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Design of Quadratic Dynamic Matrix Control for Driven Pendulum System

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Abstract

This paper proposes a Quadratic Dynamic Matrix Control (QDMC) method to control a driven pendulum system. Driven pendulum is a suspended pendulum, which has a motorized propeller at the end of the stick, so it can be controlled by controlling the voltage given to DC motor. Dynamic Matrix Control (DMC) was the first Model Predictive Control (MPC) algorithm introduced in early 1980s. These are proven methods that give good performance and are able to operate for long periods without almost any significant intervention. Today, DMC is available in almost all commercial industrial distributed control systems and process simulation software packages. The simulation results have proven that this method enhances the stability as well as ease of tuning.

Keywords: QDMC; Driven pendulum; step response; model length; control horizon; prediction horizon; MATLAB

Introduction

A simple pendulum system is a mechanical system that exhibits periodic motion. It consists of particle like bob of mass suspended by a light string of length that is fixed at the upper [1]. A Compound Pendulum is a standard topic in most physics courses because it includes some physical subjects such as the simple harmonic motion, the period of oscillation, the acceleration of gravity, the center of mass, the moment of the inertia, momentum, etc. [2]. Literature Works [3]-[5] adopted types of compound Pendulum. This type of pendulum described in this paper has a motorized propeller at the end of the pendulum so it can lift the pendulum after given voltage. This concept of pendulum system is useful and can be applied in real life. This system has many applications such as measurement, scholar tuning, coupled pendulum, entertainment etc., Kizmaz [6] proposed a sliding mode control for this system and the presented method was based on the improvement of robustness. Step response model of driven pendulum system is used for implementing QDMC algorithm.

The steps involved in implementing DMC on a process.

1. Develop a discrete step response model with length N, based on a sampled time Δt.
2. Specify the prediction (P) and control (M) horizons. N≥P≥M.

The model length should be approximately the "setting time" of the process, that is, the time required to reach a new steady state after a step input change. Prediction and control *horizons* differ in length. Usually, the prediction horizon is selected to be much longer than the control horizon.

The combination of a linear model and a quadratic objective function leads to an analytical solution for the control moves. In practice, constraints on manipulated inputs (control moves) can be very important. Fortunately DMC is easily formulated to explicitly handle constraints by using Quadratic Programming (QP); the method is known as QDMC.

Driven PEndulum System

The schematic diagram of the considered pendulum is shown in Fig.1. This pendulum is driven by DC motor. It has a motorized propeller at the end of the stick as was shown in the figure. After applied voltage, the propeller spins and generates torque *T* to pull up the pendulum.

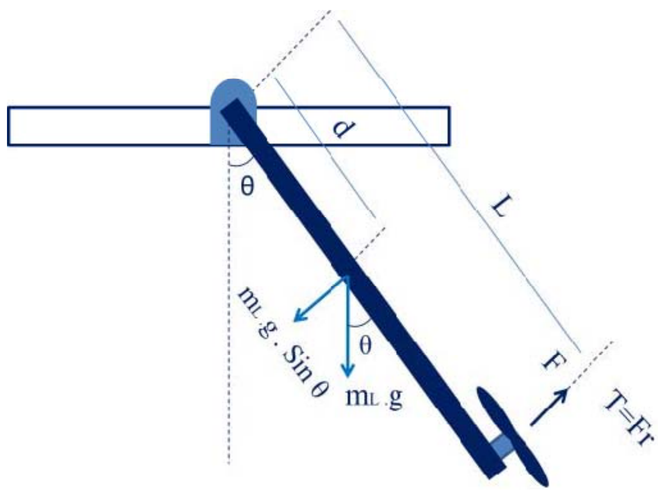

Figure 1: Schematic diagram of driven pendulum

The aim is to command the pendulum to a specified angle. The suspended point is attached to an encoder to provide the measurements of angle and angular velocity of pendulum. It is the most advantages of driven pendulum that enables us controlling its behavior with adjusting the applied voltage. Therefore, the controlled variable for this system is the angle of the pendulum settled and the manipulated variable is the voltage given to the motorized-propeller. According to Newton's laws and angular momentum, the motion equation of driven pendulum is derived.

