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Analysis of Voltage Stability using L-Index Method

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Abstract

Many papers discuss the voltage stability assessment of power system using power flow analysis methods. In this paper, a method for online monitoring of a power system based on measurements is proposed, which is aimed at detection of the voltage instability. Thereby an indicator is derived from the fundamental Kirchoff-Laws. Since in the transient process, at any time point, the electric power of the system is in balance, and the Kirchoff-Law is obeyed, this indicator will still work during the transient process. From the indicator, it is allowed to predict the voltage instability or the proximity of a collapse. The advantage of the method lies in the simple numerical calculation and strong adaptation in steady state and transient process. Through the indicator of voltage stability, it is easy to find the most vulnerable area in a system, to find the impacts of other loads, areas and power transactions.

Keywords: Voltage Stability, Voltage Collapse, Reactive Power, Steady-State Voltage Stability, Transient Analysis

Introduction

Voltage stability is a major concern in planning and operations of power systems. It is well known that voltage instability and collapse have led to major system failures; with the development of power markets, more and more electric utilities are facing voltage stability-imposed limits. The problem of voltage stability may be simply explained as inability of the power system to provide the reactive power or the egregious consumption of the reactive power by the system itself. It is understood as a reactive power problem and is also a dynamic phenomenon. [8,9] The objective of this paper is to develop a fast and simple method, which can be applied in the power system online, to Estimate the voltage stability margin of the power system. In general, the analysis of voltage stability problem of a given power system should cover the examination of these aspects:

- How close is the system to voltage instability or collapse?
- When does the voltage instability occur?
- Where are the vulnerable spots of the system?
- What are the key contributing factors?
- What areas are involved?

Voltage stability analysis often requires examination of lots of system states and many contingency scenarios. For this reason, the approach based on steady state analysis is more feasible, and it can also provide insights of the voltage reactive power problems. A number of special algorithms have been proposed in the literature for voltage stability analysis using static approached [2-4], however these approaches are laborious and does not provide sensitivity information useful in a dynamic process. Voltage stability is indeed a dynamic phenomenon. [8,9] Some utilities use Q-V curves at some load buses to determine the proximity to voltage instability [6]. One problem with Q-V curve method is that by focusing on a small number buses, the system-wide voltage stability problem will not be readily unveiled. An approaches, model analysis of the modified load flow Jacobian matrix, has been used as static voltage stability index to determine vulnerable bus's voltage stability problem [7].

This paper explores the online monitoring index of the voltage stability, which is derived from the basic static power flow and Kirchoff law. A derivation will be given. The index of the voltage stability [3] predicts the voltage problem of the system with sufficient accuracy. This voltage stability index can work well in the static state as well as during dynamic process. It can also be used to find the vulnerable spots of the system, the stability margin based on the collapse point, and the key factors for the voltage stability problem, etc

Fundamentals: Single Generator and Load System

A simple power system is considered, through which the useful index of the voltage stability is derived. As showed in Fig.1, whereby bus 1 is assumed as a generator bus, and bus 2 is a load bus whose voltage behavior will be our interest.

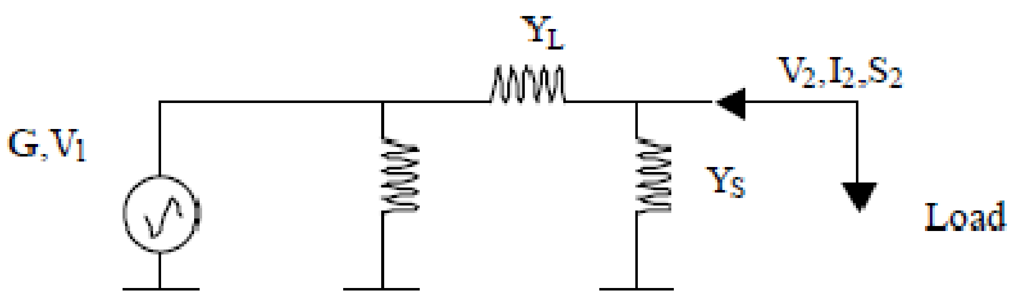

Figure 1: Single generator and load system.

