Modeling, Analysis and Simulation of Robust Control Strategy on a 10kVA STATCOM for Reactive Power Compensation

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Abstract

The STATCOM (STATic synchronous COMpensator) is being increasingly popular in power system applications. In general, reactive power compensation for power factor and stability of the grid system can be improved. A simple $d-q$ transformation and steady state and transient analysis are achieved to characterize the open loop system. The small signal scheme in H_∞ controller control the phase angle as well as modulation index of the switching pattern and with small perturbation of reference current (reactive current of load), the DC voltage nearly remains constant. The H_∞-controller design and implementation of the STATCOM is proposed without using independent control on DC-link voltage of the STATCOM. The H_∞-controller provides fast response of the STATCOM compensation of the reactive power and becoming the stable of the system. The pre-charge voltage of $600V$ on DC-link is chosen for tolerable transient variation of the relevant responses of the STATCOM. All responses are obtained trough MATLAB simulink tool box.

Index Terms: STATCOM, small signal model, H_∞ controller

Introduction

In recent years power systems have become very complex with interconnected long distance transmission lines. The interconnected grids tend to become unstable as the heavy loads vary dynamically in their magnitude and phase angle and hence power factor. Commissioning new transmission systems are extremely expensive and take

considerable amount of time to build up. Therefore, in order to meet increasing power demands, utilities must rely on power export/import arrangements through the existing transmission systems. Power electronic devices are gaining popularity for applications in the field of power transmission and distribution systems. The reactive power (VAR) compensation and control have been recognized [1] as an efficient & economic means of increasing power system transmission capability and stability. The FACTS (Flexible AC Transmission Systems) devices, such as STATCOM has been introduced more recently which employs a VSI with a fixed dc link capacitor as a static replacement of the synchronous condenser. In a traditional synchronous condenser, the field current of the synchronous motor controls the amount of VAR absorbed/injected and in a similar way; the firing instant of the 3-phase inverter controls the VAR flow into or out of the STATCOM. Large numbers of capacitor banks or any other passive elements are no more required. Only a fixed set of capacitor provides the required VAR control, with a rapid control of bus voltage and improvement of utility power factor. It offers several advantages over conventional thyristorised converters [2] in terms of speed of response. The penalty paid for this improvement is in terms of introduction of some harmonics, which requires separate handling using active filtration techniques. Moran et al [3] have shown in details how the utilization of SPWM techniques reduces harmonic distortion. It has also been shown that an increase of modulation index reduces the size of the link reactor and stress on switches which are significant issues in practical implementation. The modeling and analysis of STATCOM steady state and dynamic performance with conventional control method have been studied by Schauder and Mehta [4]. In [5] the dynamic responses and steady state behavior of STATCOM with SVPWM has been studied and the advantages of introducing SVPWM inverter with higher values of MI are highlighted. It is also shown that a PI Controller designed on the basis of a linearized model [6-8] of the STATCOM achieves very good transient response. The large interconnected power system is called upon to work with different values of 'α' the angle between the grid voltage and the fundamental component of STATCOM output voltage. Also the modulation index (M.I) m of the inverter is to be varied over a wide range. The inherently nonlinear system also experiences plant parameter variation over a wide range. In this work it is shown that the PI Controller designed on the basis of the plant model, linearized about a nominal point cannot handle the plant parameter variation over the entire range. As a remedy the plant parameter variation is included as plant uncertainty and a controller based on H_∞-optimization is used. This gives significant improvement in plant response on variation of m and α.

Starting with an established steady state open loop model of the STATCOM, the present paper goes on to develop closed loop models for investigating transient performance of the STATCOM. First, in Section II, general modeling of the STATCOM and study its open-loop responses. A small signal model is developed to establish the effect of the variation of the modulation index 'm' and the phase angle α. Thereafter, a closed loop control scheme has been proposed with two types of controller design, so that dc link voltage may be kept unchanged. This scheme is both an extension and a significant improvement of the scheme suggested by Cho et al [6].

Modeling of the STATCOM and Analysis

Operating principle

As is well known, the STATCOM is, in principle, a static (power electronic) replacement of the age-old synchronous condenser. Fig.1 (a) shows the schematic diagram of the STATCOM at PCC through coupling inductors. The fundamental phasor diagram of the STATCOM terminal voltage with the voltage at PCC for an inductive load as well as capacitive load in operation, neglecting the harmonic content in the STATCOM terminal voltage, are shown in Figs.1(b) and (c). Ideally, increasing the amplitude of the STATCOM terminal voltage $\vec{V}_{oa}$ above the amplitude of the utility voltage $\vec{V}_{sa}$ causes leading (capacitive) current $\vec{I}_{ca}$ to be injected into the system at PCC. $\vec{I}_{ca\_a}$, the real component of $\vec{I}_{ca}$, accounts for the losses in the resistance of the inductor coil and the power electronic converter. Ideally, if the system losses can be minimized to zero, $\vec{I}_{ca\_a}$ would become zero, and $\vec{I}_{ca}$ would be leading at perfect quadrature. Then, $\vec{V}_{oa}$, which is lagging and greater than $\vec{V}_{sa}$, would also be in phase with $\vec{V}_{sa}$. The STATCOM in such a case operates in capacitive mode (when the load is inductive)(as shown in Fig.1(b)). Similarly, the operation of the STATCOM can be explained to be inductive if the load is capacitive (as given in Fig.1(c)).
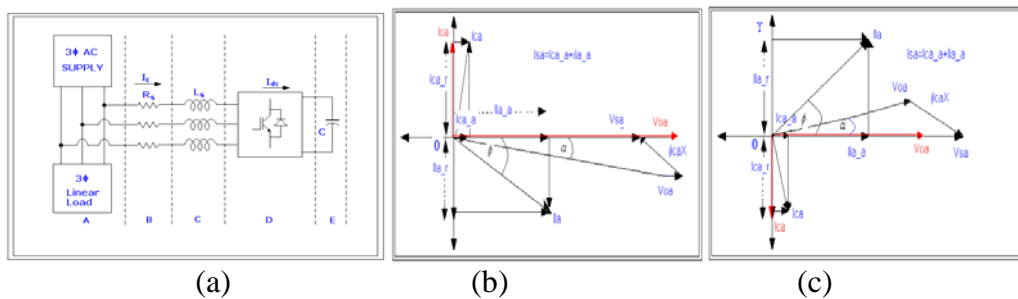

(a) (b) (c)

Figure 1: (a) Schematic diagram of STATCOM, (b) phasor diagram for inductive load operation and (c) phasor diagram for capacitive load operation.