$$\mathrm{J}.\ddot{\theta} + \mathrm{c}.\dot{\theta} + \mathrm{m_l}.\mathrm{g}.\mathrm{d}.\sin\theta = T \qquad (1)$$

Where;
Θ =angular position of the pendulum
$m_{l.}$ = weight of the pendulum
d = the distance between center of mass and pivot point
c = viscous damping coefficient
T = the trust

By considering sin $\Theta \approx \Theta$, the linearized motion equation can be written as follows

$$\mathrm{J}.\ddot{\theta} + \mathrm{c}.\dot{\theta} + \mathrm{m_l}.\mathrm{g}.\mathrm{d} = \mathrm{T} \qquad (2)$$

Transfer Function

Equation2 gives the transfer function driven pendulum.

$$\frac{\theta(s)}{T(s)} = \frac{1}{\mathrm{J}.s^2 + \mathrm{c}.s + \mathrm{m_l}.\mathrm{g}.\mathrm{d}} \qquad (3)$$

And the standard representation is

$$\frac{\theta(s)}{T(s)} = \frac{1/\mathrm{J}}{s^2 + \frac{\mathrm{c}}{\mathrm{J}}.s + \frac{\mathrm{m_l}.\mathrm{g}.\mathrm{d}}{\mathrm{J}}} \qquad (4)$$

The generated trust T in above equations is not manipulated variable for control system since the pendulum is adjusted by applied voltage. The transfer function of motorized propeller can be represented as a block diagram. (Fig.2.) the method to obtain this gain was shown in [6] as

$$K_m = \frac{\mathrm{m_l}.\mathrm{g}.\mathrm{d}.\theta}{V} \qquad (5)$$

Then the block diagram of driven pendulum is given in Fig.3.

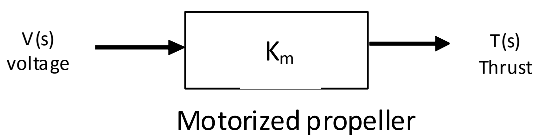

Figure 2: Block diagram of Motorized Propeller

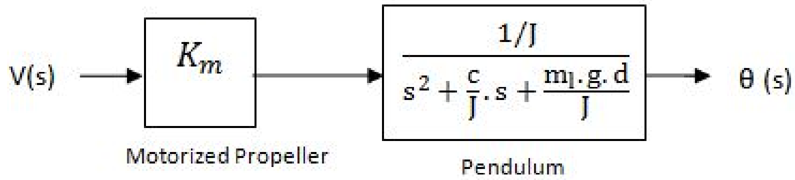

Figure 3: Block diagram of driven pendulum

Moreover, the Transfer function is obtained by

$$\frac{\theta(s)}{V(s)} = \frac{\frac{K_m}{J}}{s^2 + \frac{c}{J}.s + \frac{m_l.g.d}{J}} \qquad (6)$$

State Space
Consider this system represented in state space by

$$x_1 = \theta \,, x_2 = \dot{\theta}, x_2 = \dot{x_1} \quad (7)$$

Is written as

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{m_l.g.d}{J} & -\frac{c}{J} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_m}{J} \end{bmatrix} . u$$

$$Y = [1 \quad 0]. \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \qquad (8)$$

Quadratic Dynamic Matrix Control
QDMC considers constraints on the manipulated inputs. The input constraints can be of the following form

$$u_{min} \leq u_{k+i} \leq u_{max} \qquad (9)$$

This is suitable for minimum and maximum flow rates, for example. In addition, velocity constraints that limit the magnitude of the control mouse at each sample time have the following form

$$\Delta u_{min} \leq \Delta u_{k+i} \leq \Delta u_{max} \qquad (10)$$

Where ordinarily, $\Delta u_{min} = -\Delta u_{max}$. To use a standard quadratic program (QP), the constraints in above need to be written in terms of the control moves, Δu_{k+i} .since the previously implemented control action (u_{k-1}) is known, we can write

