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Analysis and Circuit Realization of a New Autonomous Chaotic System

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ABSTRACT

Recently many chaotic circuits are designed to generate chaos. The ability to synchronize chaotic circuits opens number of ways to use them in signal masking applications. This paper gives a brief overview of three dimensional autonomous chaotic systems which is represented by simple quadratic differential equations. A new autonomous chaotic system is introduced and compared with some existing systems. The new system has three constant variables, two equilibrium points and generates typical chaotic attractors. The new system is simpler than other system because it contains single multiplier term. System is analyzed using Matlab-Simulink and circuit is simulated using Orcad PSpice programs utilizing simple electronic components. The simulation gives satisfactory results.

Keywords: Chaotic systems, chaotic circuits, chaotic attractors, chaotic masking.

Introduction:

Chaos is a very interesting complex nonlinear phenomenon which has been intensively studied in the last four decades within the science, mathematics and engineering communities. Recently, chaos has been found to be very useful and has great potential in many technological disciplines such as in information and computer sciences, power systems protection, biomedical systems analysis, flow dynamics and liquid mixing. Chaotic signals can be generated with simple electronic circuits. Chaotic signal depends very sensitively on initial conditions, have unpredictable features and wide band spectrum. Chaotic systems are deterministic, highly sensitive to system parameters. In 1990 Pecora and Carroll suggest that chaotic systems possess a self synchronization property, hence chaotic circuits are used to transmit secure messages [1 - 3]. Chaos based system is the cheaper alternative of conventional spread spectrum system for security purpose.

Recently many chaotic systems are designed to generate chaos for various applications in secure communication, signal masking and encryption [4− 6]. The first chaotic system is the Lorenz system introduced in 1963. It is describes by three first order differential equations.

$$\begin{aligned} \dot{x} &= s(y - x) \\ \dot{y} &= r\,x - y - x\,z \\ \dot{z} &= x\,y - b\,z \end{aligned} \tag{1}$$

Where s, r and b are constant parameters. Lorenz gives two scroll butterfly chaotic attractor for s = 10, r = 28 and b = 8/3 .Lorenz system has only two quadratic nonlinearities. Lorenz system is an example of elegant way of generating chaos. It confirms that sensitivity to initial condition and the presence of period-doubling cycles leading to chaos [7]. Rossler in 1979 invented a series of chaotic systems. The most famous of these is represented by three ordinary differential equations.

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + a\,y \\ \dot{z} &= b + z\,(x-c) \end{aligned} \tag{2}$$

Where a = 0.2, b = 0.2 and c =5.7 are constant parameters .Rossler attractor has a single fold band structure and it is the simplest topological structure for a three dimensional quadratic autonomous chaotic system with single nonlinear multiplier term x z [8, 9]. In 1994 J.C. Sprott suggests 19 cases of chaotic systems: case A-S with five linear terms and two nonlinear terms. All systems have algebraic simplicity and thus serve as better example of chaotic flows [10]. In 2002 Lu, Chen and Zhang gave the new chaotic system which satisfy the condition $a_{12}a_{21} = 0$, where a_{12} and a_{21} are the corresponding elements in the constant matrix $A = [a_{ij}]_{3\times3}$ for the linear part of the system [11]. Lorenz system satisfy the condition $a_{12}a_{21} > 0$ where's Chen's system satisfy $a_{12}a_{21} < 0$.In 2004 Lu,Chen and Cheng introduced a new chaotic system of three-dimensional quadratic autonomous ordinary differential equations, which can display two 1-scroll chaotic attractors simultaneously, with only three equilibria, and two 2-scroll chaotic attractors simultaneously, with five equilibria. The system equation is given as

$$\begin{aligned} \dot{x} &= -\frac{ab}{a+b}x - y\,z + c \\ \dot{y} &= a\,y + x\,z \\ \dot{z} &= b\,z + x\,y \end{aligned} \tag{3}$$