Modeling

The modeling of the STATCOM, though well known, is reviewed in the lines below, for the sake of convenience. The modeling is carried out with the following assumptions:

1. All switches are ideal
2. The source voltages are balanced
3. R_s represents the converter losses and the losses of the coupling inductor
4. The harmonic contents caused by switching action are negligible

The 3-phase stationary *abc* coordinate vectors with 120^0 apart from each other

are converted into $\alpha\beta$ 2-phase stationary coordinates (which are in quadrature). The α axis is aligned with a axis and leading β axis and both converted into dq two-phase rotating coordinates. The Park's abc to dq transformation matrix is given in (1)

$$K = \sqrt{\frac{2}{3}} \begin{bmatrix} Cos(\varpi t) & Cos(\varpi t - 2\pi/3) & Cos(\varpi t + 2\pi/3) \\ Sin(\varpi t) & Sin(\varpi t - 2\pi/3) & Sin(\varpi t + 2\pi/3) \\ 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{1}$$

$$V_{s,abc} = \begin{bmatrix} V_{sa} \\ V_{sb} \\ V_{sc} \end{bmatrix} = \sqrt{\frac{2}{3}} V_S \begin{bmatrix} Sin(\omega t - \alpha) \\ Sin(\omega t - \alpha - \frac{2\pi}{3}) \\ Sin(\omega t - \alpha + \frac{2\pi}{3}) \end{bmatrix} \tag{2}$$

The actual proposed circuit is too complex to analyze as a whole, so that it is partitioned into several basic sub-circuits, as shown in Fig.1 (a). The 3-phase system voltage $V_{s,abc}$ lagging with the phase angle α to the STATCOM output voltage $V_{o,abc}$ and differential form of the STATCOM currents are defined in (2) and (3). where , V_s, ϖ, R_s and L_s have their usual connotations. The above voltages and currents are transformed into dq frame and given in (4) and (5)

$$L_s \frac{d}{dt}\left(i_{c,abc}\right) = -R_s i_{c,abc} + V_{s,abc} - V_{o,.abc} \tag{3}$$

$$V_{s,qdo} = V_s \begin{bmatrix} -Sin\alpha & Cos\alpha & 0 \end{bmatrix}^T \tag{4}$$

$$L_s \frac{d}{dt}\left(i_{cq}\right) = -R_s i_{cq} - w L_s i_{cd} + V_{sq} - V_{oq} \tag{5a}$$

$$L_s \frac{d}{dt}\left(i_{cd}\right) = w L_s i_{cq} - R_s i_{cd} + V_{sd} - V_{od} \tag{5b}$$

The switching function *S* of the STATCOM can be defined as given in (6) and the modulation index, being constant for a programmed PWM, is given by (7). m_c is the modulation conversion index and $\sqrt{\frac{2}{3}}$ is the multiplying factor for transformation of three phase stationary quantities to two phase dq rotating frame.

$$S = \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} = \sqrt{\frac{2}{3}} m_c \begin{bmatrix} Sin\ (wt\ ) \\ Sin\ (wt\ - \frac{2\pi}{3}) \\ Sin\ (wt\ + \frac{2\pi}{3}) \end{bmatrix} \tag{6}$$

$$MI = m = \frac{V_{o,peak}}{v_{dc}} = \sqrt{\frac{2}{3}} m_c \tag{7}$$

The STATCOM output voltages and *dq* transformation are given by,

$$V_{o.abc} = S v_{dc} \tag{8}$$

$$V_{o,qdo} = m_c \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T v_{dc} \tag{9}$$

The dc side current in the capacitor and its *dq* transformation may be written as,

$$i_{dc} = S^T i_{c,abc} \tag{10}$$

$$i_{dc} = m_c \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_{cq} & i_{cd} & i_{co} \end{bmatrix}^T \tag{11}$$

The voltage and current related in the dc side is given by

$$i_{dc} = C \frac{dv_{dc}}{dt} \tag{12}$$

Now replacing (11) in (12) yields

$$\frac{dv_{dc}}{dt} = \frac{m_c}{C} i_{cd} \tag{13}$$

The complete mathematical model of the STATCOM in *dq* frame is obtained as given in (14)

$$\frac{d}{dt}\begin{bmatrix} i_{cq} \\ i_{cd} \\ v_{dc} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_s}{L_s} & -w & 0 \\ w & -\dfrac{R_s}{L_s} & -\dfrac{m_c}{L_s} \\ 0 & \dfrac{m_c}{C} & 0 \end{bmatrix}\begin{bmatrix} i_{cq} \\ i_{cd} \\ v_{dc} \end{bmatrix} + \frac{V_s}{L_s}\begin{bmatrix} -Sin\,\alpha \\ Cos\,\alpha \\ 0 \end{bmatrix} \tag{14}$$

Steady State and transient Analysis

The steady state equations of $I_{cq}, I_{cd}, V_{dc}, P_c, Q_c$ are given in (15-16) and their responses with the parameters given in Table 1, are shown in Fig.3 and Fig.4.

$$I_{cq} = -\frac{V_s Sin\,\alpha}{R_s},\ I_{cd} = 0 \tag{15}$$

$$V_{dc} = \frac{V_s}{M}\left(Cos\,\alpha - \frac{wL_s}{R_s} Sin\,\alpha \right)$$

$$P_c = \frac{V_s^2}{2R_s}\big[1 - Cos\,(2\alpha) \big] \tag{16}$$

$$Q_c = \frac{V_s^2}{2R_s} Sin\,(2\alpha)$$

Table 1

Sl.No	Meaning	Symbol	Values
1	Fundamental Frequency	f_1	50 Hz
2	Fundamental Angular Frequency	ω_1	314 rad/sec
3	RMS line-to-line Voltage	V_s	415V
4	Effective coupling Resistance	R_s	$1.0\,\Omega$
5	Coupling Inductance	L_s	5.44mH
6	DC link capacitor	C_{dc}	$680\,\mu F$
7	Modulation Conversion Index	m_c	0.98 to 1.22
8	Resistive load	R_l	$23\,\Omega$
9	Inductive load	L_l	$60mH$, 2.06Ω

The open loop steady state responses to a step change in phase angle α of the reactive component of STATCOM current, I_{cq}, and the reactive VAR, Q_c, of the STATCOM in capacitive mode of operation are shown in Figs.2.(a) and (b) respectively. The corresponding transient responses at $\alpha = -10^0$ are given in Figs.3 (a) and (b) respectively.
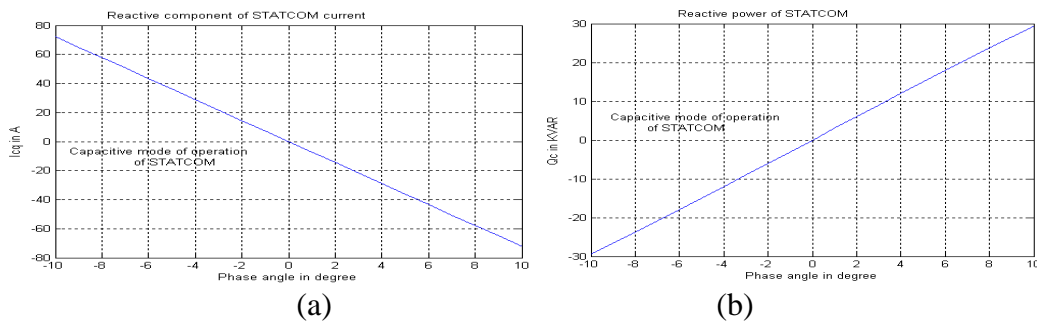

(a)   (b)

Figure 2: Steady state responses due to variation in α of STATCOM in capacitive mode: (a) Reactive component of STATCOM currents and (b) reactive power generated by STATCOM

The steady states values of I_{cq} and Q_c vary with variation in phase angle α. These values (as shown in Figs.2 (a) and (b) respectively) are $72A$ and $-29.45kVAR$ at $\alpha = -10^0$ in capacitive mode of operation of STATCOM. The corresponding transient responses (Figs.3 (a) and (b)) in generating (capacitive) mode of operation of STATCOM have high transients seem to generate $72A$ and $-29.45kVAR$ respectively after two and half power cycles. The high transient magnitude of i_{cq} of

the STATCOM unnecessarily increases the ratings of the STATCOM devices. The above calculations are on the basis of the mathematical model of the STATCOM. The practical STATCOM ratings will be limited by the choice of the switching devices (IGBT of SEMIKRON make of part no.SKM75GB123D) and the ripple current, the average current and the voltage rating of the DC link capacitor as also the current rating of the coupling inductor. The above values of the transient peak are actually out of range for the present set-up.

