

## Voltage Stability Index of Radial Distribution Networks with Distributed Generation

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### Abstract

The voltage stability problem of distribution networks is associated with a rapid voltage drop because of heavy system load. Static load which is known for different characteristics i.e. constant power, constant current, constant impedance and composite load shows a different voltage profile. This paper presents voltage stability analysis of radial distribution networks in the presence of distributed generation. The analysis is accomplished using a voltage stability index which can be evaluated at each node of the distribution system. The location of DG has the main effect voltage stability on the system. Artificial bee colony algorithm (ABC) is proposed to determine the optimal DG-unit size and location by loss sensitivity index (LSI) in order to improve the voltage stability in radial distribution system. Simulation study is conducted on 33-bus radial test system to verify the efficacy of the proposed method.

**Keywords:** distributed generation, artificial bee colony algorithm, voltage stability index, radial distribution network.

### Introduction

Voltage stability of a distribution system is one of the keen interests of industry and research sectors around the world. It concerns stable load operation, and acceptable voltage levels all over the distribution system buses. The distribution system in a power system is loaded more heavily than ever before and operates closer to the limit to avoid the capital cost of building new lines. When a power system approaches the

voltage stability limit, the voltage of some buses reduces rapidly for small increments in load and the controls or operators may not be able to prevent the voltage decay. In some cases, the response of controls or operators may aggravate the situation and the ultimate result is voltage collapse. Voltage collapse has become an increasing threat to power system security and reliability. Many incidents of system blackouts because of voltage stability problems have been reported worldwide. In order to prevent the occurrence of voltage collapse, it is essential to accurately predict the operating condition of a power system. So engineers need a fast and accurate voltage stability index (VSI) to help them monitoring the system condition. Nowadays, a proper analysis of the voltage stability problem has become one of the major concerns in distribution power system operation and planning studies.

Voltage stability concerns stable load operation, and acceptable voltage levels all over the system buses. Its instability has been classified into steady state and transient voltage instability, according to the time spectrum of the occurrence of the phenomena. A power system is said to have entered a state of voltage instability when a disturbance causes a progressive and uncontrollable decline in voltage [1, 2]. Voltage stability analysis often requires examination of lots of system states and many contingency scenarios. For this reason the approach based on steady state analysis is more feasible, and it can also provide global insight of the voltage reactive power problems [2]. The voltage stability phenomenon has been well recognized in distribution systems. Radial distribution systems having a high resistance to reactance ratio causes a high power loss so that the radial distribution system is one of the power systems, which may suffer from voltage instability [1, 3].

Voltage stability index is numerical solution which helps operator to monitor how close the system is to collapse or to initiate automatic remedial action schemes to prevent voltage collapse. The main objective of VSI is to find the distance from current operating point to the marginally stable point. The purpose of finding VSI is to find most sensitive node of the system. Voltage collapse starts at the most sensitive node and then spread out to other sensitive nodes.

DG renders a group of advantages, such as, economical, environmental and technical. The economical advantages are reduction of transmission and distribution cost, electricity price and saving of fuel. Environmental advantages entail reductions of sound pollution and emission of green house gases. Technical advantages cover wide varieties of benefit, like, line loss reduction, peak shaving, increased system voltage profile and hence increased power quality and relieved transmission and distribution congestion as well as grid reinforcement. It can also provide the stand-alone remote applications with the required power. So, optimal placement of DGs and optimal sizing attract active research interests.

Due to considerable costs, the DGs must be allocated suitably with optimal size to improve the system performance such as to reduce the system loss, improve the voltage profile while maintaining the system stability. The problem of DG planning has recently received much attention by power system researchers. Selecting the best places for installing DG units and their preferable sizes in large distribution systems is a complex combinatorial optimization problem.

Literature survey shows that a lot of work has been done on the voltage stability analysis of transmission systems [4] but very little work has been reported on the voltage stability analysis of radial distribution systems. Jasmon and Lee [5] and Guleina and Strmchnic [6] have studied the voltage stability analysis of radial distribution systems. They have represented the whole system by a single line equivalent. The single line equivalent derived by these authors [5, 6] is valid only at the operating point at which it is derived. It can be used for small load changes around this point.