Where a, b and c are constant parameters. The system displays two 1-scroll chaotic attractor for a = −10, b = −4 and c = 18.1 and when a = −10, b = −4 and c = 0 then system display two complex 2-scroll chaotic attractor. The concept of generalized Lorenz system is also extended to a new class of systems in canonical form [12]. In 2000 J.C. Sprott gives a new class of chaotic circuit defined by equation written in the form called jerk equation

$$\dddot{x} + A\ddot{x} + \dot{x} = G(x) \tag{4}$$

Where G(x) is a nonlinear function. The circuits are easily realized by resistors, capacitors and operational amplifiers [13, 14]. Elwakil and Kennedy suggested modified model of Lorenz system which contains no multipliers [15]. In 2010 Sprott gives Simple Autonomous chaotic circuit which employs an op-amp as a comparator to provide signum nonlinearity [11, 12]. The circuit is described by jerk equation

$$\dddot{x} + A\ddot{x} + B\,\dot{x} = C\,(\mathrm{sgn}(x) - x \tag{5}$$

In 2010 Pehlivan introduced new 3D chaotic system with six terms including two quadratic terms in a form very similar to the Lorenz, Chen, Lu and Yang. The system has zero initial condition thus its circuit implementation is easy .The system is defined by nonlinear ordinary differential equation

$$\begin{aligned} \dot{x} &= y - x, \\ \dot{y} &= ay - x\,z \\ \dot{z} &= x\,y - b \end{aligned} \tag{6}$$

Where a, b are constant parameters. The system generates complex 2- scroll chaotic attractors simultaneously for a = b = 0.5 [17]. The values of parameters, Equilibrium points and Eigen values of all such systems are listed in table 1 of section 2.

As chaotic systems are easily realizable by simple electronic components and recently many new systems are introduced, this paper presents the overview of some chaotic systems and presents a new system which displays a typical chaotic attractor. The system equation of new system and its analysis is given in section 2. System is simulated on Matlab-Simulink and phase portraits of the new system are obtained. Section 3 presents the electronic circuit schematic of the new system using Orcad Pspice simulation. Finally in Section 4 conclusion is given.

2. A new chaotic system and its analysis

The new system is described by following three ordinary differential equations

Where x, y and z are the state variables.

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= z \\ \dot{z} &= -a\,x - b\,y - c\,z - x^2 \end{aligned} \tag{7}$$

System equation can also be written as jerk equation

$$\dddot{x} + c\,\ddot{x} + b\,\dot{x} + ax + x^2 = 0 \tag{8}$$

The new system has one quadratic term and three positive real constants a, b and c. The system displays a typical chaotic attractor when a = 1, b = 1.1 and c =0.42.The new system has two equilibrium points (0, 0, 0) and (−1, 0, 0) to yield fixed points a>0.The Jacobian of the system is

$$J=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2x-a & -b & -c \end{bmatrix} \tag{9}$$

For (0, 0, and 0), the Jacobian becomes

$$J=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -b & -c \end{bmatrix} \tag{10}$$

The eigenvalues are obtain by solving the characteristic equation, $|\,J-\lambda I\,|=0$ which is

$$\lambda^3 + c\,\lambda^2 + b\,\lambda + a = 0 \tag{11}$$

Yielding eigenvalues of $\lambda_1 = -0.745$, $\lambda_2 = 0.162 + 1.147i$, $\lambda_3 = 0.162 - 1.147i$ for a = 1, b=1.1 and c =0.42.

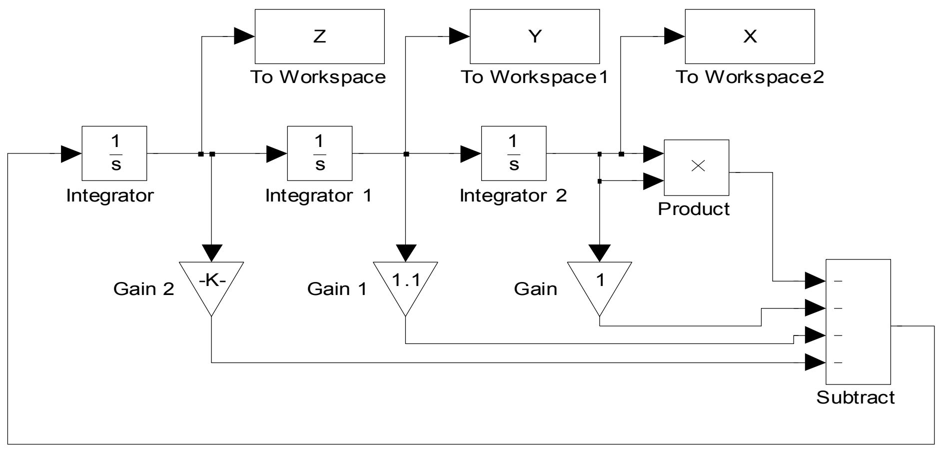
For the equilibrium point (−1, 0, 0), the Jacobian becomes