Recalling the introduction in section, the STATCOM is a voltage source behind a reactance connected in shunt to the PCC. The system demands fast injection of current, both at fundamental and harmonic frequencies. The STATCOM voltage should be capable of fast variation in order to cater to system needs. States differently, the STATCOM voltage should be a function of the current (i_{cq}^*) or reactive power (q_c^*) demanded by the power system. The most rudimentary control system structure required for this purpose is shown in Fig.4 (a). The control voltage, $v_{control}(t)$, is generated from a prior information of the series inductance, L_s, and the phase and magnitude of the PCC voltage, $v_s(t)$. If these are supplied to the reference generator, $f(.)$, only once at the beginning of the operation, it constitutes an open-loop control system. Although the series inductance may not vary significantly, the PCC voltage, $v_s(t)$, will definitely under go considerable variation in phase and magnitude. Some of the internal variables of the STATCOM, e.g. the DC-link voltage may also vary which causes a variation in the gain K shown in Fig.4 (a). An open-loop control system will be unable to maintain the STATCOM current against the disturbances. This calls for closed-loop control of the STATCOM current, which is schematically shown in Fig.4 (b). The current or reactive power control $f_1(.)$, is designed to obtain the desired closed- loop bandwidth.

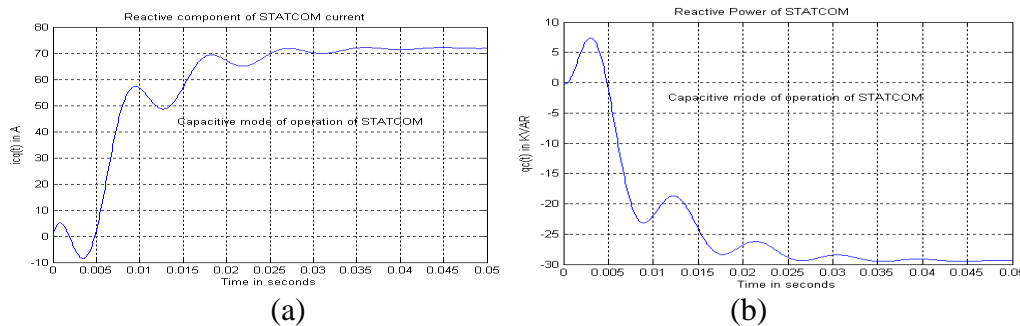(a) (b)

Figure 3: Transient responses to a step change of -10^0 in α of STATCOM in capacitive mode: (a) Reactive component of STATCOM current and (b) reactive power generated by STATCOM

(a)   (b)
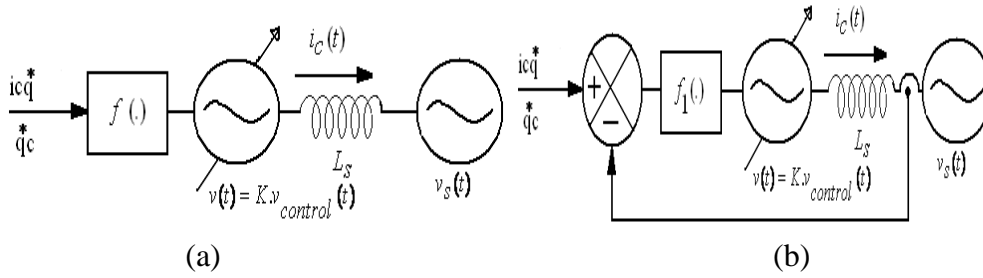
Figure 4: Schematic of STATCOM: (a) With open-loop control and (b) with closed-loop control.

Modeling of Load

The $R-L$ loads are connected at the PCC of the utility source in star connection. The a, b and c three phase stationary coordinate of the load currents are converted into $\alpha - \beta$ of two phase stationary coordinate currents and then $d-q$ of two phase rotating co-ordinate currents. The simulations of the phase a load current with voltage at PCC, active and reactive components of the load currents and active and reactive power are shown in Fig.5. Fig.5(a) is shown the lagging power factor of 0.77 of a phase load current with respect to PPC a phase voltage. This lagging power factor can be compensated to unity power factor.Fig.5(b) is shown the d and q axis current of 11.5A and 7.2A of the above load respectively which can be used as reference values at the time of controlling d and q axis current of STATCOM. Fig.5(c) is shown the active and reactive power of $6KW$ and $3KVAR$ of the above load respectively.

(a)   (b)   (c)
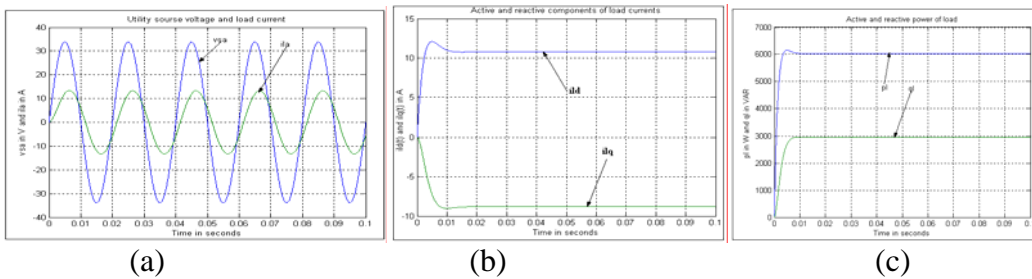
Figure 5: (a) Grid phase a voltage and load phase a current, (b) active and reactive current of the load and (c) active and reactive power of the load

Small Signal Scheme
Modeling
To know the dynamic characteristics of STATCOM system, the small signal analysis is to be done. For a given operating point, small signal equivalent circuit is derived based on the following assumption:

i.　The disturbance is small,

ii.　Hence, the second order terms (products of variations) are negligible,

iii.　The phase nominal value of angle α is small.

With the above assumptions the equations (9) and (11) can be rewritten as,

$$V_{od} + \hat{v}_{od}(t) = M_c V_{dc} + M_c \hat{v}_{dc} + \hat{m}_c V_{dc} + \hat{m}_c \hat{v}_{dc} \tag{17}$$

$$I_{dc} + \hat{i}_{dc} = M_c I_{cd} + M_c \hat{i}_{cd} \tag{18}$$

$$Sin\hat{\alpha} = \hat{\alpha}, \cos\alpha = 1 \text{ and } \alpha_0 = 0 \tag{19}$$

By using (5), (17), (18) and (19), and applying Laplace Transformation, we have,

$$\begin{bmatrix} sL_s + R_s & wL_s & 0 \\ -wL_s & sL_s + R_s & M_c \\ 0 & -M_c & sC \end{bmatrix} \begin{bmatrix} \hat{i}_{cq} \\ \hat{i}_{cd} \\ \hat{v}_{dc} \end{bmatrix} = \begin{bmatrix} -V_s\hat{\alpha}(s) \\ -V_{dc}\hat{m}_c(s) \\ 0 \end{bmatrix} \tag{20}$$

