Fuzzy Mutated Evolutionary Programming Based Algorithm for Multi-Objective Reactive Power Optimization

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Abstract

This paper presents an efficient and simple approach for solving the Multi-objective reactive power optimization problem. Fuzzy Logic has been applied in combination with Evolutionary Programming (EP). In order to have a better convergence in EP the mutation process is incorporated with Fuzzy Logic which leads to an improved technique called as Fuzzy Mutated Evolutionary Programming (FMEP). The solution method determines the optimal setting of the reactive power sources to minimize the losses and cost of the VAR sources and to improve the voltage profile. The proposed method has been tested on a standard 9 bus system and IEEE 30 bus system. The result shows the good performance of the proposed algorithm.

Keywords: Multi-Objective Reactive Power Optimization, Loss minimization, Fuzzy logic, Evolutionary programming.

Introduction

Reactive power optimization is an important aspect for the electric power system’s economical and safe operation. Reasonable distribution of reactive power compensation is the precondition of realizing voltage control. It can reduce the power loss and improve the voltage quality, and is also important for improving the performance of the system. Reactive power optimization is an uncertain non-linear integral programming problem [1-3]. This problem basically deals with determining adequate location and size of shunt capacitor/reactor bank.

In this paper, the objective function is a linear combination of several factors, such
as: investment for VAR sources, voltage deviation and transmission losses, subjected to operation constraints such as voltage profile. The problem is of complex as it has multiple objectives to be minimized simultaneously [4]. The first objective is the minimization of operation cost by reducing real power loss and improving voltage profile. The second objective is to minimize the cost of reactive power sources. So this problem is stated as a multi-objective reactive power optimization problem (MRPO).

Traditional Single-objective Optimization Algorithms like Linear programming usually provide a unique optimal solution. On the contrary, Multi-objective Evolutionary Algorithms (MOEA) independently and simultaneously can optimize several parameters tuning most traditional constraints into new objective functions.

Evolutionary programming based algorithms are increasingly applied for power system optimization problems in recent years. Major drawbacks of the problem are inconsistent convergence, large number of iterations. Since it uses a kind of random search method, it may lead to slow convergence when the number of decision variables in the problem is more [5]. In the recent trends, there has been an increasing interest in the application of fuzzy model. Fuzzy logic has been applied in combination with EP. This gives promising results especially in cases where the processes are too complex. For smooth and fast convergence in EP the mutation process is improved by fuzzy logic strategy leading to an improved EP technique termed as Fuzzy Mutated Evolutionary Programming (FMEP). This paper presents an efficient and simple approach for solving the multi-objective reactive power optimization using FMEP.

List of Symbols

\begin{itemize}
\item \( F_1 \)  \hspace{1cm} The total required investment
\item \( F_2 \)  \hspace{1cm} The total transmission active losses of the power system in MW
\item \( F_3 \)  \hspace{1cm} The maximum voltage deviation from the desired value (p.u)
\item \( K_1, K_2, K_3 \)  \hspace{1cm} Penalty factors
\item \( F_{1m} \)  \hspace{1cm} The maximum amount available for investment
\item \( V_i \)  \hspace{1cm} Actual voltage at busbar i (pu)
\item \( V_i^* \)  \hspace{1cm} Desired voltage at busbar I (p.u)
\item \( B_i \)  \hspace{1cm} The compensation at bus bar I measured in MVAr
\item \( B_m \)  \hspace{1cm} The absolute value of the maximum amount of compensation in MVAr allowed at a single bus bar of the system
\item \( \alpha \)  \hspace{1cm} Cost per MVAr of a capacitor bank
\item \( \gamma \)  \hspace{1cm} Cost per MVAr of a reactor
\item \( n \)  \hspace{1cm} number of bus bars in the electric power system.
\item \( P_g \)  \hspace{1cm} The total active power generated in MW
\item \( P_d \)  \hspace{1cm} The total load of the system in MW
\item \( V_k \)  \hspace{1cm} Voltage magnitude at node k
\item \( Y_{ki} \)  \hspace{1cm} Admittance matrix entry corresponding to node k and i
\item \( \delta_k \)  \hspace{1cm} Voltage phase angle at node k
\end{itemize}
\( \theta_{ki} \) Phase admittance matrix entry corresponding to node k and i
\( P_k \) Active power injected at node k
\( Q_k \) Reactive power injected at node k