This simple system can be described by the following equations (where the dot above a letter represents a vector):

$$\dot{I}_2 = \dot{V}_2 \dot{Y}_S + (\dot{V}_2 - \dot{V}_1)\dot{Y}_L = \frac{S_2}{\overset{*}{V}_2} \qquad (1)$$

$$\overset{*}{S}_2 = V_2^2 \dot{Y}_S + V_2^2 \dot{Y}_L - \overset{*}{V}_2 \dot{V}_1 \dot{Y}_L$$

$$= V_2^2 \dot{Y}_{22} + \dot{V}_0 \overset{*}{V}_2 \dot{Y}_{22} \qquad (2)$$

$$Here\ \dot{Y}_{22} = \dot{Y}_S + \dot{Y}_L$$

$$and\ \dot{V}_0 = -\frac{\dot{Y}_L}{\dot{Y}_L + \dot{Y}_S}\dot{V}_1 \qquad (3)$$

To solve for $\left|\dot{V}_2\right|$, it is needed to solve this complex equation (2), and it is assumed that $\frac{\overset{*}{S}_2}{\dot{Y}_{22}} = a + jb$. Express equation (2):

$$\frac{\overset{*}{S}_2}{\dot{Y}_{22}} = a + jb = V_2^2 + \dot{V}_0 \overset{*}{V}_2 \qquad (4)$$

$$= V_2^2 + V_0 V_2 \cos(\delta_0 - \delta_2) + jV_0 V_2 \sin(\delta_0 - \delta_2)$$

Here, $\dot{V}_0 = V_0 \angle \delta_0$

$$\cos(\delta_0 - \delta_2) = \frac{a - V_2^2}{V_0 V_2} \qquad (5)$$

$$\sin(\delta_0 - \delta_1) = \frac{b}{V_0 V_2} \qquad (6)$$

$(5)^2 + (6)^2$, hence,

$$V_0^2 V_1^2 = (a - V_1^2)^2 + b^2 = a^2 - 2aV_1^2 + V_1^4 + b^2 \qquad (7)$$

Solve equation (7):

$$V_2 = \sqrt{\frac{V_0^{\ 2}}{2} + a \pm \sqrt{\frac{V_0^{\ 4}}{4} + aV_0^{\ 2} - b^2}} \qquad (8)$$

When $\sqrt{\frac{V_0^{\ 4}}{4} + aV_0^{\ 2} - b^2} = 0$, the voltage at bus 2 collapses. On the other hand, equation (5) and (6) are considered again.

$$f(V_2, \delta) = V_0 V_2 \cos\delta + V_2^{\ 2} = a \qquad (5a)$$

$$g(V_2, \delta) = V_0 V_2 \sin\delta = b \qquad (6a)$$

It is very easy to get the Jacobian Matrix:

$$J = \begin{bmatrix} 2V_1 + V_0 \cos\delta & -V_1 V_0 \sin\delta \\ V_0 \sin\delta & V_1 V_0 \cos\delta \end{bmatrix} \qquad (9)$$

If the voltage at the bus 2 collapses, there will be no solution for equation (5a) and (6a), it also means that the determinant of the Matrix J should be zero (Jacobian Matrix singular)

$$\det(J) = 2V_2^{\ 2} V_0 \cos\delta + V_2 V_0^{\ 2} = 0$$

$$\Rightarrow \frac{V_2 \cos\delta}{V_0} = \mathrm{Re}\left\{\frac{\dot{V}_2}{\dot{V}_0}\right\} = -\frac{1}{2} \qquad (10)$$

When equation (2) is divided by $V_2^{\ 2}\dot{Y}_{22}$, it becomes:

$$\frac{\overset{*}{S}_2}{V_2^{\ 2}\dot{Y}_{22}} = 1 + \frac{\dot{V}_0}{\dot{V}_2} \qquad (11)$$

When the voltage collapses, it is said that $\mathrm{Re}\left\{\frac{\dot{V}_2}{\dot{V}_0}\right\} = -\frac{1}{2}$, which implies that $\frac{\dot{V}_2}{\dot{V}_0} = -\frac{1}{2} + jc$, hence

$$\left|1+\frac{\dot{V}_0}{\dot{V}_2}\right| = \left|1+\frac{1}{-\frac{1}{2}+jc}\right| = \left|\frac{\frac{1}{2}+jc}{-\frac{1}{2}+jc}\right| = 1 \qquad (12)$$