The important transfer functions of the states of the STATCOM in small signal model can be derived as

$$\frac{\hat{i}_{cq}(s)}{\hat{m}_c(s)} = \frac{V_{dc}wL_sC}{A(s)} \tag{21a}$$

$$\frac{\hat{i}_{cq}(s)}{\hat{\alpha}(s)} = \frac{-V_s(L_sCs^2 + R_sC + M_c^2)}{A(s)} \tag{21b}$$

$$\frac{\hat{i}_{cd}(s)}{\hat{m}_c(s)} = \frac{-V_{dc}C(sL_s + R_s)s}{A(s)} \tag{22a}$$

$$\frac{\hat{i}_{cd}(s)}{\hat{\alpha}(s)} = \frac{-V_{dc}wL_sCs}{A(s)} \tag{22b}$$

$$\frac{\hat{v}_{dc}(s)}{\hat{m}_c(s)} = \frac{-M_c V_{dc}(sL_s + R_s)}{A(s)} \tag{23a}$$

$$\frac{\hat{v}_{dc}(s)}{\hat{\alpha}(s)} = \frac{-M_c V_s \varpi L_s}{A(s)} \tag{23b}$$

$$A(s) = CL_s^2 s^3 + 2L_s CR_s s^2 + [C\{R_s^2 + (\varpi L_s)^2\} + M^2 L_s]s + M^2 R_s \tag{24}$$

Open loop responses

The responses of the states $\hat{i}_{cq}, \hat{i}_{cd}$ and $\hat{v}_{dc}$ (as in (21-23)) are simulated in MATLAB using the parameters given in Table-1, with a variation of phase angle $\hat{\alpha} = -5u(t)$ (in degree) and a change of modulation conversion index $\hat{m}_c = 0.1u(t)$. Fig.6 and Fig.7 show responses for a step change in modulation conversion index $\hat{m}_c$ and phase angle

$\hat{\alpha}$ respectively in capacitive mode. It may be noted from the results that a change in $\hat{m}_c$ has no steady state effect on the reactive component of STATCOM currents (Fig.6 (a)) in capacitive mode. It however causes v_{dc} to reduce by $35V$ in the steady state as shown in Fig.6 (b). However, due to a change in $\hat{\alpha}$ by an amount given above, the STATCOM draws additional steady state reactive current of $37A$ (Fig.7 (a)) at steady state. Neither does, v_{dc}, the DC-link voltage remain at its previous steady state value. It increases in magnitude (by $50V$) for a change in $\hat{\alpha}$ (Fig.7 (b)). Hence the open loop dynamics suggest that it is required to have closed loop control of reactive component of STATCOM currents as well as DC-link voltage of STATCOM in order to reduce transient surges and to keep V_{dc} constant.
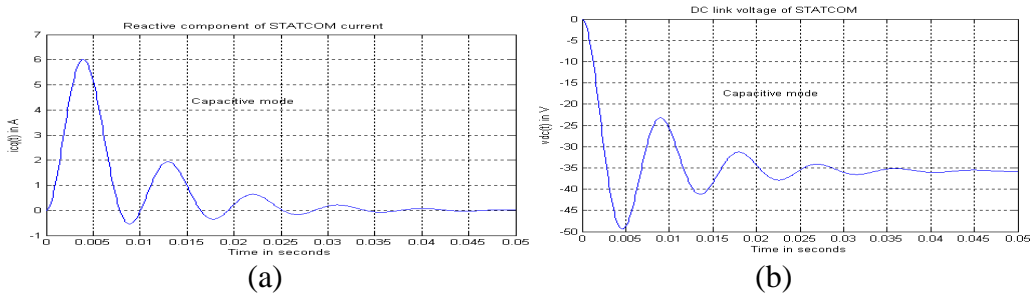

(a)   (b)

Figure 6: Transient response for change in $\hat{m}_c$ in capacitive mode: (a) In $\hat{i}_{cq}$ and (b) $\hat{v}_{dc}$
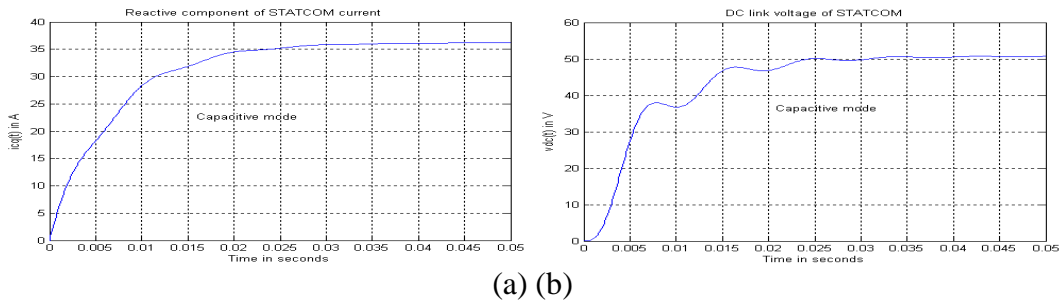

(a) (b)

Figure 7: Transient response for change in $\hat{\alpha}$ in capacitive mode: (a) In $\hat{i}_{cq}$ and (b) $\hat{v}_{dc}$

Design of robust controller
The state space model of STATCOM (14) is nonlinear in phase angle α. The models of the STATCOM in linear model are valid for small changes in α and m_c about the nominal values. In any practical set-up the variables do vary beyond small ranges. Then the small signal models may be in adequate to represent the system behaviour. The controllers designed on the basis of these models may not give desired performance when applied to the original system. A way to overcome this problem is

to design 'robust' controllers. To the best knowledge of the author no work is reported in the literature on robust control of the nonlinear STATCOM system catering to linear loads. The problem of controlling d and q axes components of the currents and DC link voltage in case of the actual nonlinear system may be stated as follows: given the set of equations (14), one has to design a controller(s) whose output u (please see Fig.8(a)) will stabilize the system and yield desired performance in the face of uncertainty(s) and perturbation(s). There is no standard method of designing a stabilizing controller for a nonlinear system. One way that is usually followed is to perform the design for a linearized system about nominal operating conditions as done in previous sections. The design objectives are met if the controller is robust enough to perform well for the other operating conditions too. In the present work perturbations to the plant are modeled as multiplicative uncertainties and a robust controller is designed using the theory of H_∞ optimization [9]. This section gives a brief description of the theory of the uncertainty modeling, the robust stability criterion and design procedure of the robust controller for the mixed sensitive problem as well as a graphical design technique termed loop shaping.

Modeling
Uncertainty model
A robust controller is to be designed for the nominal plant based on the Mixed Sensitivity problem in H_∞-optimal Control Theory [9] as explained in brief below. Let us consider the block-diagram in Fig. 8(a). A nominal transfer function $P(s)$ belongs to a bounded set of transfer functions, $K(s)$ is the H$_\infty$ -controller, $W_1(s)$ is the sensitivity weighting function and $W_2(s)$ is the complementary sensitivity weighting function, $y(t)$ is the measured output, $u(t)$ the controlled input, $w(t)$ the exogenous input (which may include the disturbances) and $z(t)$ is the controlled output. We can write,
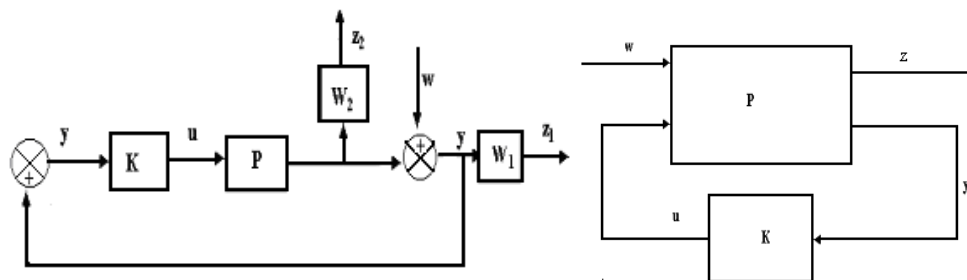

