Hybrid PID/H\(_\infty\) Control of Active Magnetic Bearings

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Abstract

Many advanced controllers have been proposed and designed for controlling active magnetic bearing (AMB) systems with uncertainty and disturbances. The main objectives of the advanced control design were to provide both stability and performance along with high degree of robustness. The simulation studies show that the H infinity controllers provide higher level of robustness, performance and stability for AMB systems. But the higher order controller requires higher starting current than conventional PID controllers. In this paper a comparison between the control effort required by H\(_\infty\) controller and PID controllers are done. The simulation studies shows that the PID control effort is less and smooth during starting of levitation but has more settling time and higher steady state current. H infinity controllers provide rapid action with higher starting current and less steady state current. Machines with high frequency of switching between on/off states will be suffered with higher power losses if it is levitated using AMBs under H infinity control. Better power dissipation mechanism need to be devised which adds to the cost of AMBs. In this paper a hybrid control approach is proposed in which the AMBs are started using a well tuned PID controller and sequentially switched to H infinity controller as it achieves its operating point. Simulation studies were done which shows that PID controller provides less starting current and H infinity controller provides high degree of disturbance rejection and robustness while running.

Keywords: H\(_\infty\) control; Loop Shaping; PID controller; magnetic bearings, hybrid control.
Introduction

Active magnetic bearing is an advanced mechatronic system which uses magnetic fields to levitate and support a rotating shaft without any physical contact [1],[14]. Since there exists zero friction between the moving and stationary part this technology is becoming popular than other bearing technologies. Because of its advantages, ordinary bearings have been replaced by magnetic bearings in many applications like high speed machining, fly wheel energy storage systems, turbo machineries, turbines etc [6],[7],[15]. But magnetic bearings are inherently unstable systems. There exists a right hand side pole as understood by the linearized model of the system as shown in equation (1).

\[
G(s) = \frac{K_i}{ms^2 - K_s}
\]  

(1)

Many modern control strategies have been proposed to stabilize system including fuzzy control, neural network control, adaptive vibration control [2],[13], H\(_\infty\) control \(\mu\) control [3],[14]. Analytical and experimental studies shows that the popular H\(_\infty\) controller can provide a perfect control over different kinds of disturbances[5],[11] with good stability and high degree of robustness. But in the implementation point of view the H\(_\infty\) controller attracts less interest since the controllers are of higher order and degree. More over simulation studies done in this work show that H infinity controllers require more starting current than PID controllers. But while running, the control effort is less in H\(_\infty\) control and rapid disturbance rejection is achieved than a PID controller. High starting currents will cause reduction in terminal voltage and also affect other machines connected to the supply. Higher starting current makes the power amplifier switches to be designed at high ratings. This increases the cost of design of power converters for AMB under H infinity control. More over higher power losses due to higher starting current may surpass the advantages provided by H infinity controller in large machines. In this paper as a solution for the above mentioned problem a hybrid control approach is proposed in which AMBs are started using a well tuned PID controller and when it achieves its operating point the controller is switched to H infinity controller. Analysis shows that a smooth transition is attained with less starting current and high degree of robustness and disturbance rejection while running.

The paper is organized as described below. The section II describes about the H\(_\infty\) controller synthesis for magnetic bearings, followed by the PID controller in section III. Section IV shows the simulation results and analysis of the H infinity control strategies with its comparison with ordinary PID control. Section V provides the analysis of the proposed hybrid control approach and simulation results.

Robust H\(_\infty\) control Theory

The robust controller is synthesized in order to make the H\(_\infty\) norm of the plant to be as low as possible. There are many techniques available for designing H\(_\infty\) controller [4-5],[8-12] In this work two sensitivity method using loop shaping is adopted. In order
to obtain this condition three weight functions are added to the plant for loop shaping. The weight functions are lead-lag compensators which can shape the frequency response of the system in the desired way. Loop shaping is done to make the frequency response of the plant with the weight functions to come in the desired manner. Loop shaping can be done in many ways. In loop shaping the parameters of the weight functions are changed to make the frequency response of the whole system to remain within limits. The control synthesis requires the plant transfer function, controller transfer function and the various weight functions to augment together. Thus an augmented plant model is made as shown in Fig (1).

![Figure 1: Augmented Plant model for the synthesis of $H\infty$ controller.](image)

The normalized frame of the plant is

$$\begin{bmatrix} w, e \\ w, u \\ w, y \\ e \end{bmatrix} = \begin{bmatrix} w, - w, G \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix}$$

(2)

Where, $U = K*e$

After determining the weights $W_i$ and $W_s$, the generalized plant can be determined as

$$P = \begin{bmatrix} w, - w, G \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & D_{11} & D_{12} \\ C_2 & D_{21} & D_{22} \end{bmatrix}$$

(3)

The equation (9) in terms of (7) & (8) is
Then the mixed sensitivity problem is to find a rational function controller $K(s)$ and make the closed loop system stable and satisfy

$$\gamma = \min \| P \| = \min \begin{bmatrix} W_nS \\ W_nR \\ W_nT \end{bmatrix} = 1$$  \hspace{1cm} (5)

Where $P$ is the transfer function from $w$ to $Z$ i.e

$$|P_{zw}| = \gamma$$  \hspace{1cm} (6)

Where $|P_{zw}| = \rho$ is the cost function. According to the minimum gain theorem, make the $H_\infty$ norm of $|P_{zw}|$ less than unity.

$$\min \| P_{zw} \| = \min \begin{bmatrix} W_nS \\ W_nR \\ W_nT \end{bmatrix} \leq 1$$  \hspace{1cm} (7)

Thus a stabilizing controller $K(s)$ is achieved by solving the algebraic Riccati equation.