The effect of DG capacity and location on voltage stability analysis of radial distribution system is investigated in this paper. The analysis process is performed using a steady state voltage stability index. This index can be evaluated at each node of radial distribution system.

### Voltage Stability Index

A new steady state voltage stability index is proposed by M. Charkravorty et.al in [7] for identifying the node, which is most sensitive to voltage collapse. Fig. 1 shows the electrical equivalent of radial distribution system.

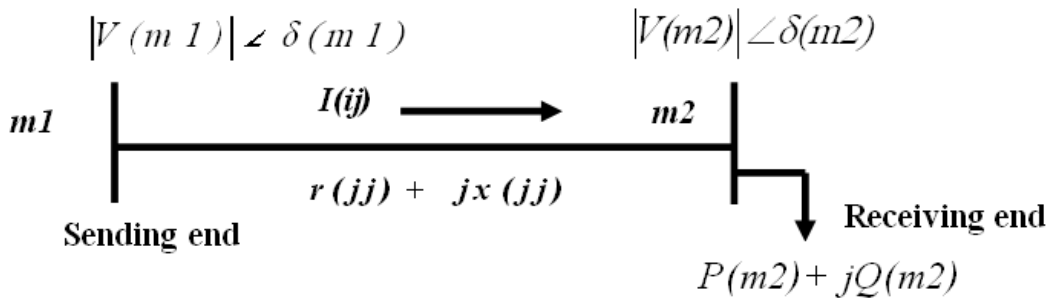


Fig.1: Simple two node system

From Fig. 1, the following equation can be written:

$$I(ij) = \frac{V(m1) - V(m2)}{r(jj) + jx(jj)} \quad (1)$$

Where

$jj$  = branch number,

$m1$  = sending end node,

$m2$  = receiving end node,

$I(ij)$  = current of branch  $ij$ ,

$V(m1)$  = voltage of node  $m1$ ,

$V(m2)$  = voltage of node  $m2$ ,

$P(m2)$  = total real power load fed through node  $m2$ ,

$Q(m2)$  = total reactive power load fed through node  $m2$ .

From Eq. (1)

$$|V(m2)|^4 - b(jj)|V(m2)|^2 + c(jj) = 0 \quad (2)$$

Let,

$$b(jj) = \{ |V(m1)|^2 - 2P(m2)r(jj) - 2Q(m2)x(jj) \} \quad (3)$$

$$c(jj) = \{ P^2(m2) + Q^2(m2) \} \{ r^2(jj) + x^2(jj) \} \quad (4)$$

The solution of Eq. (2) is unique. That is

$$|V(m2)| = 0.707 \left[ b(jj) + \{ b^2(jj) - 4c(jj) \}^{\frac{1}{2}} \right]^{\frac{1}{2}} \quad (5)$$

$$b^2(jj) - 4c(jj) \geq 0 \quad (6)$$

From Eqs. (3), (4) and (6) we get

$$\left\{ |V(m1)|^2 - 2P(m2)r(jj) - 2Q(m2)x(jj) \right\}^2 - 4 \{ P^2(m2) + Q^2(m2) \} \{ r^2(jj) + x^2(jj) \} \geq 0$$

After simplification we get

$$\left\{ |V(m1)|^4 \right\} - 4 \{ P(m2)x(jj) - Q(m2)r(jj) \}^2 - 4 \{ P(m2)r(jj) + Q(m2)x(jj) \} |V(m1)|^2 \geq 0 \quad (7)$$

Let

$$SI(m2) = \left\{ |V(m1)|^4 \right\} - 4.0 \{ P(m2)x(jj) - Q(m2)r(jj) \}^2 - 4.0 \{ P(m2)r(jj) + Q(m2)x(jj) \} |V(m1)|^2 \quad (8)$$

Where

$SI(m2)$  = voltage stability index of node  $m2$ .

For stable operation of the radial distribution networks,  $SI(m2) \geq 0$ .

The node at which the value of the stability index is minimum, is more sensitive to the voltage collapse.