$$u_k = u_{k-1} + \Delta u_k$$

$$u_{k+1} = u_{k-1} + \Delta u_k + \Delta u_{k+1} \quad (11)$$

and so on. Since the manipulated input constraints are enforced over the control horizon of *M steps*,

$$\begin{bmatrix} u_{min} \\ u_{min} \\ \vdots \\ u_{min} \end{bmatrix} \le \begin{bmatrix} u_{k-1} \\ u_{k-1} \\ \vdots \\ u_{k-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} \Delta u_k \\ \Delta u_{k+1} \\ \vdots \\ \Delta u_{k+M-1} \end{bmatrix} \le \begin{bmatrix} u_{max} \\ u_{max} \\ \vdots \\ u_{max} \end{bmatrix} \quad (12)$$

Most standard QP code use a "one-sided" form

$$\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} \Delta u_k \\ \Delta u_{k+1} \\ \vdots \\ \Delta u_{k+M-1} \end{bmatrix} \ge \begin{bmatrix} u_{min} - u_{k-1} \\ u_{min} - u_{k-1} \\ \vdots \\ u_{min} - u_{k-1} \end{bmatrix}$$

And

$$-\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \dots & 1 \end{bmatrix} \begin{bmatrix} \Delta u_k \\ \Delta u_{k+1} \\ \vdots \\ \Delta u_{k+M-1} \end{bmatrix} \ge \begin{bmatrix} u_{k-1} - u_{max} \\ u_{k-1} - u_{max} \\ \vdots \\ u_{k-1} - u_{max} \end{bmatrix}$$

Which have the form $A\, \Delta u_f \ge b$.

The velocity constraints are implemented as bounds on the control moves

$$\begin{bmatrix} \Delta u_{min} \\ \Delta u_{min} \\ \vdots \\ \Delta u_{min} \end{bmatrix} \le \begin{bmatrix} \Delta u_k \\ \Delta u_{k+1} \\ \vdots \\ \Delta u_{k+M-1} \end{bmatrix} \le \begin{bmatrix} \Delta u_{max} \\ \Delta u_{max} \\ \vdots \\ \Delta u_{max} \end{bmatrix} \quad (13)$$

The majority of constrained MPC problems can be solved based on the input constraints considered above. For completeness, however, we also show how constraints on the process outputs can be included. It may be desirable to force the predicted process outputs to be within a range of minimum and maximum values

$$y_{min} \le \hat{y}^c{}_{k+i} \le y_{max} \qquad (14)$$

Here we first rewrite equation,

$$\hat{y}^c = s_f \Delta u_f + s_{past} \Delta u_{past} + s_N u_P + \hat{d}$$

$$\hat{y}^c = s_f \Delta u_f + f \qquad (15)$$

Where f, the free response of the "corrected-predicted output" (if no current and future control moves are made) is

$$f = s_{past} \Delta u_{past} + s_N u_P + \hat{d} \quad (16)$$

So that we can write from (16.31)

$$y_{min} - f \leq s_f \Delta u_f \leq y_{max} - f \quad (17)$$

Or, in terms of one-sided in equalities

$$\begin{aligned} s_f \Delta u_f &\geq y_{min} - f \\ -s_f \Delta u_f &\geq -y_{min} + f \qquad (18) \end{aligned}$$

Using shorthand matrix-vector notation, the quadratic programming problem is stated as

$$\min_{\Delta u_f} \emptyset = \frac{1}{2} . \Delta u_f^T H \Delta u_f + c^T \Delta u_f$$

$$s.t. \quad A\Delta u_f \geq b$$

$$\Delta u_{min} \leq \Delta u_f \leq \Delta u_{min} \qquad (19)$$

Simulation Results

Mathematical model of system is obtained based on an experimental set up by [6].