$$J=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a+2 & -b & -c \end{bmatrix} \tag{12}$$

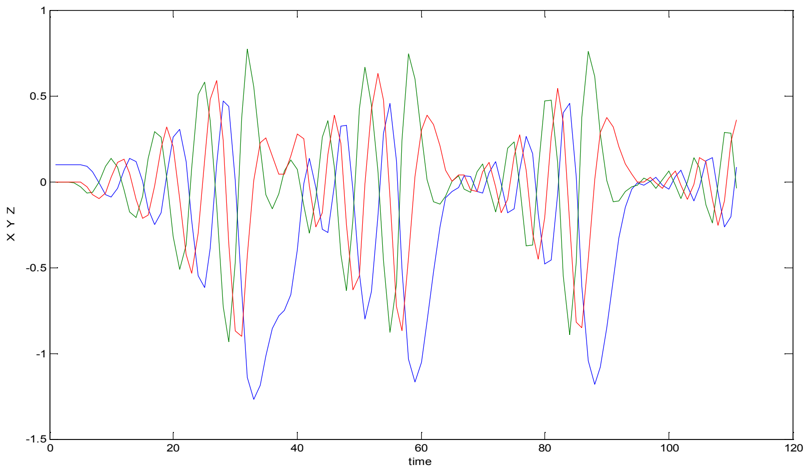
And the characteristic equation is

$$\lambda^3 + c\,\lambda^2 + b\,\lambda + a - 2 = 0 \tag{13}$$

Yielding eigenvalues of $\lambda_1 = 0.589$, $\lambda_2 = -0.504 + 1.20i$, $\lambda_3 = -0.504 - 1.20i$ for a = 1, b=1.1 and c =0.42.Some eigenvalues have positive real parts this implies chaos [18]. The system model is simulated in Matlab-Simulink as shown in Figure 1(a) and the time domain plots of x, y; z chaotic signals are shown in Figure1 (b).

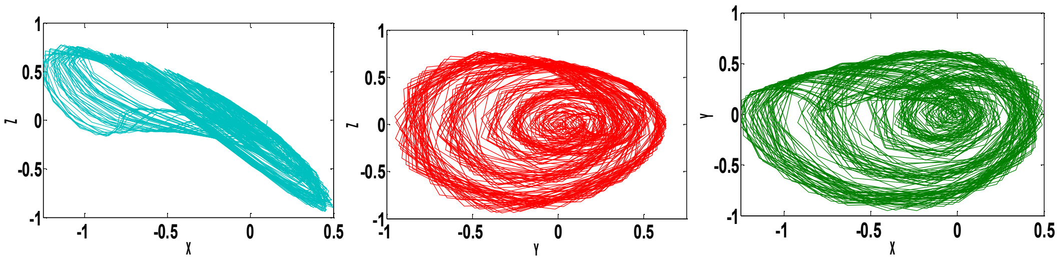


a



(b)

Figure 1. (a) Matlab – simulink model of new system, (b) Plot of x, y and z in time domain.



(a) (b) (c)

Figure 2. (a) X y, (b) y z, and (c) x z chaotic attractors

The new system is designed for initial condition x = 0.1, y = 0 and z = 0. The system is realized with three integrators; one multiplier and one subtractor block in Matlab-Simulink. The x y, y z and x z chaotic attractors of new system are shown in figure 2.

The new system does not belong to generalized Lorenz –like system and it has typical chaotic attractors.