## Load Modelling

In distribution systems, voltages vary widely along system feeders as there are fewer voltage control devices. Therefore, the V–I characteristics of load are more important in distribution system load flow studies. The real and reactive power loads of node ‘ $i$ ’ is given as:

$$PL(i) = PL_o(i) \left[ c1 + c2|V(i)| + c3|V(i)|^2 \right] \quad (9)$$

$$QL(i) = QL_o(i) \left[ d1 + d2|V(i)| + d3|V(i)|^2 \right] \quad (10)$$

Static load models are typically categorized as follows

Constant power load model (constant P): A static load model where the power does not vary with changes in voltage magnitude. It is also known as constant MVA load model. For constant power load,  $c1=d1=1$ ,  $c2=c3=d2=d3=0$ .

Constant current load model (constant I): A static load model where the power varies directly with voltage magnitude. For constant current load,  $c2=d2=1$ ,  $c1=c3=d1=d3=0$ .

Constant impedance load model (constant Z): A static load model where the power varies with the square of the voltage magnitude. It is also referred to as constant admittance load model. For constant impedance load,  $c3=d3=1$ ,  $c1=c2=d1=d2=0$ .

Composite load model: A composition of 40% constant power, 30% of constant current and 30% of constant impedance loads are considered.

### Artificial Bee Colony Algorithm (Abc)

Artificial Bee Colony (ABC) is one of the most recently defined algorithms by Dervis Karaboga in 2005, motivated by the intelligent behavior of honeybees. ABC as an optimization tool provides a population based search procedure in which individuals called food positions are modified by the artificial bees with time and the bee's aim is to discover the places of food sources with high nectar amount and finally the one with the highest nectar. In this algorithm [8, 9], the colony of artificial bees consists of three groups of bees: employed bees, onlookers and scouts. First half of the colony consists of the employed artificial bees and the second half includes the onlookers. For every food source, there is only one employed bee. In other words, the number of employed bees is equal to the number of food sources around the hive. The employed bee whose food source has been abandoned becomes a scout [10].

Thus, ABC system combines local search carried out by employed and onlooker bees, and global search managed by onlookers and scouts, attempting to balance exploration and exploitation process [11].

The ABC algorithm creates a randomly distributed initial population of solutions ( $f = 1, 2, \dots, E_b$ ), where ' $f$ ' signifies the size of population and ' $E_b$ ' is the number of employed bees. Each solution  $x_f$  is a D-dimensional vector, where D is the number of parameters to be optimized. The position of a food-source, in the ABC algorithm, represents a possible solution to the optimization problem, and the nectar amount of a food source corresponds to the quality (fitness value) of the associated solution. After initialization, the population of the positions (solutions) is subjected to repeated cycles of the search processes for the employed, onlooker, and scout bees (cycle = 1, 2, ..., MCN), where MCN is the maximum cycle number of the search process. Then, an employed bee modifies the position (solution) in her memory depending on the local information (visual information) and tests the nectar amount (fitness value) of the new position (modified solution). If the nectar amount of the new one is higher than that of the previous one, the bee memorizes the new position and forgets the old one. Otherwise, she keeps the position of the previous one in her memory. After all employed bees have completed the search process; they share the nectar information

of the food sources and their position information with the onlooker bees waiting in the dance area. An onlooker bee evaluates the nectar information taken from all employed bees and chooses a food source with a probability related to its nectar amount. The same procedure of position modification and selection criterion used by the employed bees is applied to onlooker bees. The greedy-selection process is suitable for unconstrained optimization problems. The probability of selecting a food-source  $p_f$  by onlooker bees is calculated as follows:

$$P_f = \frac{\text{fitness}_f}{\sum_{f=1}^{E_b} \text{fitness}_f} \quad (11)$$

Where  $\text{fitness}_f$  is the fitness value of a solution  $f$ , and  $E_b$  is the total number of food-source positions (solutions) or, in other words, half of the colony size. Clearly, resulting from using (11), a good food source (solution) will attract more onlooker bees than a bad one. Subsequent to onlookers selecting their preferred food-source, they produce a neighbor food-source position  $f+1$  to the selected one  $f$ , and compare the nectar amount (fitness value) of that neighbor  $f+1$  position with the old position. The same selection criterion used by the employed bees is applied to onlooker bees as well. This sequence is repeated until all onlookers are distributed. Furthermore, if a solution  $f$  does not improve for a specified number of times (limit), the employed bee associated with this solution abandons it, and she becomes a scout and searches for a new random food-source position. Once the new position is determined, another ABC algorithm (MCN) cycle starts. The same procedures are repeated until the stopping criteria are met.