From equation (11) and (12), an indicator of the voltage stability is defined as:

$$\left|1+\frac{\dot{V}_0}{\dot{V}_2}\right| = \left|\frac{\overset{*}{S}_2}{V_2^{\,2}\,\dot{Y}_{22}}\right| = \frac{S_2}{V_2^{\,2}Y_{22}} \qquad (13)$$

Thereby an indicator has been derived which can be used for monitoring the voltage stability problem of the system and for assessing the degree of risk for a potential voltage collapse. When 0 2 *S* = , the indicator will be zero YL Load V2,I2,S2 YS G,V1 and indicates that there will be no voltage problem. When *S*2 = 1 , the voltage at load bus will collapse. One example of a single generator and load system was constructed to demonstrate the correctness of the indicator

$$V_1 = 1.0\angle 0^\circ, Y_S = 0, Y_L = -j4(X = j0.25),$$

$$PowerFactor = 0.97 lagging, \phi = 14.07^\circ$$

Continuously change the load at bus 2, and keep the power factor of the load to find the collapse point

Generalization to An N-Bus System

As shown in the basic theory of the multi-bus power system, all the buses can be divided into two categories: Generator bus (PV bus and Slack bus) and Load bus (PQ bus). Because the voltage stability problem is reactive power relative problem, and the generator bus can provide the reactive power to support the voltage magnitude of the bus, it is absolutely necessary that the all of buses be distinguished. The power system can be expressed in the form through Kirchoff Law:

$$I_{System} = \begin{bmatrix} I_L \\ I_G \end{bmatrix} = \begin{bmatrix} Y_{LL} & Y_{LG} \\ Y_{GL} & Y_{GG} \end{bmatrix} \begin{bmatrix} V_L \\ V_G \end{bmatrix} = Y_{System} V_{System} \quad (14)$$

Subscript L means Load bus, and G means Generator bus.

$$\begin{bmatrix} V_L \\ I_G \end{bmatrix} = \begin{bmatrix} Z_{LL} & -Z_{LL} Y_{LG} \\ Y_{GL} Z_{LL} & Y_{GG} - Y_{GL} Z_{LL} Y_{LG} \end{bmatrix} \begin{bmatrix} I_L \\ V_G \end{bmatrix} \quad (15)$$

Here, $Z_{LL} = Y_{LL}{}^{-1}$

For any load bus $j \in L$, through the equation (15), the voltage of the bus is known as:

$$\dot{V}_j = \sum_{i \in L} Z_{ji} \dot{I}_i + \sum_{k \in G} A_{jk} \dot{V}_k \quad (16)$$

$$A = -Z_{LL} Y_{LG}$$

which can be expressed as the form:

$$V_j^2 + \dot{V}_{0j} \overset{*}{V}_j = \frac{\overset{*}{S}_j{}'}{Y_{jj}{}'} \quad (17)$$

Substituting equivalent $\dot{V}_{0j}$, $\overset{*}{S}_j{}'$ and $Y_{jj}{}'$, we have

$$\dot{V}_{0j} = -\sum_{k \in G} A_{kj} V_k \quad (18)$$

$$Y_{jj}{}' = \frac{1}{Z_{jj}} \quad (19)$$

$$\dot{S}_j{}' = \left(\sum_{i \in L} \frac{\overset{*}{Z}_{ji} \dot{S}_i}{\overset{*}{Z}_{jj} \dot{V}_i}\right) * \dot{V}_j = \dot{S}_j + \left(\sum_{\substack{i \in L \\ i \neq j}} \frac{\overset{*}{Z}_{ji} \dot{S}_i}{\overset{*}{Z}_{jj} \dot{V}_i}\right) * \dot{V}_j \quad (20)$$

Hence, we see that the voltage of the load bus j is affected by an equivalent complex power $\overset{*}{S}_j{}'$ and by an equivalent generator part $\dot{V}_{0j}$.

