$\gamma_{kI}^{-1} = \left(X_{kI}^{-1}\right)^T \left(X_{kI}^{-1}\right) \tag{20}$$

The covariance matrix for interfering signals with equal powers is given as

$\left(\dfrac{\Gamma_{kI}}{N}\right)$ for k=*l*, i=j

$R_{X_{kI}^{-1}} = E\left[X_{kI,i}^{-1} X_{lI,j}^{-1}\right] = \dfrac{\rho'_{k,l}}{N}\sqrt{\Gamma_{kI}\Gamma_{lI}}$ for k≠ *l*, but i = j=1, 2, 3,…..

0 otherwise (21)

and

$2m_{ks}\Gamma_{kN}$ for i=j, k=l $E\left[X_{kN,i}^{-1}, X_{lN,j}^{-1}\right] = \rho''_{k,l}\sqrt{2m_{ks}\Gamma_{kN}\, 2m_{ls}\Gamma_{lN}}$ for k≠*l*

but i=j=1, 2, 3,……..

0 otherwise (22)

Here, $0 \le \rho'_{k,l} = \rho'_{l,k} < 1$ for k≠ *l*

Let the eigen values of $R_{X_{kI}^{-1}}$ be $\lambda_1, \lambda_2, \lambda_3, \ldots\ldots\ldots.. \lambda_{N'}$,

and the eigen values of $R_{X_{kN}^{-1}}$ be $\lambda'_1, \lambda'_2, \lambda'_3, \ldots\ldots\ldots\ldots\ldots \lambda'_{N'}$,

Where, $$N' = 2\sum_{k=1}^{L} m_{ks} = 2\sum_{k=1}^{L} m_{kI} \tag{23}$$

Let $\xi_1^{-1}, \xi_2^{-1}, ...\xi_{N'}^{-1}$ are orthogonal vectors corresponding to eigen values λ_1, λ_2, λ_3, …… $\lambda_{N'}$ and $\xi_1^{'-1}, \xi_2^{'-1}, ...\xi_{N'}^{'-1}$ are orthogonal vectors corresponding to $\lambda_1^{'}, \lambda_2^{'}, \lambda_3^{'} ....\lambda_{N'}^{'}$.

From Karhunen – Loeve (KL) expansion of X_{kI}^{-1} and X_{kN}^{-1} we have

$$X_{kI}^{-1} = \sum_{n=1}^{N'} \sqrt{\lambda_n} \xi_n^{-1} . w_n^{-1} \tag{24}$$

and

$$X_{kn}^{-1} = \sum_{n=1}^{N'} \sqrt{\lambda_n^{'}} \xi_n^{'-1} . w_n^{'-1} \tag{25}$$

Where w_n^{-1} and $w_n^{'-1}$ are independent Gaussian random variables with zero mean and unit variance.

From eqn. (11) and (12) we get

$$\gamma_{kI}^{-1} = \sum_{n=1}^{N'} \lambda_n w_n^{-2} \tag{26}$$

and

$$\gamma_{kN}^{-1} = \sum_{n=1}^{N'} \lambda_n^{'} w_n^{'-2} \tag{27}$$

So,

$$\gamma_{tot}^{-1} = \sum_{n=1}^{N'} \lambda_n w_n^{-2} + \sum_{n=1}^{N'} \lambda_n^{'} w_n^{'-2} \tag{28}$$

Now,

$$\psi_{\left(w_n^{-2} + w_n^{'-2}\right)}(j\omega) = E\left[e^{j\omega\left(w_n^{-2} + w_n^{'-2}\right)}\right] = E\left[e^{j\omega w_n^{-2}}\right] E\left[e^{j\omega w_n^{'-2}}\right] = \psi_{w_n^{-2}}(j\omega) . \psi_{w_n^{'-2}}(j\omega) \tag{29}$$

Equation (29) may be written as

$$\psi_{\left(w_n^{-2} + w_n^{'-2}\right)} = \frac{1}{(1 - 2j\omega)^{\frac{1}{2}}} . \frac{1}{(1 - 2j\omega)^{\frac{1}{2}}} = \frac{1}{(1 - 2j\omega)} \tag{30}$$