**Formulation of Multi-Objective Reactive Power Optimization Problem**

A general Multi-objective Optimization problem includes a set of n decision variables, a set of k objective functions, and a set of m restrictions [6-7]. Objective functions and restrictions are functions of decision variables. This can be expressed as:

\[
\text{Optimize } Y = F(X) = [F_1(X) F_2(X) \ldots F_k(X)]
\]

Subjected to

\[
e(X) = [e_1(X) e_2(X) \ldots e_m(X)] \geq 0
\]

Where
\( X = [x_1 x_2 x_3 \ldots x_m] \in X \)
\( Y = [y_1 y_2 y_3 \ldots y_k] \in Y \)

Here X is known as decision vector and Y as objective vector. X denotes the decision space and the objective space are denoted by Y. Depending on the problem at hand “optimize” could mean minimize or maximize. The set of restrictions \( e(X) \geq 0 \) determines the set of feasible solutions \( X_f \) and its corresponding set of feasible objective vectors \( Y_f \). From this definition, it follows that every solution consists of an n X’s, that yields an objective vector Y, where every X must satisfy the set of restrictions \( e(X) \geq 0 \). The optimization problem consists in finding the X that has the “best”.

The multi-objective reactive power optimization is also of the above form and can be stated as:

The objective function:

\[
\text{Min } F = K_1F_1 + K_2F_2 + K_3F_3
\]

\[
F_1 = \sum_{i=1}^{n} k_i B_i
\]

\( k = \alpha \text{ if } 0 \leq B_i \leq B_m \)
\( \gamma \text{ if } -B_m \leq B_i \leq 0 \)

Subjected to:
\( F_1 \leq F_{1m} \)
\( F_2 = P_g - P_d \geq 0 \)
\( F_3 = \max_i (V_i - V_i^*) = \|V - V^*\| \geq 0 \)

The summary of the problem is

\[
\text{Min } F = [F_1 F_2 F_3]
\]

Where
\[
F = \left[ \sum_{i=1}^{n} k_i B_i \right] \left[ P_g - P_d \right] \left\| V - V^* \right\|_2
\]

Subjected to \( F_1 \leq F_{1m} \) and the load flow equations.
\[ P_k = V_{ki} \sum_{i=1}^{n} Y_{ki} V_i \cos(\delta_k - \delta_i - \theta_{ki}) \]
\[ Q_k = V_{ki} \sum_{i=1}^{n} Y_{ki} V_i \sin(\delta_k - \delta_i - \theta_{ki}) \]

In order to represent the amount of reactive compensation to be allocated at each bus bar \( i \), an unknown vector \( B \) has been used as a decision vector. It indicates the size of each reactive bank in the power system.

\[ B = [B_1, B_2, \ldots, B_n] \]
\[ B_k \in \mathbb{R} \]
\[ |B| \leq B_m \]

(10)

The solution of the above problem consists of a set of decision vectors \( B \) for which the corresponding objective function \( F \) can be improved in any dimension without any violation of the constraints. The set of decision vectors are known as Pareto optimal \((P)\). The corresponding set of objective function can be calculated and are known as Pareto Front \((PF)\). It is normally enough to find Pareto Optimal set \((P_{known})\) with its corresponding Pareto Front, close enough to the optimal solution.