Where,

d=0.03m, m_{pl}=0.36 kg, g=9.8m/sec^2, j=0.0106 Kgm^2, c=0.0076 Nms/rad, Km=0.0296

the transfer function is given by

$$\frac{\theta(s)}{V(s)} = \frac{2.7922}{s^2 + 0.7191s + 9.9989} \qquad (20)$$

To implement QDMC algorithm for driven pendulum system, following procedure is as follows:

The step response of driven pendulum system is as follows:

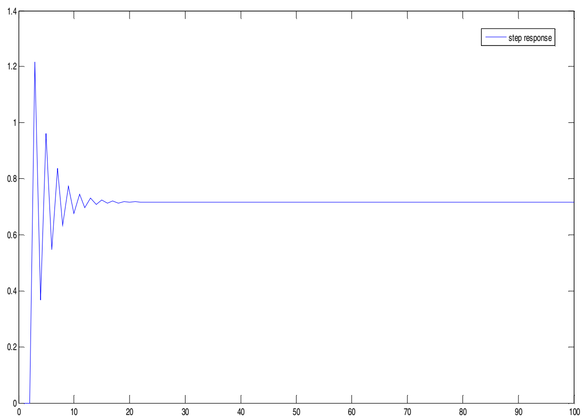

Figure 4: The step response of driven pendulum system For N = 100, M = 1, P = 4.

QDMC implemented by considering the following parameters:

Model length (N) =100,

Control horizon (M) =1,

Prediction horizon (P) =4,

Constraints on manipulated inputs

$$\Delta u_{min} \leq \Delta u_{k+i} \;\leq \Delta u_{max}$$

The response of driven pendulum system for unit step change in input is as follows with various constraints on manipulated variable.

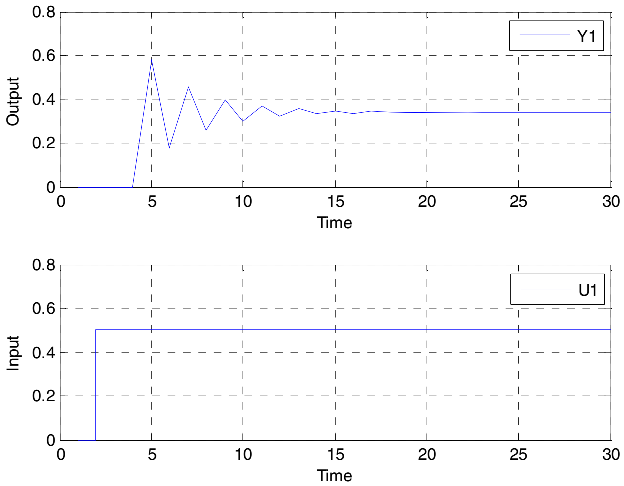

Figure 5: System response with QDMC with in the Constraints $u_{max} = 0.5, u_{min} = -0.5$

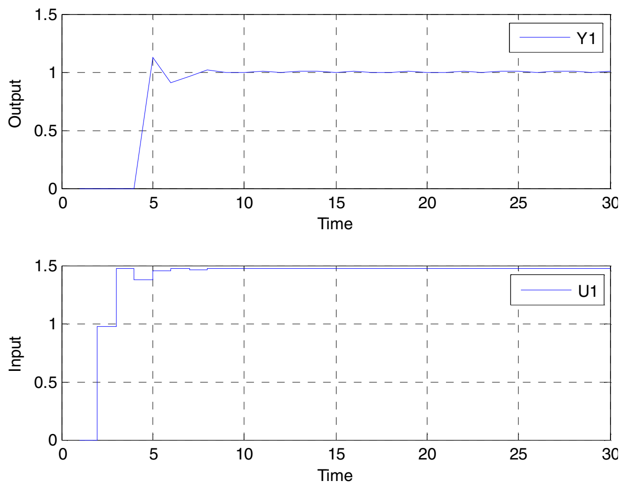

Figure 6: System response with QDMC with in the Constraints $u_{max} = 1.5, u_{min} = -1.5$

Conclusion

QDMC algorithm was implemented for a linearized driven pendulum system with various constraints on manipulated variable. The simulation was done for the system with controller. The Response characteristics of driven pendulum improved with QDMC algorithm. QDMC is more effective way to enhance the stability of time domain performance of the driven pendulum system.

In future, one can compare these results with sliding mode controller. Various control methods can be designed for driven pendulum such as Fuzzy control, MRAC, etc.

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