To compare the equation (17) and (2), we can observe that they have an identical form, and the voltage stability of the multi-bus system has been equivalent to a simple single generator and load system. The indicator of the voltage stability of the load bus j will be easily obtained:

$$Indicator_j = \left|1 + \frac{\dot{V}_{0j}}{\dot{V}_j}\right| = \left|\frac{\overset{*}{S}{}_j'}{V_j^2 \overset{*}{Y}{}_{jj}'}\right| = \frac{S_j'}{V_j^2 Y_{jj}} \quad (21)$$

For the system

$$Indicator_{system} = \underset{i \in L}{Max}(Indicator_j) \quad (22)$$

Thereby it is clear that the indicator of the voltage stability at a load bus mainly influenced by the equivalent load ¢*j S* , which has two parts: the load at bus j itself, and the 'contributions' of the other load buses (showed at equation 20). When the load at a load bus changes, it will influence the indicator. On the other words, the voltage stability problem is a system-wide problem, not a local problem. Through equation (20), the contribution of any other load bus on the load bus j can be numerically updated and computed. It is a very important concept for the deregulated power market, and will help the customers and ISO to evaluate the responsibility of voltage stability problem.

Interpretation

The indicator is an effective quantitative measurement for the system to find how far is the current state of the system to the voltage collapse point. All the derivations are correct as long as that all the generator bus of the system have the enough reactive power supply to maintain the magnitude of voltage as constant. If some of the generators are unable to maintain the voltage magnitude, these generator buses will become load buses, the load bus congregation will expand; the indicator will have a discrete jump, which will be shown at the following demonstration.

Test System and Results

The WSCC 9 bus system is taken as a sample system to illustrate the applicability of the indicator L to a multi-bus system. The test system is shown in Figure 2

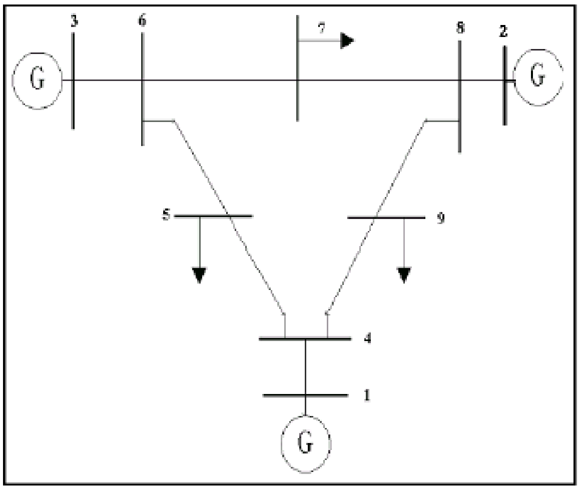

Figure 2: The IEEE 9 bus systems.

Fig 2 Case scenarios of the test system
The normal base loading at load buses are:

Bus 5: 90 + j 30 MVA

Bus 7: 100 + j 35 MVA

Bus 9: 125 + j 50 MVA

Buses 1 to 3 are generation buses; there are no generators or loads at buses 4, 6 and 8.Three case scenarios have been simulated to study the steady state voltage collapse at the load buses and their respective L index.

Case I

- Increase loading of bus 5 from zero to the voltage collapse point, keeping the load at other buses fixed at the normal value. Observe the effect on index L (5).
- Observe the effect on index L (7) at bus 7 when load at bus 5 is increasing and approaching collapse.
- Observe the effect on index L (9) at a bus 9, when load at bus 5 is increasing and approaching collapse.

Case II

- Increase loading of bus 7 from zero to the voltage collapse point keeping the load at other buses fixed at the normal value.

Case III

- Increase loading of bus 9 from zero to the voltage collapse point keeping the load at other buses fixed at the normal value.

(Note: Power factor is kept constant throughout the loading of buses.)

Case IV

- Increasing load at bus 5 and observing the index (L) for bus 5, 7 and bus 9 and identify the first collapse point.