Table 1. System Equation, Equilibria and Eigenvalues for different Chaotic Systems.

System	Equation	Parameter	Equilibria	Eigenvalues
Lorenz system	$\dot{x}$= s(y − x) $\dot{y}$= r x − y −x z $\dot{z}$= x y − b z	s=10 r = 28 b = 8/3	(0,0,0), ($\pm 6\sqrt{2}$, $\pm 6\sqrt{2}$, 27)	−22.8277,−2.6667,11.8277, −13.8546,0.0940±0.1945i
Rossler system	$\dot{x}$ = −y −z, $\dot{y}$ = x + ay $\dot{z}$ = b +z(x−c)	a = 0.2 b = 0.2 c = 5.7	(0.0070,−0.0351, 0.0351) (5.6930,−28.4649,28.4649)	−5.6870,0.0970±0.995i, -4.6×10^{-6},0.1930±5.4280i
Lu, Chen And Cheng System	$\dot{x} = -\frac{ab}{a+b}x - yz + c$ $\dot{y}$ = ay +xz (3) $\dot{z}$ = b z + x y	a =−10 b = −4 c = 18.1	(−6.335,0,0) ($2\sqrt{10}$,±4.7829, ±7.5624)	−14.0094,0.0094,2.8571, −13.3021, 1.0796±8.223i
Pehlivan system	$\dot{x}$= y −x, $\dot{y}$ = ay − xz $\dot{z}$ = xy −b	a =0.5 b = 0.5	($\pm\sqrt{0.5}$, $\pm\sqrt{0.5}$,0.5)	−1,0.25±0.9682i
New system	$\dot{x}$ = y $\dot{y}$ = z $\dot{z} = -ax - by - cz - x^2$	a = 1 b = 1.1 c =0.42	(0, 0, 0) (−1, 0, 0)	−0.745, 0.162 ± 1.147i, 0.589 , −0.504 ± 1.20i

3. Circuit Realization on ORCAD- PSpice

Circuit realization of new system is shown in figure 3. The circuit is easily implemented with 5 op-amps, 3 capacitors, one multiplier and 10 resistors. The new system is simpler than other system because it requires only one multiplier. Circuit is designed for parameter values a = 1, b = 1.1 and c = 0.42 .The initial condition is x = 0.1, y = 0 and z = 0.