From eqn. (21), we get find that

$$\psi_{\gamma_{tot}^{-1}}(j\omega) = \prod_{n=1}^{N'} \psi_{\left(w_n^{-2} + w_n^{'-2}\right)} j\omega\left(\lambda_n + \lambda_n^{'}\right)$$

$$= \frac{1}{\prod_{n=1}^{N'} \left(1 - 2j\omega\left(\lambda_n + \lambda_n'\right)\right)} \tag{31}$$

Symbol Error Probability

The decision variables can be expressed as

$$U_1 = \sum_{k=1}^{L} \left| \left(2E_s \alpha_{ks} + n_{k1}\right) + 2\sum_{i=1}^{N} E_i \alpha_{ki1} \right|^2 \tag{32}$$

$$U_m = \sum_{k=1}^{L} \left\{ \left|n_{km}\right|^2 + \left| 2\sum_{i=1}^{N} E_i \alpha_{kim} \right|^2 \right\} \tag{33}$$

Where m=2, 3,……M

Where $\left|n_{km}\right|$ and $\left|2\sum_{i=1}^{N} E_i \alpha_{kim}\right|^2$ are complex valued zero mean Gaussian random variables with variances $2E_s N_0$ and $2E_i E\left[\alpha_{kim}^2\right]$ or $2E_i\Omega_i^2$ respectively.

For M-ary FSK, U_m is the decision variable which is transmitted, the c.f. is given as

$$\psi_{U_m}\left(j\omega \middle| \gamma_{tot}^{-1}\right)$$

$$= \frac{1}{\left[1 - 4j\omega E_s N_0 - \sum_{i=1}^{N} 4j\omega E_i \Omega_i\right]^L} \times \exp\left\{ \frac{4j\omega E_s N_0 + \sum_{i-1}^{N} 4j\omega E_i \Omega_i^2}{1 - 4j\omega E_s N_0 - \sum_{i=1}^{N} 4j\omega E_i \Omega_i^2} \times \gamma_{tot}^{-1} \right\} \tag{34}$$

The unconditional characteristic function can be obtained by averaging over the statistics

$$\psi_{U_{M-1}}\left(j\omega\right) = \frac{1}{\left[1 - 4j\omega E_s N_0 - \sum_{i=1}^{N} 4j\omega E_i \Omega_i\right]^L} \times \psi_{tot^{-1}}\left(\frac{4j\omega E_s N_0 + \sum_{i=1}^{N} 4j\omega E_i \Omega_i}{1 - 4j\omega E_s N_0 - \sum_{i=1}^{N} 4j\omega E_i \Omega_i} \right) \tag{35}$$

Following the method [14], the probability density function of U_2

$$P\left(U_2 < u_1 \middle| U_1 = u_1\right) = 1 - \exp\left(\frac{-u_1}{4E_s N_0 + \sum_{i=1}^{N} 4E_i \Omega_i^2} \right) \times \sum_{k=0}^{L-1} \frac{1}{k!} \left(\frac{u_1}{4E_s N_0 + \sum_{i=1}^{N} 4E_i \Omega_i^2} \right)^k \tag{36}$$

Joint probability $P(U_2<U_1, U3<U_1, \ldots\ldots U_m<U_1)$

$$=\left[P\left(u_2 < u_1 \middle| U_1 = u_1\right)\right]^{M-1}$$

$$=\left[1-\exp\left(\frac{-u_1}{4E_sN_0+\sum_{i=1}^{N}4E_i\Omega_i^2}\right)\times\sum_{k=0}^{L-1}\frac{1}{k!}\left(\frac{u_1}{4E_sN_0+\sum_{i=1}^{N}4E_i\Omega_i^2}\right)^k\right]^{M-1} \tag{37}$$

Thus, Joint probability after expansion may be written as.

$$\left[P\left(u_2 < u_1 \middle| U_1 = u_1\right)\right]^{M-1}$$

$$= 1-\sum_{i=1}^{M-1}\binom{M-1}{\mathrm{i}}\mathrm{e}^{\frac{-iu_1}{4E_SN_0+\sum_{i=1}^{N}4E_i\Omega_i^2}}\times\left(\sum_{K=0}^{L-1}\frac{1}{k!}\times\left(\frac{u_1}{4E_sN_0+\sum_{i=1}^{N}4E_i\Omega_i^2}\right)^k\right)^i \tag{38}$$