In order to determine a neighboring food-source position (solution) to the old one in memory, the ABC algorithm alters one randomly chosen parameter and keeps the remaining parameters unchanged. In other words, by adding to the current chosen parameter value the product of the uniform variant  $[-1, 1]$  and the difference between the chosen parameter value and other “random” solution parameter value, the neighbor food-source position is created. The following expression verifies that:

$$x_{fg}^{new} = x_{fg}^{old} + u(x_{fg}^{old} - x_{mg}) \quad (12)$$

Where  $m \neq 1$  and both are  $\in \{1, 2, \dots, E_b\}$ . The multiplier  $u$  is a random number between  $[-1, 1]$  and  $j \in \{1, 2, \dots, D\}$ . In other words,  $x_{fg}$  is the  $g^{th}$  parameter of a solution  $x_f$  that was selected to be modified. When the food-source position has been abandoned, the employed bee associated with it becomes a scout. The scout produces a completely new food-source position as follows:

$$x_{fg}^{(new)} = \min(x_{fg}) + u \left[ \max(x_{fg}) - \min(x_{fg}) \right] \quad (13)$$

Where (13) applies to all  $g$  parameters and  $u$  is a random number between  $[-1, 1]$ .

If a parameter value produced using (12) and/or (13) exceeds its predetermined limit, the parameter can be set to an acceptable value. In this paper, the value of the parameter exceeding its limit is forced to the nearest (discrete) boundary limit value associated with it. Furthermore, the random multiplier number  $u$  is set to be between  $[0, 1]$  instead of  $[-1, 1]$

Thus, the ABC algorithm has the following control parameters: 1) the colony size  $CS$ , that consists of employed bees  $E_b$  plus onlooker bees  $E_b$ ; 2) the limit value, which is the number of trials for a food-source position (solution) to be abandoned; and 3) the maximum cycle number  $MCN$ .

The proposed ABC algorithm for finding size of DG at selected location is as follows:

*Step-1:* Initialize the food-source positions  $x_i$  (solutions population), where  $f = 1, 2, \dots, E_b$ . The  $x_f$  solution form is as follows.

*Step-2:* Calculate the nectar amount of the population by means of their fitness values using

$$fitness = \frac{1}{1 + powerloss} \quad (14)$$

*Step-3:* Produce neighbor solutions for the employed bees by using Eqs. (12) and evaluate them as indicated by Step 2.

*Step-4:* Apply the selection process.

*Step-5:* If all onlooker bees are distributed, go to Step 9. Otherwise, go to the next step.

*Step-6:* Calculate the probability values  $P_f$  for the solutions  $x_f$  using Eqs. (11)

*Step-7:* Produce neighbor solutions for the selected onlooker bee, depending on the value, using Eqs. (12) and evaluate them as Step 2 indicates.

*Step-8:* Follow Step 4.

*Step-9:* Determine the abandoned solution for the scout bees, if it exists, and replace it with a completely new solution using Eqs. (13) and evaluate them as indicated in Step 2.

*Step-10:* Memorize the best solution attained so far.

*Step-11:* If cycle = MCN, stop and print result. Otherwise follow Step 3.

## Results and Analysis

To check the effectiveness of the proposed method, 33-bus radial distribution network [7] is considered. First, load flow [12] is conducted for 33-bus bus test system for base case. The loss sensitivity factors [13] at different buses have been evaluated to select appropriate buses for DG planning. These sensitivity factors reflect how the feeder power losses change if more real power is injected at a particular bus and it also allows obtaining the candidate buses to locate DG. Loss sensitivity factors are evaluated for the base case first to decide the first appropriate location. Artificial bee colony algorithm (ABC) is proposed to determine the optimal DG-unit size in order to improve the voltage stability in radial distribution system. Critical loading condition

for different types of load and different values of substation voltage results before and after DG placement is shown in Table 1 and 2. The Control parameters of ABC method are colony size ( $C_s$ ) is 30 and  $MCN$  is 40.