Figure 8: (a) Mixed Sensitivity Problem and (b) modification of Mixed Sensitivity Problem

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} \qquad (25)$$

The augmented plant is: $G = \begin{bmatrix} W_1 & W_1 P \\ 0 & W_2 P \\ I & P \end{bmatrix}$ (26)

which relates: $\begin{bmatrix} z_1 \\ z_2 \\ y \end{bmatrix} = G \begin{bmatrix} w \\ u \end{bmatrix}$ (27)

The block diagram of Fig.8(a) of the mixed sensitivity problem is modified to the general control problem as shown in Fig.8(b). The closed loop transfer function (Fig.8(a)) from w to z is denoted by $T(s)$. The control problem is to find a controller $K(s)$ such that

The closed-loop system is internally stable
The ∞-norm of the closed loop transfer function $T(s)$ satisfies: $\| T \|_\infty < \gamma$ for some small $\gamma > 0$

The perturbed transfer function, resulting from variations in operating conditions, can be expressed in the form of the 'uncertainty set' as shown in Fig.9 and given in

$$\tilde{P}(s) = P(s)\left(1 + \Delta W_2(s)\right)$$ (28)

Here, Δ is a variable transfer function satisfying $\| \Delta \|_\infty < 1$. It may be recalled that the infinity norm $(\infty - norm)$ of a SISO transfer function is the least upper bound of its absolute value. This is also written as $\| \Delta \|_\omega = \sup_\omega | \Delta(j\omega) |$ and is the largest value of the gain on a Bode magnitude plot. The uncertainties, which are the variations of system operating conditions, are thus modeled through $\tilde{P}$ in equation (2.46). In the multiplicative uncertainty model in equation (28), ΔW_2 is the normalized plant perturbation away from 1. If $\| \Delta \|_\infty < 1$, then
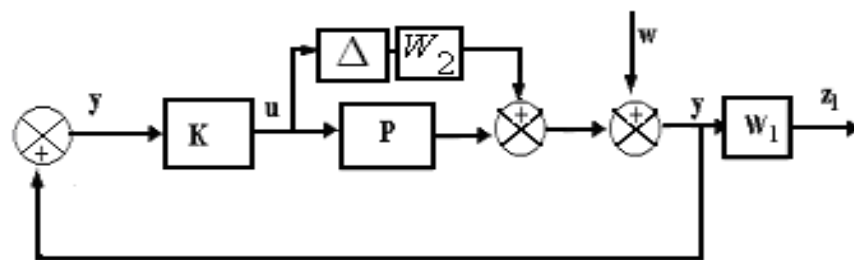

Figure 9: Multiplicative perturbation model.

$$\left| \left(\frac{\tilde{P}(j\omega)}{P(j\omega)} - 1 \right) \right| \leq | W_2(j\omega) | \text{ for all } \omega$$ (29)

So, $|W_2(j\omega)|$ provides the uncertainty profile. In the frequency plane it is the upper bound of all the normalized perturbed plant transfer functions away from 1. For the optimum case the weighting function can be determined as

$$W_2(j\omega) = \frac{\tilde{P}(j\omega) - P(j\omega)}{P(j\omega)} \tag{30}$$

Robust stability and performance

Let us consider $K(s)$ some controller which is assumed to provide overall system stability if it provides internal stability for every plant in the uncertainty set. If L denotes the loop transfer function $(L = PK)$, then the sensitivity function S is written

as $S = \dfrac{1}{1+L}$ (31) The complimentary sensitivity function or the input-output transfer

function is $\quad T = 1 - S = \dfrac{PK}{1+PK} \tag{32}$

For a multiplicative perturbation model, the robust stability condition is met if and only if $\parallel W_2 T \parallel_\infty < 1$. This implies that $|\dfrac{W_2(j\omega)L(j\omega)}{1+L(j\omega)}| < 1,$ for all $\omega \tag{33}$

or, $|\Delta(j\omega)L(j\omega)| < |1 + L(j\omega)|$, for all ω, $\parallel \Delta \parallel_\infty < 1 \tag{34}$

The nominal performance condition for an internally stable system is given as $\parallel W_1 S \parallel_\infty < 1$, where W_1 is a real-rational, stable, minimum phase transfer function (also called a weighting function). If $P(s)$ is perturbed to $\tilde{P}(s) = P(s)(1 + \Delta W_2(s))$ and S is perturbed to $\tilde{S} = \dfrac{1}{1+(1+\Delta W_2 L)} = \dfrac{S}{1+\Delta W_2 T} \tag{35}$

then the robust performance condition should therefore be [9]

$$\parallel W_2 T \parallel_\infty < 1 \text{ and } \parallel \frac{W_1 S}{1+\Delta W_2 T} \parallel < 1, \text{ for all } \parallel \Delta \parallel < 1 \tag{36}$$

Combining all the above, it can be shown that a necessary and sufficient condition for robust stability and performance is $\parallel |W_1 S| + |W_2 T| \parallel_\infty < 1 \tag{37}$

This is the Mixed Sensitivity problem for controller design.

Proposed control scheme

Fig.10 is shown the proposed control strategy of small signal model of the STATCOM. H_∞-mixed sensitivity reduced order controller ($H_\infty - MSROC$) (2.37) is applied at the reactive component of the STATCOM currents with reference reactive component of the load currents. The output of the controller generates a change in phase angle $\hat{\alpha}$ and it can be converted to a change of modulation conversion index

$\hat{m}_c$ by the transfer function G_a [(39)]. Three phase of the STATCOM output voltages are generated by 120^0 apart from each other with the angle summing of a change in phase angle $\hat{\alpha}$, initial value of phase angle α_0 and synchronizing θ from PLL [11]. Then three-phase voltages of STATCOM are multiplied with the peak value of the grid side voltage and the sum of a change in modulation conversion index $\hat{m}_c$ as well as the initial value of the modulation conversion index M_c. The gate pulses for the converter are generated with the 3-phase STATCOM terminal reference voltages and SVPWM principle. The mathematical modeling of the system for designing the controller is given in next section.
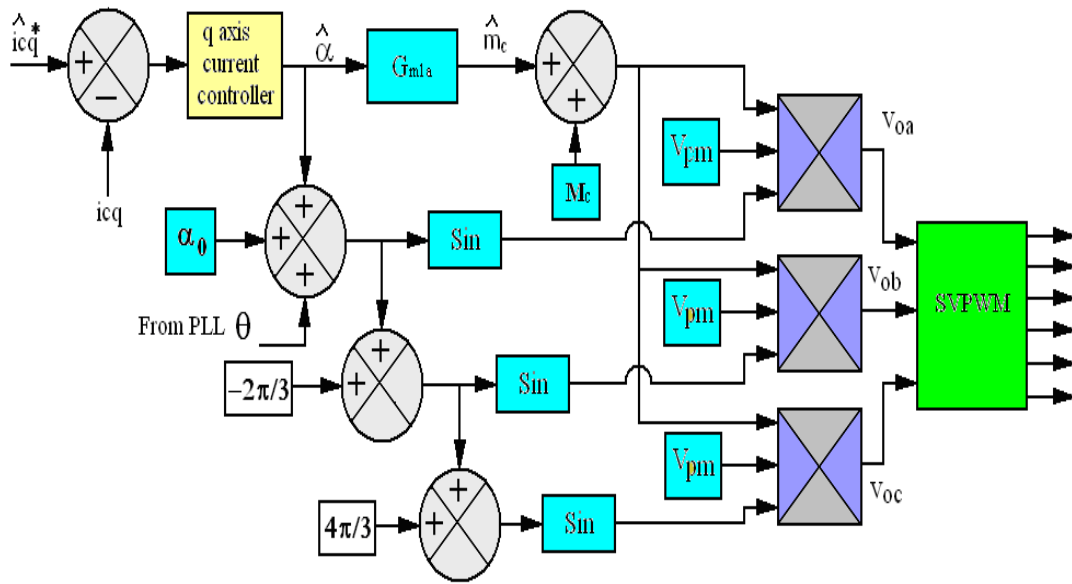
Figure 10: Proposed control scheme of small signal model of the STATCOM.