The conditional probability of error P_c is given by

$$P_c=\int_0^\infty\left[P\left(u_2 < u_1 \middle| U_1 = u_1\right)\right]^{M-1}P(u_1)du_1 \tag{39}$$

From eqn. (38) and (39) we get

$$P_c=\int_0^\infty 1-\sum_{i=1}^{M-1}\binom{M-1}{\mathrm{i}}\mathrm{e}^{\frac{-iu_1}{4E_SN_0+\sum_{i=1}^{N}4E_i\Omega_i^2}}\times\left(\sum_{K=0}^{L-1}\frac{1}{k!}\times\left(\frac{u_1}{4E_SN_0+\sum_{i=1}^{N}4E_i\Omega_i^2}\right)^k\right)^i P\left(u_1\right)du_1 \tag{40}$$

The multinominal expansion of $\left(\sum_{k=0}^{L-1}\left(\frac{u_1}{4E_SN_0+\sum_{i=1}^{N}4E_i\Omega_i^2}\right)^k\times\frac{1}{k!}\right)^i$ [13, 15] can be written as

$$\left(\sum_{k=0}^{L-1}\frac{1}{k!}\times\left(\frac{u_1}{4E_SN_0+\sum_{i=1}^{N}4E_i\Omega_i^2}\right)^k\right)^i$$

$$= i! \sum_{\substack{(l_0, l_1, \ldots, l_{L-1}) \\ l_0, l_1, \ldots, l_{L-1} \geq 0 \\ l_0 + l_1 + \ldots + l_{L-1} = i \\ l_1 + 2l_2 + \ldots + (L-1) l_{L-1} = p}} \prod_{q=0}^{L-1} \frac{1}{l_q! (q!)^{l_q}} \left(\frac{u_1}{4E_s N_0 + \sum_{i=1}^{N} 4E_i \Omega_i^2} \right)^p \tag{41}$$

From the definition of characteristic function we have probability density

$$P(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi(jv) e^{-jvx} dv \tag{42}$$

Substituting eqn. (42) into (40) together with (41) we find

$$P_c = \int_0^\infty 1 - \sum_{i=1}^{M-1} \binom{M-1}{\mathrm{i}} \mathrm{e}^{\frac{-iu_1}{4E_s N_0 + \sum_{i=1}^{N} 4E_i \Omega_i^2}} \times i! \sum_{\substack{(l_0, l_1, \ldots, l_{L-1}) \\ l_0, l_1, \ldots, l_{L-1} \geq 0 \\ l_0 + l_1 + \ldots + l_{L-1} = i \\ l_1 + 2l_2 + \ldots + (L-1) l_{L-1} = p}} \prod_{q=0}^{L-1} \frac{1}{l_q! (q!)^{l_q}} \left(\frac{u_1}{4E_s N_0 + \sum_{i=1}^{N} 4E_i \Omega_i^2} \right)^p$$

$$\times \left\{ \frac{1}{2\pi} \int_{-\infty}^{\infty} \psi_{U_M}(j\omega) e^{-j\omega u_1} du_1 \right\} du_1 \tag{43}$$

$$P_c = 1 - \sum_{i=1}^{M-1} \binom{M-1}{\mathrm{i}} (-1)^{i+1} \sum_{p=0}^{i(L-1)} i! \left[\prod_{q=0}^{L-1} \frac{1}{l_q! (\mathrm{q}!)^{l_q}} \right] \frac{p! \left(4\mathrm{E_s} N_0 + \sum_{i=1}^{N} 4E_i \Omega_i^2 \right)}{2\pi}$$

$$\times \int_{-\infty}^{\infty} \frac{\psi_{U_M(j\omega)}}{\left[1 + 4j\omega E_s N_0 + \sum_{i=1}^{N} 4j\omega E_i \Omega_i^2 \right]^{p+1}} d\omega \tag{44}$$

Symbol Error Probability = $P_e = 1 - P_c$

$$= \sum_{i=1}^{M-1} \binom{M-1}{\mathrm{i}} (-1)^{i+1} \sum_{p=0}^{i(L-1)} i! \left[\prod_{q=0}^{L-1} \frac{1}{l_q! (\mathrm{q}!)^{l_q}} \right] \frac{p! \left(4\mathrm{E_s} N_0 + \sum_{i=1}^{N} 4E_i \Omega_i^2 \right)}{2\pi}$$

$$\times \int_{-\infty}^{\infty} \frac{\psi_{U_M(j\omega)}}{\left[1 + 4j\omega E_s N_0 + \sum_{i=1}^{N} 4j\omega E_i \Omega_i^2 \right]^{p+1}} d\omega \tag{45}$$