The weight functions used for the synthesis is

$$W_N = \frac{s + w_b}{s + w_b + A}$$  \hspace{1cm} (8)

$$W_T = \frac{Ls+1}{2(0.5Ls+1)}$$  \hspace{1cm} (9)

Where $W_N$ is the performance weighting function, ‘$w_b$’ is the cut off frequency, ‘$M$’ is the gain for high frequency disturbances and ‘$A$’ is the gain for low frequency control signal and $L$ is a constant. $w_b$ is the frequency which differentiates the high frequency disturbance signal and the low frequency control signal. The plots, Fig 1 and Fig 2 show the nature of $\frac{1}{W_N(jw)}$, $S$ and $\frac{1}{W_T(jw)}$, $T$ to be satisfied for the synthesis of $H_\infty$ controller. This can be made by closely shaping the $\frac{1}{W_N(jw)}$ and $\frac{1}{W_T(jw)}$ plot.

The loop shaping technique consists of adjusting the various parameters of the weight functions.
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Figure 2: Required nature of frequency plots of $1/w_s$, $S$ for $H_\infty$ control synthesis.

Figure 3: Required nature of frequency plots of $1/w_t$, and $T$ for $H_\infty$ control synthesis.

The matlab commands ‘`augw`’ is used for augmenting the plant model and ‘`hinfsyn`’ command is used to synthesis the $H_\infty$ controller $K(s)$.

For an AMB prototype having the plant transfer function as

$$G(s) = \frac{27}{12.5s^2 - 188400} \quad (10)$$

The controller $k(s)$ is designed as

$$K(s) = \frac{7.56e012s^3 + 7.653e016s^2 + 9.354e018s + 8.559e018}{s^4 + 7.49e004s^3 + 2.249e009s^2 + 1.668e013s + 1.668e012}$$

taking the value of weight functions as
PID Control

A well-tuned PID controller is designed for levitating the bearing at initial start-up. The Simulink model of the PID controlled AMB plant is shown in Fig (4). Using Zeigler Nichols method, the coefficients of the PID controller are selected for minimum starting current and good running performance. The PID coefficient values of the AMB example (1) discussed in this paper are
\[ K_p = 1200 \]
\[ K_i = 205714 \]
\[ K_d = 63 \]

**Figure 4:** PID control of Active Magnetic Bearings.

Hybrid PID/H∞ Control

Fig (5) shows the starting characteristics of the AMB system under the PID control and H∞ control methods.

The plot of the control current as in Fig (6) shows that the higher performance of H∞ during starting is achieved by a higher and rapid change in current, which may be hard to realize in practice. The slope of rise of increase in current is very large than a PID control. More over the currents remains in saturation for a long time. Fig (7) & (8) shows that when a similar step change is given as disturbance, the H∞ controller brings the plant back to stable condition with less control effort when compared to PID controller.
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Figure 5: Starting response of AMB system with $H_\infty$ & PID control.

Figure 6: Control current requirement for $H_\infty$ & PID control.

The control effort requirement is given in Fig (9) & (10). The disturbance responses were simulated for various values of load ranging from 10% to 50%. PID controller which had shown good starting response for a step input change went to saturation and the control effort was more than $H_\infty$ controller.

Figure 7: Disturbance response with $H$ infinity control.
Figure 8: Disturbance rejection control current for $H_\infty$ control.

Figure 9: Disturbance response with PID control.

Figure 10: PID Disturbance rejection control current.
Hence it can be inferred that $H\infty$ controllers are well suited for disturbance rejection properties. In systems like magnetic bearings where operating conditions are unknown and any disturbance may occur, disturbance rejection property becomes the governing criterion for controller selection. In that perspective $H\infty$ controller is most suitable control technique. But starting currents also becomes a governing criterion when the system is having frequent start-up schedules. To reduce the control effort at starting a hybrid control action may be used as proposed in this paper. This method uses the advantages of the two popular control strategies PID control and $H\infty$ control. The hybrid control subsystem is shown in Fig (11), Fig (12). The controller switching is done at steady state operation of the system.

**Figure 11:** Hybrid PID/$H\infty$ control of Active magnetic Bearing.

**Figure 12:** Hybrid PID/$H\infty$ controller Subsystem.
Simulation result
The hybrid model given in fig (11) & fig (12) is simulated to see its effect during starting and normal running operations. A sequence of operation is simulated containing starting, switching of the controllers and disturbance. Fig (13) shows the response of the system during starting with PID control and after switching to H\(_\infty\). A smooth transition is observed at the switching time. When a disturbance signal is given during normal operation the system could maintain high robustness. The fig (14) shows the control effort required in the whole sequence of operation. The control effort is reduced during starting because of PID control and disturbance rejection current is less due to H\(_\infty\) control.

![Figure 13: Position response of Hybrid PID/H Infinity for sequence of starting, controller switching and disturbance rejection operations.](image1)

![Figure 14: Effect of Hybrid PID/H Infinity control for sequence of starting, controller switching and disturbance rejection operations.](image2)
Conclusion

In this paper a solution for reducing the requirement of high starting current for a robust $H_\infty$ controller is proposed. The proposed hybrid control model includes a well tuned PID controller for starting operations and a robust $H_\infty$ controller for higher robustness and performance while running at other operating conditions. A suitable switching logic will switch the controller from PID to $H_\infty$ control. Thereby a control mechanism which provides stable operation with high degree of robustness and performance along with least starting control effort can be achieved. The simulation results with PID, $H_\infty$, hybrid PID/$H_\infty$ shows that hybrid control is having better performance and is best suited for magnetic bearings with frequent starting operations.

References