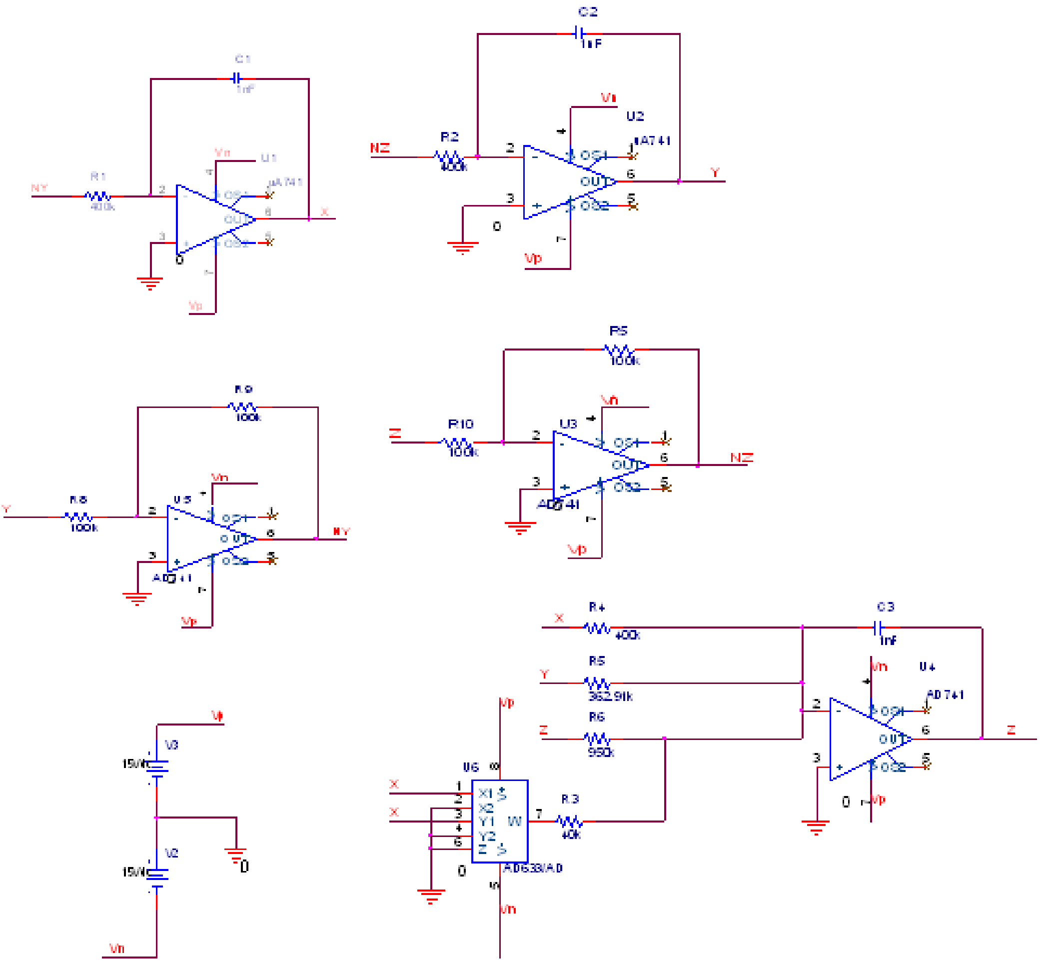
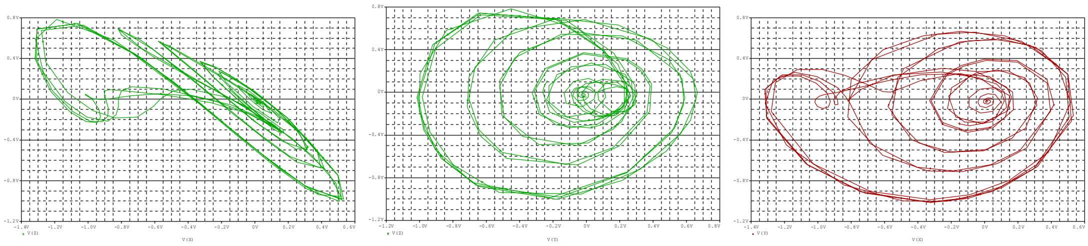


Figure 3. Circuit Schematic of new system.

Circuit Equation for the new system is given below.

$$\begin{aligned} \dot{x} &= \frac{1}{C1R1}\,y \\ \dot{y} &= \frac{1}{C2R2}\,z \\ \dot{z} &= -\frac{1}{C3R4}\,x - \frac{1}{C3R5}\,y - \frac{1}{C3R6}\,z - \frac{1}{10C3R3}\,x^2 \end{aligned} \qquad (14)$$

AD741 op-amp and AD633/AD multiplier is used with $R_1 = R_2$ =400k, R_3 = 40k, R_4 = 400k, R_5 = 362.91k and R_6 = 950k.The values of C_1, C_2, C_3 is 1nF. The Orcad Pspice results are shown in figure4.



(a) (b) (c)

Figure 4. Pspice simulation result of new system (a) x y, (b) y z, and (c) x z chaotic attractors.

D.C. Power supply of ±15V is used for Op-amp and multiplier IC AD633.Plots of Chaotic Attractors obtained with Orcad PSpice is same as the plots obtain with Matlab-Simulink.

4. Conclusions

Over the last decade many chaotic circuits are designed to generate chaos from three dimensional autonomous ordinary differential equations. Synchronization of chaotic system is the most emerging topic for the researchers. This article gives the comparative overview of some chaotic systems in terms of system equations, parameters, equilibria and eigen values. A new chaotic system is introduced which generates typical chaotic attractors with two equilibrium points. The new system is simulated using Matlab-Simulink and Orcad-Pspice program. The new system is simple to design because it contains single multiplier term. This paper presents the basic ideas and techniques to generate chaos purposely and helps to investigate much simpler three dimensional quadratic autonomous system.

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