Substituting eqn. (31) into eqn. (35) and solving the c.f. of U_{M-1} can be written as

$$\psi_{U_{M-1}}(j\omega)=\frac{1}{\left[1-4j\omega E_s N_0-\sum_{i=1}^{N}4j\omega E_i\Omega_i^2\right]^L}\times\frac{1}{\prod_{n=1}^{N}\left(1-2\left(\frac{4j\omega E_s N_0+\sum_{i=1}^{N}4j\omega E_i\Omega_i}{1-4j\omega E_s N_0-\sum_{i=1}^{N}4j\omega E_i\Omega_i}\right)\times\left(\lambda_n+\lambda_n^{'}\right)\right)}$$

or

$$\psi_{U_{M-1}}(j\omega)=\frac{\left[1-\left(4j\omega E_s N_0+\sum_{i=1}^{N}4j\omega E_i\Omega_i\right)\right]^{N-L}}{\prod_{n=1}^{N}\left[1-\left[1+2\left(\lambda_n+\lambda_n^{'}\right)\right]\times\left(4j\omega E_s N_0+\sum_{i=1}^{N}4j\omega E_i\Omega_i\right)\right]} \tag{46}$$

Using Cauchy’s integral formula for n$^{\text{th}}$ derivatives

$$f^{n}(z_0)=\frac{n!}{2\pi\,\mathrm{i}}\oint\frac{f(z)}{(z-z_0)^{n+1}}dz \tag{47}$$

Into eqn. (46), we get the probability of symbol error as

$$P_e=\sum_{i=1}^{M-1}\binom{M-1}{\mathrm{i}}(-1)^{i+1}\sum_{p=0}^{i(L-1)}i!\left[\prod_{q=0}^{L-1}\frac{1}{l_q!(\mathrm{q}!)^{l_q}}\right]\frac{d^p}{dz^p}\left[\psi_{U_{M-1}}\left(\frac{z}{4E_s N_0+\sum_{i=1}^{N}4E_i\Omega_i}\right)\right]_{z=-i} \tag{48}$$

From eqn. (46) replacing jω by $\frac{z}{\left(4E_s N_0+\sum_{i=1}^{N}4E_i\Omega_i^2\right)}$ we have

$$\psi_{U_{M-1}}\left(\frac{z}{\left(4E_s N_0+\sum_{i=1}^{N}4E_i\Omega_i^2\right)}\right)=\frac{(1-z)^{N-L}}{\prod_{n=1}^{N}\left(1-z\left[1+2\left(\lambda_n+\lambda_n^{'}\right)\right]\right)}=\mathrm{G(z)} \tag{49}$$

Let

$$H_{Nak}(z)=\ln G(z) \tag{50}$$

Using Faadi Bruno’s formula [16]

$$\frac{d^p}{dz^p}G(z)=G(z)\sum_{\substack{(l_0,l_1,\ldots l_{L-1})\\ l_0,l_1,\ldots\ldots l_{L-1}\geq 0\\ l_0+l_1+\ldots+l_{L-1}=i\\ l_1+2l_2+\ldots(L-1)l_{L-1}=p}}\prod_{q=0}^{p}\left[\frac{H^q_{Nak}(z)}{q!}\right]^{l_q} \tag{51}$$

Here,

$$H^q(z)=\frac{-(N-L)(q-1)!}{(1-z)^q}+\sum_{n=1}^{N}\frac{(q-1)!\left[1+2\left(\lambda_n+\lambda_n^{'}\right)\right]^q}{\left[1-z\left(1+2\left(\lambda_n+\lambda_n^{'}\right)\right)\right]^q} \tag{52}$$