**Solution using Evolutionary Programming**

Evolutionary programming [Yang et al, 1996] searches for the optimal solution by evolving a population of candidate solutions over a number of generations or iterations. The evolution of solutions is carried out through mutation by Gaussian distribution and Competitive selection. The steps for the solution using evolutionary programming are follows [8-9]:

*Initialization of parent population*

The initial population \( I_{ij} = [V_{ij}, V_{i2} \ldots V_{ij}, T_1, T_2, \ldots, T_n, Q_{i1}, Q_{i2} \ldots Q_{ik}] \) are generated randomly within the feasible range. The distributions of the initial trial parents are uniform.

Where,
\[ i = 1, 2 \ldots N_p, \]
\[ j = 1, 2 \ldots N_V, \]
\[ k = 1, 2 \ldots N_q \]

Evaluate the fitness for each individual variable and store the maximum fitness value as \( f_{\text{max}} \).

*Mutation*

The initial parent population produces ‘\( N_p \)’ number of offspring vectors from each parent \( I_{ij} \) by adding a Gaussian random variable with zero mean and a standard deviation proportional to the scaled values of the parent trial solution. After adding a Gaussian random number to parents, the element of offspring may violate the limit constraints. After correcting the limit violations obtain the fitness value for each individual in the offspring population.
**Competition and Selection**

The Np parent trial vector $I_{ij}$ and their corresponding offspring $I_{ij}'$, contend to survive within the competing pool. After competing, the 2Np trial solutions, including the parents and the offspring, are ranked in descending order of the score obtained. The first Np trial solutions will survive and are transcribed along with their fitness functions into the survivor set as the basis of the next generation.

**Next Generation**

Mutation, Competition and Selection are repeated until the maximum generation or iteration count is reached. The best solution at the end of the process gives the optimal solution.

**Solution using Fuzzy Mutated Evolutionary Programming**

The fuzzy logic has been incorporated with evolutionary programming to solve various power system problems [10-11]. In this particular incorporation the mutation process is improved by fuzzy logic strategy leading to an improved evolutionary programming technique termed as Fuzzy Mutated Evolutionary Programming (FMEP).

The value of the variance in the mutation process depends on three factors:

$$\frac{f_{pi}}{f_{pmax}}$$

$$\beta$$

$$V_{max} - V_{min}$$ and $$Q_{max} - Q_{min}$$

The first factor has the major influence on the value of variance. If the value of it is low then the width of the normal distribution curve shrinks and if it is high the width of the normal distribution curve increases. The second factor $\beta$ is normally fixed throughout the process. But it will lead to premature convergence as a fixed mutation scaling factor may not converge. Therefore an adoptive scaling factor has to be used. The third factor is the search range between $V_{max}$ and $V_{min}$ and $Q_{max}$ and $Q_{min}$. The search range will vary in each iteration. The value of $\beta$ and the search range plays an important role in better convergence. The relationship between the factors seems to be arbitrary, complex and ambiguous to determine, hence fuzzy logic strategy where the search criteria are not precisely bounded would be appropriate than a crisp relation. Thus a fuzzy logic strategy has been implemented in order to obtain an adoptive scaling factor or the variance. As the fuzzy logic has been incorporated with the mutation process of Evolutionary Programming the technique is termed as Fuzzy Mutated Evolutionary Programming.

The following steps are involved in the fuzzy implementation in mutation process:

The inputs and outputs of the fuzzy logic is decided. The inputs are $\frac{f_{pi}}{f_{pmax}}$ and $V_{max} - V_{min}$ or $Q_{max} - Q_{min}$. The second input is the active search range pertaining to each element of parent individual in the present iteration from any of its corresponding maximum or minimum limits. The scaling factor $\beta$ is resolved in the fuzzy logic control. The output of the fuzzy logic control is the variance.

Fuzzification of input and output using triangular membership is done using five fuzzy linguistic sets.
Fuzzy rules are formulated. Defuzzification of output using centroid method is performed. The centroid C is scaled to obtain variance value of each element in the parent population.