P5(M.W)	0	100	200	250	300	375
V5(p.u)	1.014	0.971	0.909	0.867	0.809	0.635
L55	0.1662	0.2439	0.3177	0.3802	0.5171	0.8959

Case V

- Increasing load at bus 7 and observing the index (L) for bus 5, 7 and bus 9 and identify the first collapse point.

Case VI

- Increasing load at bus 9 and deserving the index (L) for bus 5, 7 and bus 9 and identify the first collapse point.

Case I: Increasing the load at bus5 and observing the various indicators
Case I (a): increasing load at bus5 and observing the indicator L at bus 5.

- As shown in graph index L approaches one at the collapse point. For this simulation, the load at bus7 is taken as 100+j35 MVA and the load at bus9 is taken to be 125+j50 MVA.
- The collapse occurs when the load at bus5 is about 350+j115.5

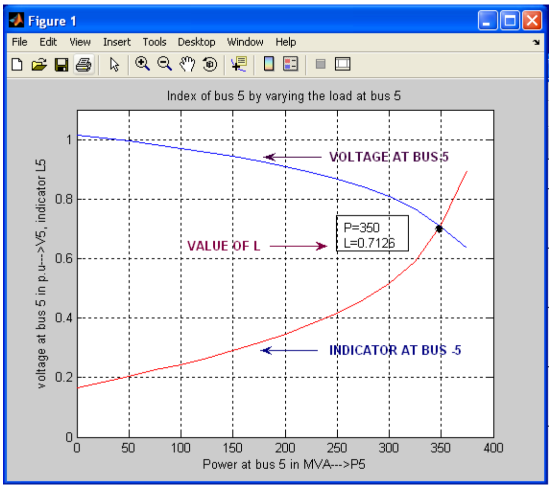

Figure 2.1: indicator for bus 5 with increased loading at bus 5.

Case I (b): increasing load at bus5 and observing the indicator L at bus 7.
As shown in graph. The voltage collapse point would not identified with index of L7 for the increasing of load at bus 5. For this simulation, load at Bus 5 is varied, the load at bus7 is taken as 100+j35 MVA and the load at bus9 is taken to be 125+j50 MVA

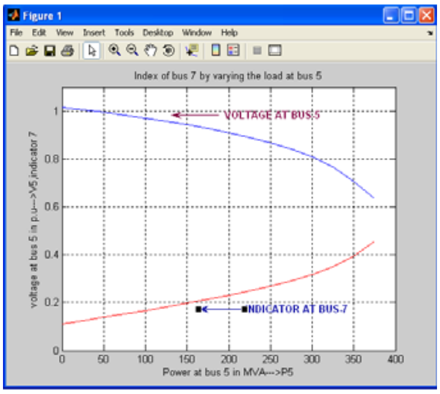

Figure 2.2: indicator for bus 7 with increased loading at bus 5.

Table 2.2: Data for variation of load at bus5 and for voltage at bus5 and for indicator L (7) .

P5(M.W)	0	100	200	300	350	375
V5(p.u)	1.014	0.971	0.909	0.809	0.707	0.635
L75	0.1124	0.1665	0.2305	0.3171	0.3941	0.4541

Case I (c): increasing load at bus5 and observing the indicator L at bus 9.
As shown in graph index L approaches one at the collapse point. For this simulation, the load at bus7 is taken as 100+j35 MVA and the load at bus9 is taken to be 125+j50 MVA. The collapse occurs when the load at bus5 is about 375+j123.75

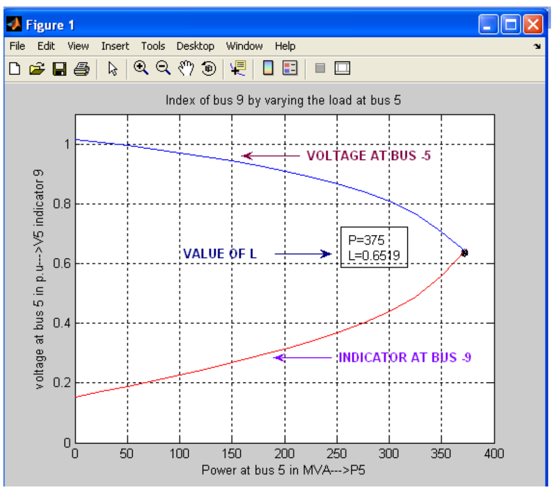

Figure 2.3: indicator for bus 9 with increased loading at bus 5.