Substituting eqn. (52) and (49) into eqn. (51) we get the p^{th} derivative of G(z) as

$$\frac{d^p}{dz^p}G(z)=$$

$$\frac{(1-z)^{N-L}}{\prod_{n=1}^{N}\left[1-z\left(1+2\left(\lambda_n+\lambda_n^{'}\right)\right)\right]}\times\sum_{\substack{(l_0,l_1,\ldots l_{L-1})\\ l_0,l_1,\ldots l_{L-1}\geq 0\\ l_0+l_1+\ldots+l_{L-1}=i\\ l_1+2l_2+\ldots(L-1)l_{L-1}=p}}\prod_{q=0}^{p}\left[\sum_{n=1}^{N}\left(\frac{(q-1)!\left[1+2\left(\lambda_n+\lambda_n^{'}\right)\right]^q}{\left[1-z\left(1+2\left(\lambda_n+\lambda_n^{'}\right)\right)\right]^q}-\frac{(N-L)(q-1)!}{(1-z)^q}\right)\times\frac{1}{q!}\right]^{l_q} \tag{53}$$

So, symbol error probability can be obtained by substituting eqn. (53) for z=-i into eqn.(48)

$$P_e=\sum_{i=1}^{M-1}\binom{M-1}{i}(-1)^{i+1}\sum_{p=0}^{i(L-1)}i!\sum_{\substack{(l_0,l_1,\ldots l_{L-1})\\ l_0,l_1,\ldots\ldots l_{L-1}\geq 0\\ l_0+l_1+\ldots+l_{L-1}=i\\ l_1+2l_2+\ldots(L-1)l_{L-1}=p}}\left[\prod_{q=0}^{L-1}\frac{1}{l_q!(q!)^{l_q}}\right]$$

$$\times\frac{(1+i)^{N-L}}{\prod_{n=1}^{N}\left[1+i\left(1+2\left(\lambda_n+\lambda_n^{'}\right)\right)\right]}\times\sum_{\substack{(l_0,l_1,\ldots l_{L-1})\\ l_0,l_1,\ldots l_{L-1}\geq 0\\ l_0+l_1+\ldots+l_{L-1}=i\\ l_1+2l_2+\ldots(L-1)l_{L-1}=p}}\prod_{q=0}^{p}\left[\sum_{n=1}^{N}\left(\frac{(q-1)!\left[1+2\left(\lambda_n+\lambda_n^{'}\right)\right]^q}{\left[1+i\left(1+2\left(\lambda_n+\lambda_n^{'}\right)\right)\right]^q}-\frac{(N-L)(q-1)!}{(1+i)^q}\right)\times\frac{1}{q!}\right]^{l_q} \tag{54}$$

All the calculations have been performed using equation (54).Numerical computation have been performed using MATLAB.

Numerical results and discussion

Symbol Error Probability (SEP) for different values of Symbol to Interference Ratio (SIR) for fixed SNR= 7, 9, and 12 dB for diversity order L=2 and M=4 is presented in fig. (1a). Here parameters used are $2m_1$=1, $2m_2$=2, δ=1, ρ_{SNR} =0.9 ρ_{SIR}=0.8, $N^{'}$=3 and number of interferers N=2. The same is presented using the above parameters for N=6 in fig. (1b). From the fig. (1a), we infer that SEP decreases with the increase of SIR for a fixed value of SNR. For fixed value of SIR, SEP also decreases with the increase of SNR. From fig.(1b) we find the value of SEP is large for N=6 as compared to N=2.