Design of controller

If the DC-link voltage of the STATCOM is to be kept constant, then the variation $\hat{V}_{dc}$ should be zero. Hence, from equation (20)

$$\hat{V}_{dc}(s) = \frac{-M_c V_{dc}(sL_s + R_s)\hat{m}_c(s) - M_c V_s \omega_1 L_s \hat{\alpha}(s)}{A(s)} \Rightarrow \hat{m}_c(s) = -\frac{V_s \omega_1 L_s}{V_{dc}(sL_s + R_s)}\hat{\alpha}(s) \quad (38)$$

In the present work the authors have used the above relation (equation (38)), to suggest an important modification to the scheme suggested by Chou.et al [6]. This is a significant contribution of the present work. Fig.11 shows the proposed block diagram of the closed-loop control system with a $H_\infty - MSROC$ in order to control the reactive component of the STATCOM currents with independent control on DC-link voltage.
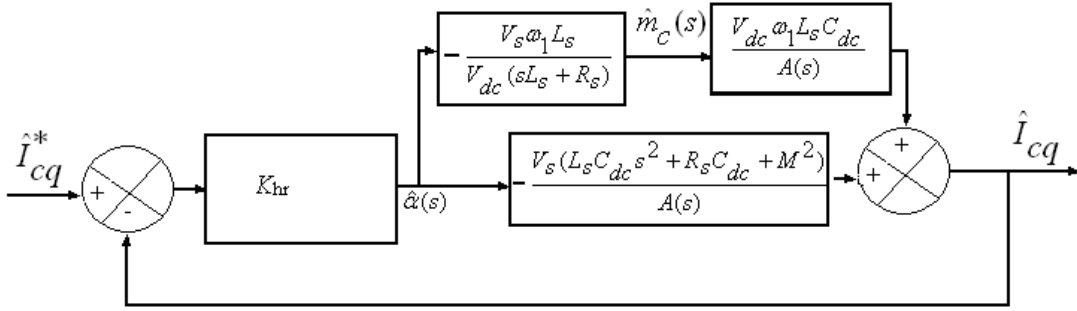
Figure 11: Block diagram of closed loop system with $H_\infty - MSROC$.

The forward path transfer function G_{m2a} is

$$G_a = \left(-\frac{V_s \omega_1 L_s}{V_{dc}(sL_s + R_s)}\right)\left(\frac{V_{dc}\omega_1 L_s C_{dc}}{A(s)}\right) \qquad (39)$$

In this section $\hat{I}_{cq}(s)$ has been modeled as from (21) is

$$\hat{I}_{cq}(s) = \frac{-V_s\left(L_s C_{dc}s^2 + R_s C_{dc} + M_c^2\right)}{A(s)}\hat{\alpha}(s) + \frac{V_{dc}\omega_1 L_s C_{dc}}{A(s)}\hat{m}_c(s) \qquad (40)$$

To achieve fast dynamic response without controlling the DC link voltage, then the variation of $\hat{V}_{dc}(s)$ should be zero. By using equation (39), equation (40) may be modified to:

$$\hat{I}_{cq}(s) = \left[\frac{-V_s\left(L_s C_{dc}s^2 + R_s C_{dc} + M_c^2\right)}{A(s)} - \frac{V_s \omega_1^2 L_s^2 C_{dc}}{A(s)(sL_s + R_s)}\right]\hat{\alpha}(s)$$

So that the nominal plat transfer function is,

$$P(s) = \frac{\hat{I}_{cq}(s)}{\hat{\alpha}(s)} = -\left[\frac{V_s\left(L_s C_{dc}s^2 + R_s C_{dc} + M_c^2\right)}{A(s)} + \frac{V_s \omega_1^2 L_s^2 C_{dc}}{A(s)(sL_s + R_s)}\right] \qquad (41)$$

The mixed sensitivity robust controller [9] is now designed for the nominal plant based on Mixed Sensitivity problem. The sensitivity weighting function $W_1(s)$ is following basis:

If the plant is $P(s)$ and the loop transfer function (as mentioned earlier) is

$L(s)(=P(s)K(s)) = \dfrac{\omega_c}{s}$, and the sensitivity function is $S(s) = \dfrac{s}{s + \omega_c}$ then the

weighting function $W_1(s)$ for sensitivity $S(s)$ is $W_1(s) = \dfrac{s + \omega_c}{s + \xi\omega_c}$, where ξ is the

damping ratio.

Here the nominal plant, $P(s)$ is taken at (i) $m_c = 0.98$ at $\alpha = 0^0$ and the perturbed plant, $\tilde{P}(s)$ is taken at (ii) $m_c = 1.22$ and $\alpha = 0^0$. The weighting function $W_2(s)$ can be determined as: $W_2(s) = \dfrac{\tilde{P}(s) - P(s)}{\tilde{P}(s)}$ (42)

The weighting function $W_1(s)$ can be determined after choosing 5% over shoot, $\xi = 0.69$ and $\omega_c = 409.8$ rad/sec and steady state error of 0.1 [9]. So $W_1(s)$ will be $W_1(s) = \dfrac{(0.2s + 409.85)}{(s + 40.98)}$ (43).However, both weighting functions can be finalized in such a way that both weighting functions obey $|W_1^{-1}(s)| + |W_2^{-1}(s)| \geq 1$ for all frequencies. Hence, by trial and error adjustment of pole and zero of the transfer function of the weighting functions, $W_1(s)$ and $W_2(s)$ will be

$$W_1(s) = \frac{10}{(s + 0.4)} \text{ (44) and } W_2(s) = \frac{s}{(s + 10^2)} \quad (45)$$

The Bode plots of $|W_1^{-1}(s)|$ and $|W_2^{-1}(s)|$ shown in Fig.12 indicate $|W_1^{-1}(s)| + |W_2^{-1}(s)| \geq 1$ at all frequencies. The weighting functions together with the nominal plant $P(s)$ are used in the Robust Control Toolbox in MATLAB to yield the mixed sensitivity H_∞-Controller K_h given in (46). The controller is of 5th order transfer function.
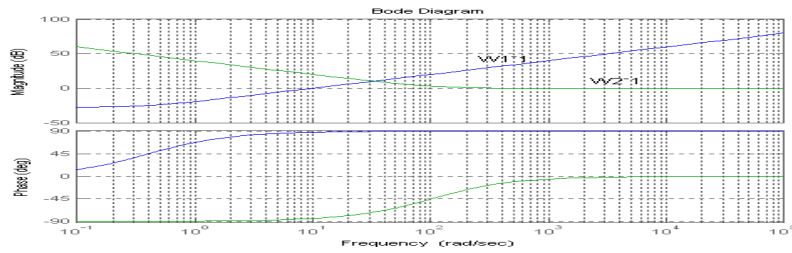

Figure 12: Bode plots of $|W_1^{-1}(s)|$ and $|W_2^{-1}(s)|$ for small signal model.

$$K_h(s) = \frac{-8.6 \times 10^7 s^4 - 4.3 \times 10^{10} s^3 - 3.7 \times 10^{13} s^2 - 7.8 \times 10^{15} s - 4.4 \times 10^{17}}{s^5 + 4.6 \times 10^{10} s^4 + 1.3 \times 10^{13} s^3 + 1.2 \times 10^{16} s^2 + 9.9 \times 10^{18} s + 4 \times 10^{17}} \quad (46)$$

It may be noted that the small signal model of the STATCOM used here includes an improvement over existing work of Cho et al [6]. The design of the controller is another contribution of the author.