Table 2.3: Data for variation of load at bus5 and for voltage at bus5 and for indicator L (9).

P5(M.W)	0	100	200	300	350	375
V5(p.u)	1.014	0.971	0.909	0.809	0.707	0.635
L95	0.1532	0.2262	0.3144	0.4392	0.5567	0.6519

Case II: Increasing the load at bus7 and observing the various indicators
Case II (a): increasing load at bus7 and observing the indicator L at bus 5.
As shown in graph index L approaches one at the collapse point. For this simulation, the load at bus5 is taken as 90+j30 MVA and the load at bus9 is taken to be 125+j50 MVA. The collapse occurs when the load at bus7 is about 425+j148.75

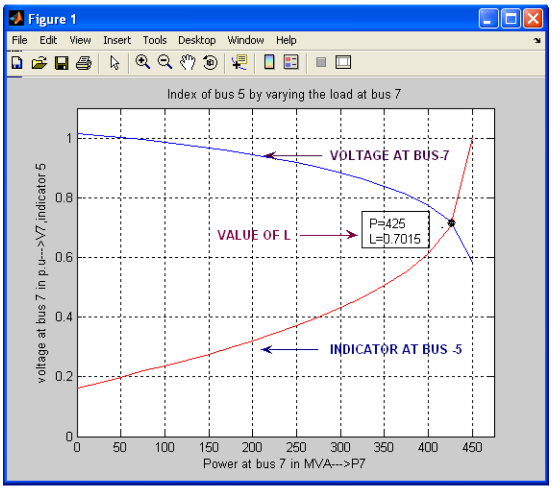

Figure 2.4: indicator for bus 5 with increased loading at bus 7.

Table 2.4: Data for variation of load at bus5 and for voltage at bus5 and for indicator L (9).

P5(M.W)	0	50	100	200	300	375
V5(p.u)	1.014	0.994	0.971	0.909	0.809	0.635
L95	0.1532	0.1885	0.2262	0.3144	0.4392	0.6519

Case II: Increasing the load at bus7 and observing the various indicators
Case II (a): increasing load at bus7 and observing the indicator L at bus 5.
As shown in graph index L approaches one at the collapse point. For this simulation, the load at bus5 is taken as 90+j30 MVA and the load at bus9 is taken to be 125+j50 MVA. The collapse occurs when the load at bus7 is about 425+j148.75

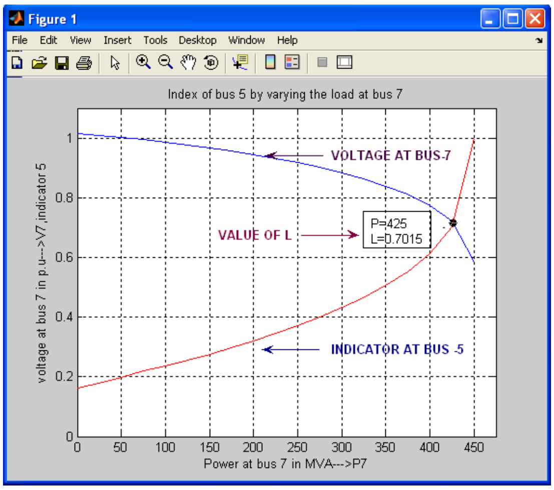

Figure 2.5: indicator for bus 5 with increased loading at bus 7.

Table 2.5: Data for variation of load at bus7 and for voltage at bus7 and for indicator L (5).