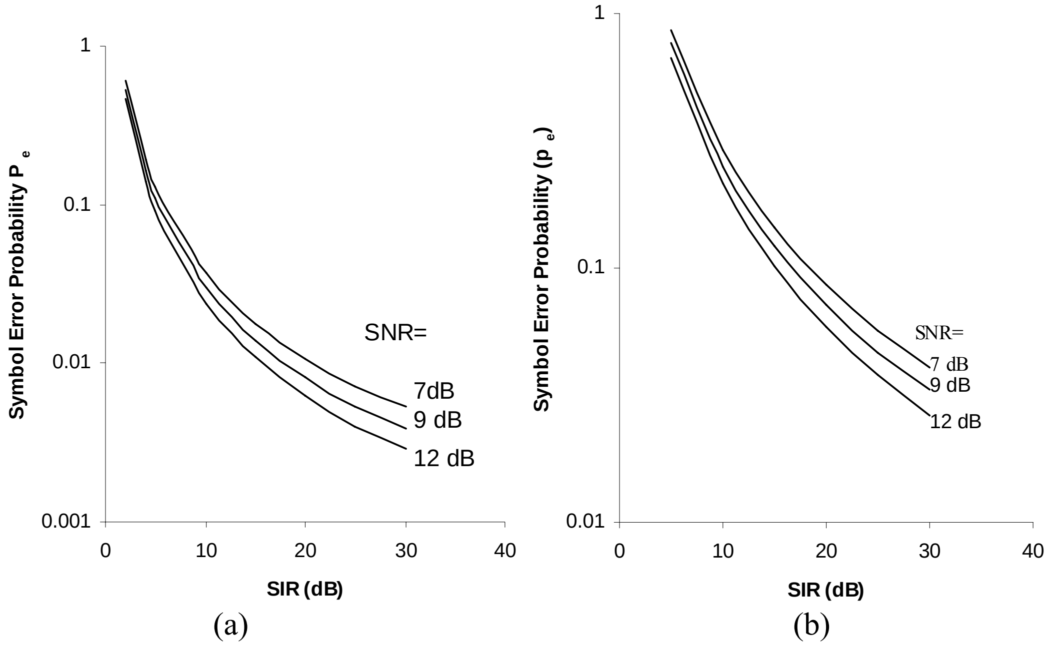

Figure 1: Symbol Error Probability for different values of SIR for fixed SNR =7, 9 and12dB. [a] for number interferers N=2 [b] for number interferers N=6

SEP is computed for the number of interferers N=2, 4 and 6 by selecting the L=2, M=4, $2m_1$=1, $2m_2$ = 2, $N^{'}$=6, SNR =12dB, ρ_{SNR} =0.5, ρ_{SIR}=0.8 for different values of SIR and depicted in fig. (2). From the curves, we see that for large N, SEP is large. SEP is numerically calculated for L=3, M=4 with SNR =2dB, 5dB and 7dB for different values of SIR and produced in fig. (3). Parameters selected in this calculation are as follows.

The average value SIR and SNR i.e. Γ_{kI} or Γ_{kN} is defined as

$\Gamma_k = \Gamma_1 e^{-ik\delta}$ where k=1, 2, 3 ……. L, δ>0

δ is a parameter

Symbol error probability for L=2, 3 for SNR =5dB SIR= 10dB, M=4; and SNR =12dB, SIR= 8dB, M=4 for different values of δ is illustrated in figure SEP increases with the increase of δ for different values of L.

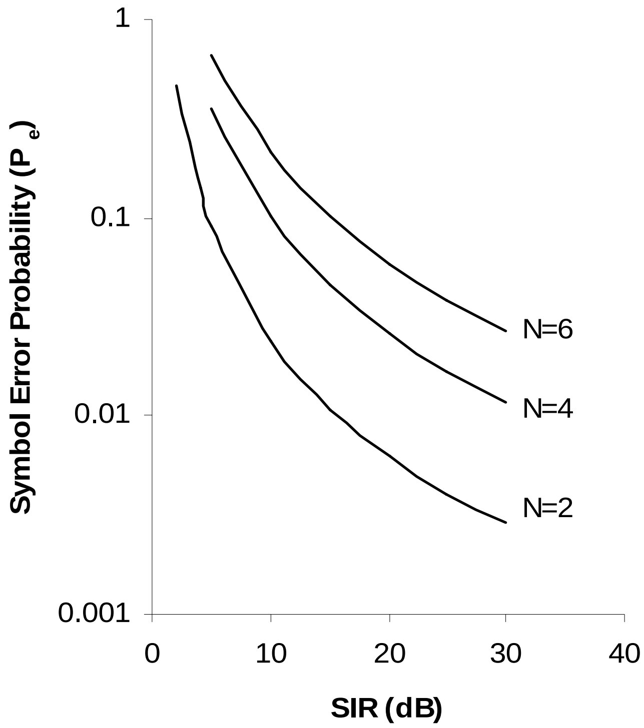

Figure 2: SEP versus SIR for interferers N=2, 4 and 6 for SNR =12dB.