The step response of the closed loop system is shown in Fig.13 (a) and the output

settles at 0.007 seconds. The system can compensate $\mp 20A$ quadrature current as shown in Fig.13 (b). It is a good agreement with the reference quadrature current.
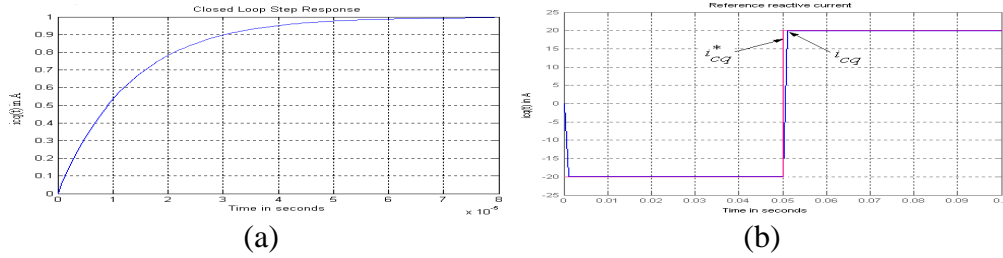

(a)   (b)

Figure 13: Response using $H_\infty - MSROC$: (a) Step response of i_{cq} and (b) step response of i_{cq} with reference reactive current.

Robust control techniques generally yield high-order controllers. The controller K_h (46) of 5th order is reduced by Hankel norm method [12]. The reduced-order controller of 2nd order is given in (47). Fig.14 shows the comparison between the dynamics of the reduced-order controller with reference to the full order one. The closed loop step response of the full order controller and reduced-order controller are found to be in good agreement. The same is the case for frequency response in Fig.14 (b) respectively.

$$K_{hr}(s) = \frac{-8.68 \times 10^7 s - 2.06 \times 10^{10}}{s^2 + 4.6 \times 10^{10} s + 1.88 \times 10^{10}} \tag{47}$$
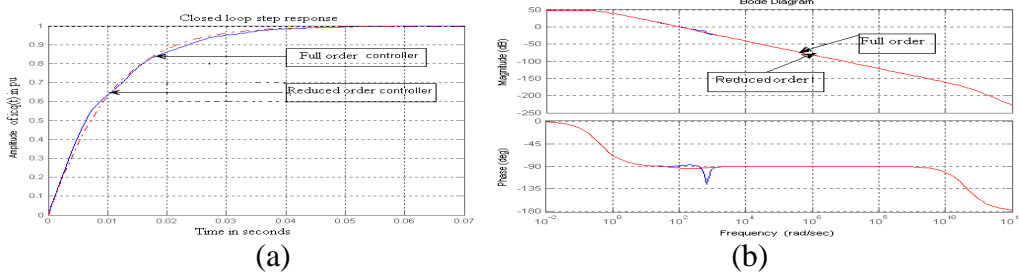

(a)   (b)

Figure 14: Response of i_{cq} for full order and reduced-order $H_\infty - MSROC$: (a) Step response of i_{cq} and (b) Bode plots.

Simulation

The $H_\infty - MSROC$ in (47) with the proposed scheme is simulated using MATLAB SIMULINK environment on variation of initial DC-link (pre-charge) voltage.

An initial DC link voltage of $100V$, the STATCOM with this controller operates well with larger current spike of $110A$ as shown in Fig.15 (a) of grid phase 'a'

voltage with current .The power factor of the grid source with $R - L$ load improves from 0.77 to unity in three power cycles over the open loop situation in Fig.5(a). The same dynamics of the STATCOM phase 'a' current is given in Fig.15 (b) with a current spike of $100A$. The current surges in all cases are more than the acceptable limit as per the specifications of the STATCOM.
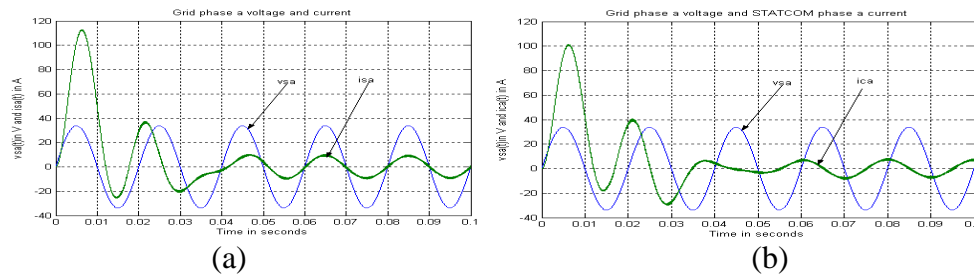

(a)   (b)

Figure 15: Responses of small signal model of STATCOM with $H_\infty - MSROC$ at initial DC link voltage of $100V$: (a) Grid phase 'a' voltage and current (b) Grid phase 'a' voltage and STATCOM phase 'a' current

The same model is simulated with changing the initial DC link voltage of $550V$. It is seen that with R-L load the power factor improves from 0.77 to unity after two cycles with a current spike of $45A$ as given in Fig.16 (a) (for grid phase 'a') on compared to Fig.5(a). Fig.16 (b) shows the dynamics of the STATCOM current with transient surge of $30A$. Figs.16 (c) and (d) show the dynamics of d and q axes currents of the STATCOM and their transients ($42A$ of active and $-50A$ of reactive current) die out after two power cycles respectively .The active with reactive power generated by the STATCOM is shown in Fig.16 (e) and the transients ($18kW$ of active power and $20kVAR$ of reactive power) die out in two power cycles as shown in Fig.16 (f). It is noted that the dynamics of the active currents of STATCOM is similar to the active power dynamics. Incase of reactive currents the dynamics is similar in nature but in phase opposition. The prototype STATCOM can not be designed taking the above transients of the relevant waveforms, since that will mean excessive surge current ratings of the solid-state devices (which are otherwise unnecessary with respect to steady state ratings)