P7(M.W)	0	100	200	400	450
V7(p.u)	1.014	0.986	0.944	0.775	0.723
L57	0.1627	0.2357	0.3215	0.6129	0.9976

Case II (a): increasing load at bus7 and observing the indicator L at bus 7.
As shown in graph index L approaches one at the collapse point. For this simulation, the load at bus5 is taken as 90+j30 MVA and the load at bus9 is taken to be 125+j50 MVA. The collapse occurs when the load at bus7 is about 435+j152.25

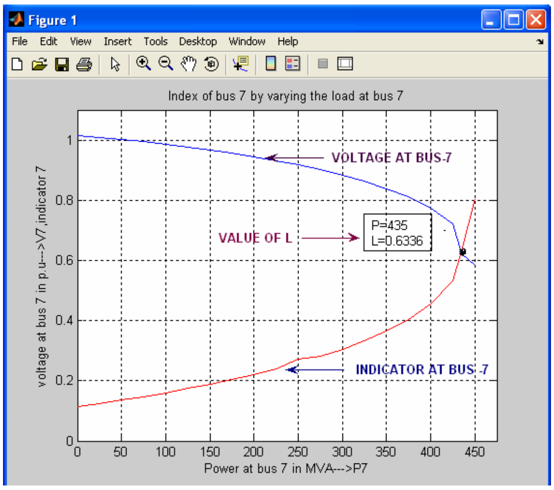

Figure 2.6: indicator for bus 7 with increased loading at bus 7.

Table 2.6: Data for variation of load at bus7 and for voltage at bus7 and for indicator L (7).

P7(M.W)	0	100	200	300	450
V7(p.u)	1.014	0.986	0.944	0.833	0.723
L77	0.1127	0.1608	0.2216	0.3045	0.5356

Case II (c): increasing load at bus7 and observing the indicator L at bus 9.
As shown in graph index L approaches one at the collapse point. For this simulation, the load at bus5 is taken as 90+j30 MVA and the load at bus9 is taken to be 125+j50 MVA. The collapse occurs when the load at bus7 is about 425+J148.75

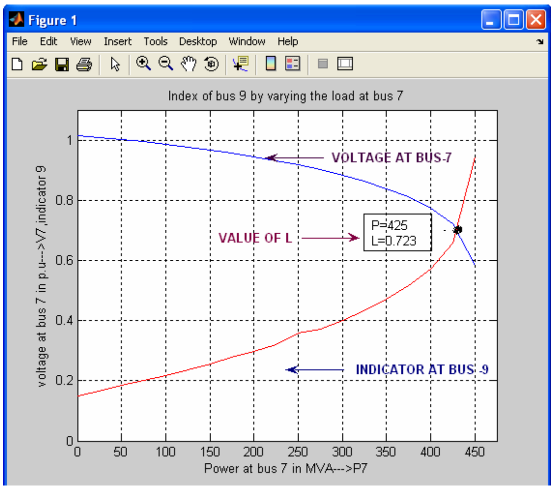

Figure 2.7: indicator for bus 9 with reased loading at bus 7.

Table 2.7: Data for variation of load at bus7 and for voltage at bus7 and for indicator L (9).

P7(M.W)	0	100	200	300	450
V7(p.u)	1.014	0.986	0.944	0.833	0.723
L97	0.1508	0.2185	0.2983	0.4009	0.6572

Case III: Increasing the load at bus9 and observing the various indicators
Case III (a): increasing load at bus9 and observing the indicator L at bus 9. As shown in graph index L approaches one at the collapse point. For this simulation, the load at bus5 is taken as 90+j30 MVA and the load at bus7 is taken to be 100+j35 MVA. The collapse occurs when the load at bus9 is about 367.8+j147.12

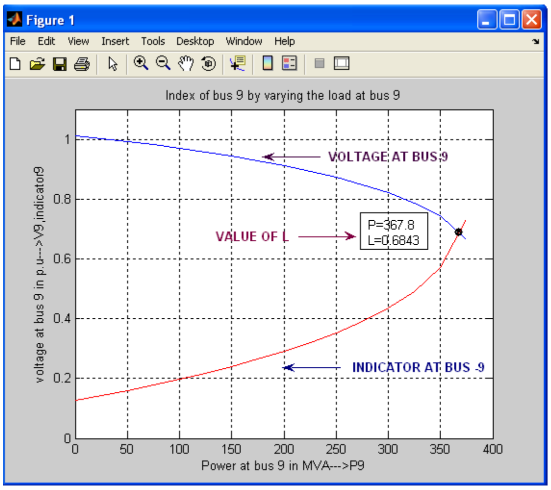

Figure 2.8: indicator for bus 9 with increased loading at bus 9.