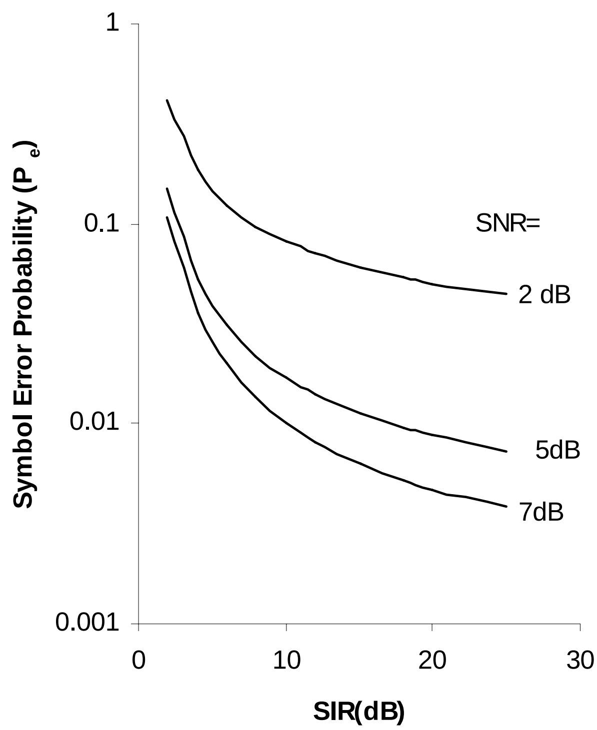

Figure 3: SEP versus SIR with SNR = 2, 5 and 7dB.

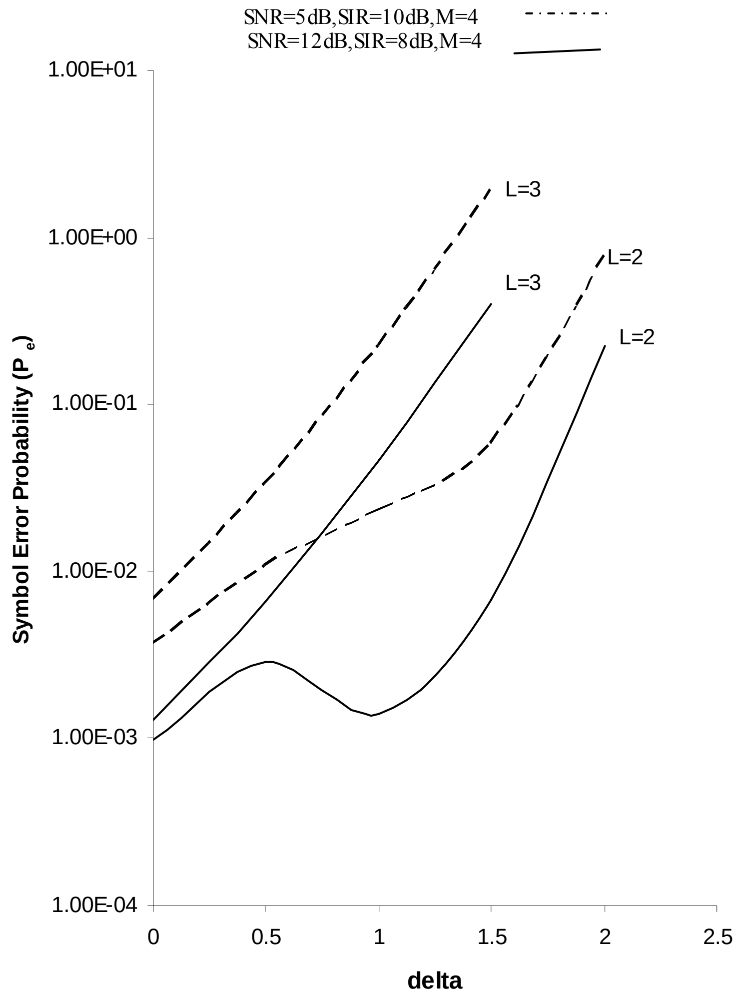

Figure 4: SEP versus delta (δ) with SNR =5dB, SIR = 10dB; and SNR =12dB, SIR=8dB with L=2, 3.

Conclusion

The closed form expression for symbol Error Probability provides direct solution for arbitrary SIR and SNR. The numerical analysis through this expression is simple. There is no need for any iteration and integration process. Thus, it saves computational time as well as space. At the same time, no approximation is employed during the derivation of expression. So, we get exact analysis of SEP through the expression. With the flexibility in the choice of parameters like SIR, SNR and number of interferers; co- channel reduction factor i.e. the ratio of radius of cell and distance between two cochannel cell and transmitted power at each co-channel cell can be adjusted as practical designer requires.

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