No separate voltage controller is needed for this model. The beauty of the model is that the voltage get controlled indirectly and come to steady state at $640V$ after surges of $970V$ respectively as shown in Fig.16(g). This is a contribution of the present work. It is noted that the system in case of this model is not so sluggish but the transient surges are not within reasonable limits.
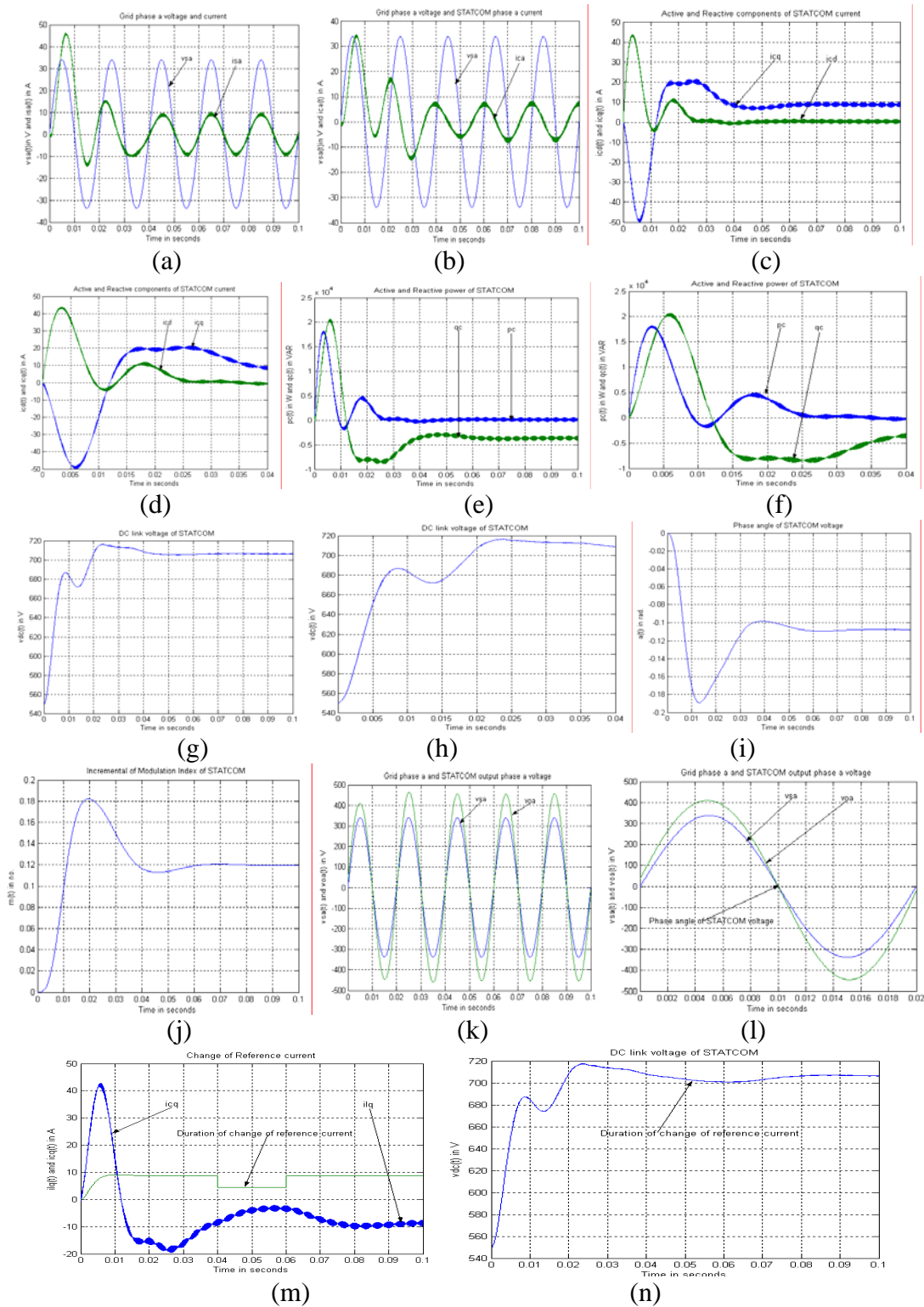
Figure 16: Responses of small signal model of STATCOM with $H_\infty - MSROC$ at initial DC link voltage of $550V$: (a) Grid phase 'a' voltage and current, (b) Grid

phase 'a' voltage and STATCOM phase 'a' current, (c) Active and reactive components of STATCOM current, (d) Active and reactive components of STATCOM current (in expansion), (e) Active and reactive power generated by STATCOM, (f) Active and reactive power generated by STATCOM (in expansion), (g) DC link voltage of STATCOM, (h) DC link voltage of STATCOM (in expansion), (i) Variation of incremental of phase angle $\hat{\alpha}$, (j) Variation of incremental of modulation conversion index $\hat{m}_c$, (k) Grid phase 'a' voltage and STATCOM output phase 'a' voltage, (l) Grid phase 'a' voltage and STATCOM output phase 'a' voltage (in expansion), (m) Change of STATCOM reactive current due to change of reference current (load reactive current), (n) DC link voltage of STATCOM due to change of reference current (load reactive current)

Figs.16 (i) and (j) show the corresponding dynamics of incremental phase angle $\hat{\alpha}$ and modulation index $\hat{m}_c$ respectively. Figs.16 (k) and (l) show the dynamics of the grid phase 'a' voltage with STATCOM output phase 'a' voltage and latter is greater with lagging very small angle than the former respectively. Application of small perturbation of reference reactive current as shown in Fig.16 (m) (reactive current of load), a very small change in DC link voltage is obtained. While the reference current returns to its original value the DC link voltage also returns to its steady state value as shown in Fig.16 (n).

Again the same proposed model is simulated with an initial DC link voltage of $600V$. The relevant response is obtained with a lesser current spike of $20A$ and compensates after one power cycle as shown Fig.3.13 of the grid phase 'a' voltage and current. The peak and rms values of the grid current are the basis of the design and selection of the devices and the peripherals of STATCOM system.
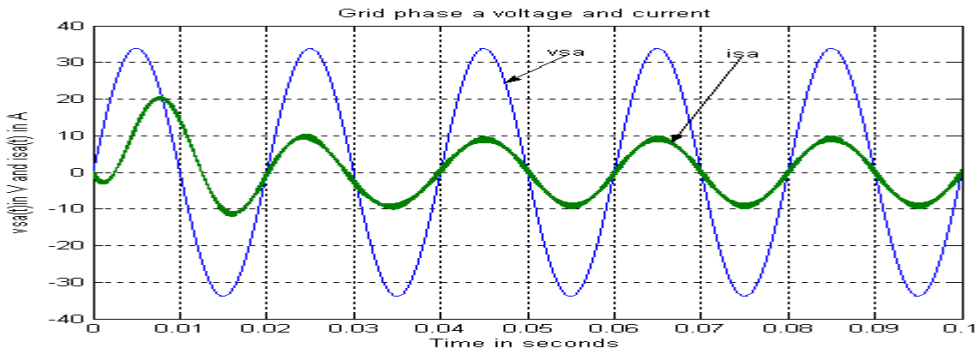

Figure 17: Grid phase a voltage and current of small signal model of STATCOM with $H_\infty - MSROC$ at initial DC link voltage of $600V$

Conclusion
The STATCOM (STATic synchronous COMpensator) has been modeled in a simple

$d-q$ transformation. Its steady state and transient analysis have been achieved to characterize the open loop response of the STATCOM. A small signal control scheme of the STATCOM has been proposed without using direct DC-link capacitor voltage control. The controller has been designed on basis of mixed sensitivity method. This scheme controls reactive current of the STATCOM and the DC-link voltage gets controlled indirectly. This is an achievement of the present work. The phase angle as well as modulation index of the switching pattern is controlled indirectly. Different pre-charge voltages are applied on the DC-link capacitor to get proper reasonable transient variations and settling times of the relevant responses of the STATCOM. These results give the proper idea for designing the devices and peripheral components of the STATCOM. The pre-charge voltage of 600V is chosen on basis of the tolerable and acceptable transient variations and settling times of the relevant responses of the STATCOM. With small perturbation of reference current (reactive current of load), the DC voltage nearly remains steady state. This scheme also controls active and reactive power indirectly. All responses are obtained trough MATLAB SIMULINK tool box.

Acknowledgement

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