**Table 1:** Critical loading condition for different types of load without DG

Load type	Substation voltage (p.u)	Critical loading condition			
		TPL (MW)	TQL (MVA <sub>r</sub> )	SI <sub>min</sub>	V <sub>min</sub> (p.u)
Constant Power (CP)	1.000	13.456	8.3306	0.4598	0.5796
	1.025	14.139	8.7538	0.4624	0.5908
	1.050	14.838	9.1862	0.4631	0.6043
Constant current (CI)	1.000	26.971	16.698	0.4851	0.6559
	1.025	27.640	17.112	0.4850	0.6723
	1.050	28.308	17.526	0.4847	0.6888
Constant impedance (CZ)	1.000	58.697	36.340	0.5063	0.6170
	1.025	58.722	36.356	0.5067	0.6324
	1.050	58.723	36.356	0.5065	0.6478
Composite load	1.000	21.220	13.138	0.4755	0.6701
	1.025	22.026	13.637	0.4758	0.6864
	1.050	22.836	14.138	0.4761	0.7027

A DG is connected at node 6, it increase and support the voltage and stability in the system. The optimal size of DG is shown in Table 2 for different load models and different values of substation voltage. The connection point of DG influences the voltage stability in the system. DG strongly supports the voltage at nearby nodes and has less impact on distant nodes.

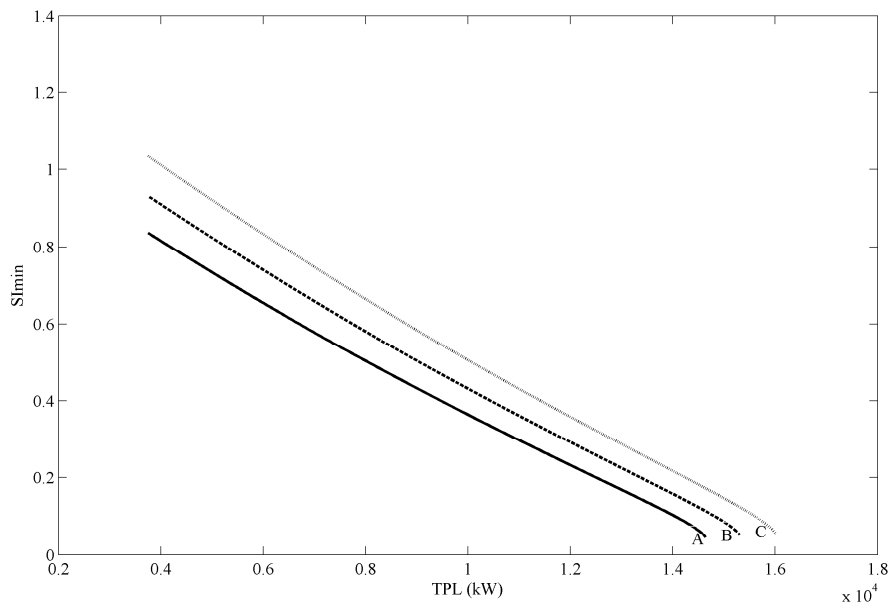
From Table 2, it is seen that the critical loading for constant impedance load is the maximum and that for constant power load is minimum before and after DG placement. The critical loading for constant current lies between these two and that for the composite load solely depends on the percentage composition of the three loads. The stability index and consequently the voltage are minimum for constant power load and maximum for constant impedance load and that for constant current load is in between these two. Consequently the voltage is minimum for constant impedance load and maximum for constant power load and that for constant current load is in between these two. Similarly, the composition of loads governs the position of the stability index for the composite load.