Table 2.8: Data for variation of load at bus9 and for voltage at bus9 and for indicator L (9).

P9(M.W)	0	100	200	300	375
V9(p.u)	1.001	0.970	0.929	0.849	0.663
L99	0.1265	0.1982	0.2644	0.4343	0.7305

Case III (b): increasing load at bus9 and observing the indicator L at bus 5.
As shown in graph index L approaches one at the collapse point. For this simulation, the load at bus5 is taken as 90+j30 MVA and the load at bus7 is taken to be 100+j35 MVA. The collapse occurs when the load at bus9 is about 379.6+151.84

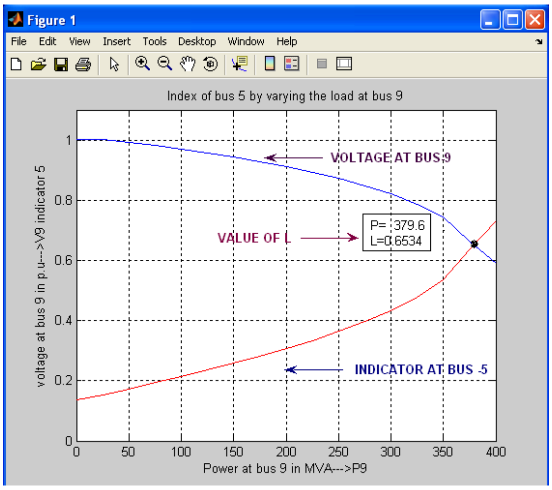

Figure 2.9: indicator for bus 5 with increased loading at bus 9.

Table 2.9: Data for variation of load at bus9 and for voltage at bus9 and for indicator L (5).

P9(M.W)	0	100	200	300	377
V9(p.u)	1.001	0.970	0.929	0.849	0.663
L59	0.1352	0.2142	0.3070	0.4338	0.7302

Case III(c): increasing load at bus9 and observing the indicator L at bus 7.
As shown in graph. The voltage collapse point would not identified with index of L7 for the increasing of load at bus 9. For this simulation, load at Bus 5 is varied, the load at bus5 is taken as 90+j30MVA and the load at bus7 is taken to be 100+j35 MVA

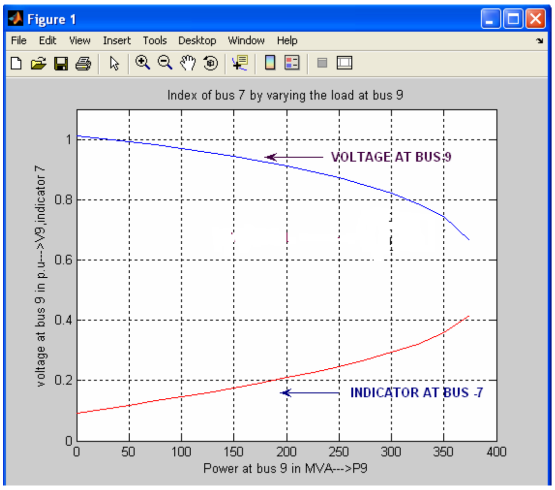

Figure 2.10: indicator for bus 7 with increased loading at bus 9.

Table 2.10: Data for variation of load at bus9 and for voltage at bus9 and for indicator L (7).

P9(M.W)	0	100	200	300	375
V9(p.u)	1.001	0.970	0.929	0.849	0.663
L79	.0926	0.1461	0.2094	0.2929	0.4178

Conclusion

A real time measurement based voltage stability indicator for monitoring of the power systems is presented. We verify our approach by both static and dynamic simulations. We conclude that

- The indicator can predict the voltage stability problem correctly and properly by using both steady-state data as well as dynamic data

- The indicator can be used for both static and dynamic voltage problems
- Through the indicator, it is very easy to locate the vulnerable locations of the system
- The indicator can correctly predict the collapse point of the system.
- In the transaction based power system operation, through the indicator, it is easy to find the responsibility and obligation of the every customer and power supplier.
- The indicator has simple structure and can be handled easily.

The needed information can be obtained through local measurements and data exchanges of among preset buses.

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