**Sample System Studies and Results**
The proposed Fuzzy Mutated Evolutionary Programming algorithm has been implemented on a standard 9-bus system and IEEE 30-bus system. The EP based algorithms are implemented in MATLAB 6.5 and the numerical tests are carried out on Pentium IV, 2.5 GHz processor.

**Standard 9-bus system**
The one line diagram of the system is shown in Figure 1. The system consists of 3 Generator buses, 3 load buses and 9 transmission lines. The reactive power compensations are at buses 5, 6 and 8. The lower voltage magnitude is 0.95 and upper voltage limit is 1.1 for all PV buses and 1.05 for all PQ buses.

![Figure 1: Standard 9- Bus System.](image)

**Table 1:** Optimum Results using EP and FMEP for standard 9- bus system.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>EP</th>
<th>FMEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_2$ (p.u)</td>
<td>1.033</td>
<td>1.034</td>
</tr>
<tr>
<td>$V_3$ (p.u)</td>
<td>1.012</td>
<td>1.012</td>
</tr>
<tr>
<td>$Q_{inj.5}$ (MVAr)</td>
<td>139.78</td>
<td>131.06</td>
</tr>
<tr>
<td>$Q_{inj.6}$ (MVAr)</td>
<td>117.228</td>
<td>101.92</td>
</tr>
<tr>
<td>$Q_{inj.8}$ (MVAr)</td>
<td>104.87</td>
<td>111.596</td>
</tr>
<tr>
<td>Objective Value</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$F_1$ (MVAr)</td>
<td>361.89</td>
<td>344.562</td>
</tr>
<tr>
<td>$F_2$ (MW)</td>
<td>8.34</td>
<td>8.308</td>
</tr>
<tr>
<td>$F_3$ (p.u)</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Time (sec)</td>
<td>27.33</td>
<td>27.00</td>
</tr>
</tbody>
</table>
The parameters used in the EP algorithm are as follows:
1. Population size, \( N_p = 200 \)
2. Scaling factor \( \beta = 0.001 \) (constant).
3. No. of iterations = 25

The convergence characteristic is shown in Figure 2. The convergence characteristic is drawn with minimum fitness value from the combined population across the iteration index. From Figure 2 it can be observed that the optimal solution is obtained at the End of 25\(^{th}\) iteration. There is no significant convergence after the 8\(^{th}\) iteration. The optimal solution is given in Table 1.

**Table 2. Data for Fuzzy Mutation**

<table>
<thead>
<tr>
<th>FUZZY SET</th>
<th>INPUT 1</th>
<th>INPUT 2</th>
<th>INPUT 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Xsmall</td>
<td>0.00001 to 0.00004</td>
<td>0.1% - 15 %</td>
<td>0.001 – 0.005</td>
</tr>
<tr>
<td>Small</td>
<td>0.00003 to 0.006</td>
<td>13% - 50 %</td>
<td>0.004 - 0.06</td>
</tr>
<tr>
<td>Medium</td>
<td>0.005 to 0.05</td>
<td>45% - 65%</td>
<td>0.04 – 0.08</td>
</tr>
<tr>
<td>Large</td>
<td>0.03 to 0.5</td>
<td>60% - 80%</td>
<td>0.075 – 0.09</td>
</tr>
<tr>
<td>Xlarge</td>
<td>0.4 to 1</td>
<td>80% - 100%</td>
<td>0.085 – 0.1</td>
</tr>
</tbody>
</table>

**Figure 2.** Convergence characteristics using EP and FMEP for Standard 9-bus system

The problem formulated was implemented with new proposed FMEP method. The data for fuzzy mutation process is given in Table 2. The convergence characteristic with the proposed FMEP is shown in Figure 2. From Figure 2 it is observed that the
fitness function value converges smoothly to optimum value without any abrupt oscillations, thus ensuring convergence reliability of the proposed FMEP. The optimal solution obtained with the proposed method is given in Table 1. The results were found to be better than simple EP.