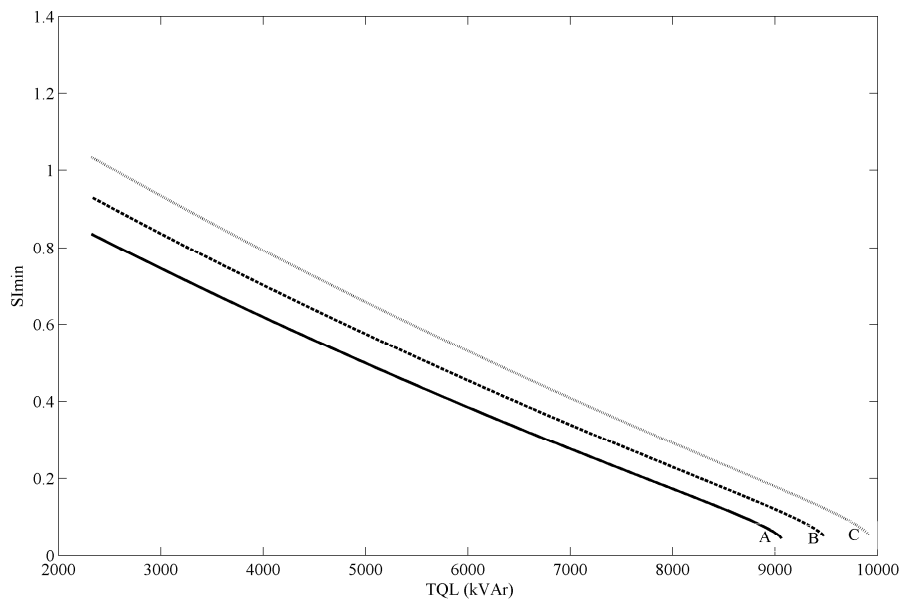
**Table 2:** Critical loading condition for different types of load with DG

Load type	Substation voltage (p.u)	Critical loading condition				DG Size (MW)
		TPL (MW)	TQL (MVA <sub>r</sub> )	SI <sub>min</sub>	V <sub>min</sub> (p.u)	
Constant Power (CP)	1.000	14.637	9.063	0.0515	0.4581	2.5207
	1.025	15.305	9.476	0.0575	0.4732	2.5160
	1.050	16.011	9.913	0.0608	0.4779	2.5122
Constant current (CI)	1.000	28.234	17.480	0.0335	0.4247	2364.53
	1.025	28.902	17.894	0.0369	0.4351	2427.27
	1.050	29.571	18.308	0.0405	0.4455	2487.50
Constant impedance (CZ)	1.000	58.756	36.376	0.0273	0.4046	2230.46
	1.025	58.757	36.377	0.0302	0.4147	2343.71
	1.050	58.759	36.377	0.0332	0.4249	2459.92
Composite load	1.000	22.624	14.007	0.0360	0.4310	2378.86
	1.025	23.404	14.490	0.0400	0.4429	2432.01
	1.050	24.221	14.996	0.0437	0.4528	2486.22