When the results obtained by using EP and FMEP were compared FMEP shows better convergence than EP. Though both the algorithm has fetched a voltage deviation zero and the losses as 8.3 FMEP has consistently reduced the cost of investment.

**IEEE 30-bus system**

Figure 3 shows the IEEE 30-bus system which consists of 6 generator buses, 24 load buses and 41 transmission lines of which 4 branches (6-9), (6-10), (4-12) and (28-27) are with tap setting transformer. The transmission line parameters of this system and the base loads are taken out from [1]. The lower voltage magnitude is 0.95 p.u. and upper voltage limit is 1.1 p.u. for all PV buses and 1.05 p.u. for all PQ buses. The reactive power compensations are at 10 and 24. The lower limit of transformer tap setting is 0.95 p.u. and the upper limit is 1.1 p.u.. The reactive power sources lower limit and upper limits are -0.12 p.u. and 0.36 p.u. respectively.

The IEEE 30-bus system is implemented with the proposed Fuzzy Mutated Evolutionary programming (FMEP) and Evolutionary Programming (EP). To prove the effectiveness of the algorithm, the relationship between the best fitness value of the results and the average fitness are plotted against the generation number in Figure 4. From the figure it is observed that the proposed method converges smoothly. From the table 3 it is observed that the proposed method keeps all the control values with the limits.

![Figure 3. IEEE 30-bus system](image-url)
Table 3. Optimum Results using EP and FMEP for IEEE 30- bus system

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>EP</th>
<th>FMEP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Control Variables</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$V_1$ (p.u)</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>$V_2$ (p.u)</td>
<td>1.0384</td>
<td>1.0265</td>
</tr>
<tr>
<td>$V_5$ (p.u)</td>
<td>1.0206</td>
<td>1.0432</td>
</tr>
<tr>
<td>$V_8$ (p.u)</td>
<td>1.0457</td>
<td>1.0416</td>
</tr>
<tr>
<td>$V_{11}$ (p.u)</td>
<td>0.9679</td>
<td>1.0172</td>
</tr>
<tr>
<td>$V_{13}$ (p.u)</td>
<td>0.9945</td>
<td>1.0865</td>
</tr>
<tr>
<td>$T(6-9)$</td>
<td>1.0376</td>
<td>1.0012</td>
</tr>
<tr>
<td>$T(6-10)$</td>
<td>0.9924</td>
<td>0.9865</td>
</tr>
<tr>
<td>$T(4-12)$</td>
<td>1.0424</td>
<td>1.0226</td>
</tr>
<tr>
<td>$T(27-28)$</td>
<td>0.95</td>
<td>1.05</td>
</tr>
<tr>
<td>$Q_{inj.10}$ (MVAr)</td>
<td>0.1386</td>
<td>0.1554</td>
</tr>
<tr>
<td>$Q_{inj.24}$ (MVAr)</td>
<td>0.0405</td>
<td>0.0279</td>
</tr>
<tr>
<td>$F_2$ (MW)</td>
<td>5.01</td>
<td>4.99</td>
</tr>
<tr>
<td>$F_3$ (p.u)</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>Time (sec)</td>
<td>28.33</td>
<td>27.89</td>
</tr>
</tbody>
</table>

Figure 2. Convergence characteristics using EP and FMEP for IEEE 30-bus system

Conclusion
In this paper a new method based on Evolutionary Programming and Fuzzy logic is proposed for solving multi-objective reactive power optimization problem. For smooth and better convergence in EP the mutation process is improved by using fuzzy logic strategy leading to an improved algorithm. The method is successfully applied for optimization of three objective functions for optimal reactive power dispatch in the power system. The simulation results of the obtained method for the standard 9
bus system and IEEE 30 bus system are compared with results of the Evolutionary Programming, which shows better convergence. The new method has shown promising results in the multi-objective optimization problems with multiple constraints. The proposed algorithm has the potential to be applied to other power engineering problems.

References