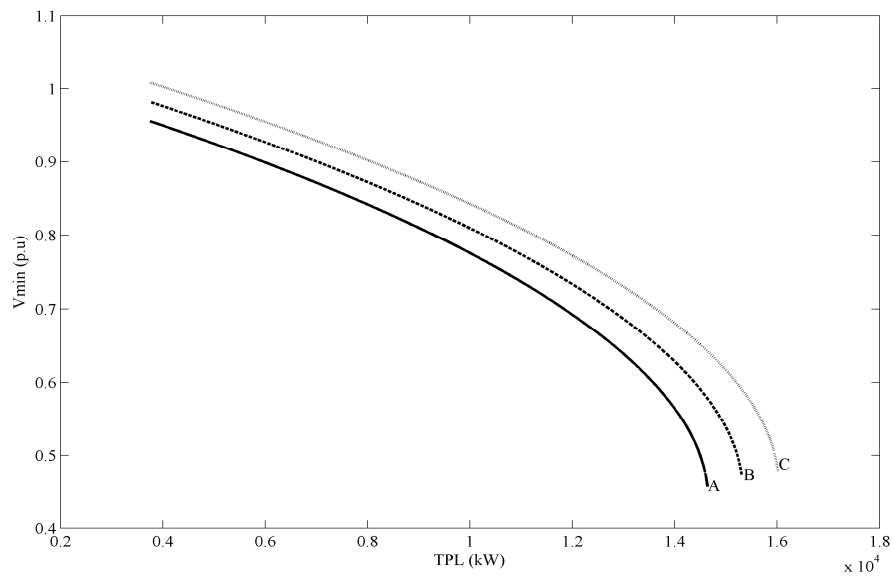
The total real power load, total reactive power load, minimum voltage stability index and minimum voltage without DG for constant power load at 1.0 p.u substation voltage are 13.456 MW, 8.3306 MVA<sub>r</sub>, 0.4598 and 0.5796 p.u , they are improved to 14.637 MW, 9.063 MVA<sub>r</sub>, 0.0515 and 0.4581 p.u after DG placement. For constant current load, The total real power load, total reactive power load, minimum voltage stability index and minimum voltage without DG for 1.0 p.u substation voltage are 26.971 MW, 16.698 MVA<sub>r</sub>, 0.4851 and 0.6559 p.u , after DG placement they are improved to 28.234 MW, 17.480 MVA<sub>r</sub>, 0.0335 and 0.4247 p.u . For constant impedance load, The total real power load, total reactive power load, minimum voltage stability index and minimum voltage without DG for 1.0 p.u substation voltage are 58.697 MW, 36.34 MVA<sub>r</sub>, 0.5063 and 0.6170 p.u , after DG placement they are improved to 58.756 MW, 36.376 MVA<sub>r</sub>, 0.0273 and 0.4046 p.u . The total real power load, total reactive power load, minimum voltage stability index and minimum voltage without DG for composite load at 1.0 p.u substation voltage are 21.22 MW, 13.138 MVA<sub>r</sub>, 0.4755 and 0.6701 p.u , they are improved to 22.624 MW, 14.007 MVA<sub>r</sub>, 0.0360 and 0.4310 p.u after DG placement.



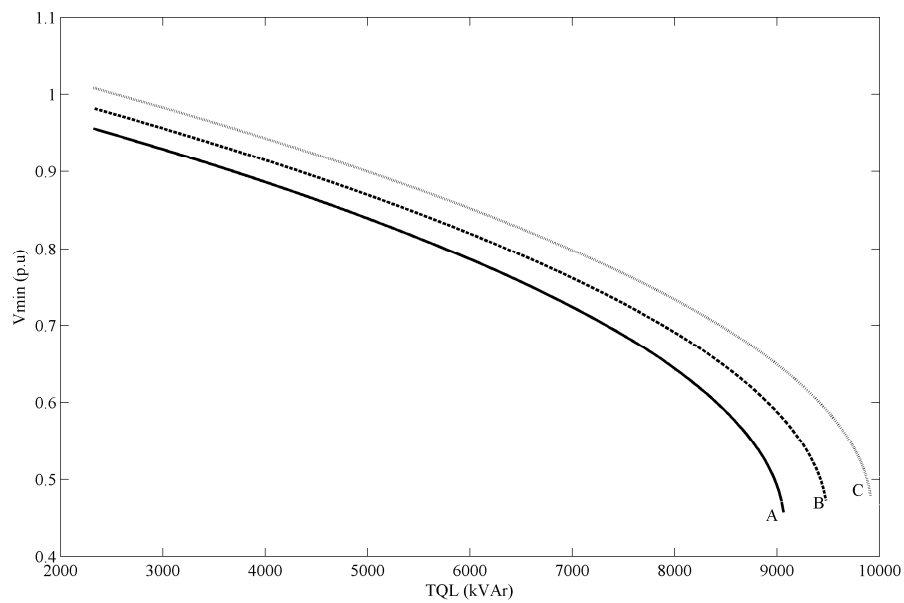
**Fig.2** Plot of  $TPL$  vs  $S_{Imin}$  with DG



**Fig. 3** Plot of  $TQL$  vs  $S_{Imin}$  with DG



**Fig. 4** Plot of *TPL* vs *Vmin* with DG



**Fig.5** Plot of *TQL* vs *Vmin* with DG

The plots of total real power load versus voltage stability index with DG is shown in Fig. 2 and the total reactive power load versus voltage stability index with DG for constant power load at different substation voltage is shown Fig. 3. Figs 4 and 5 show

the plots of total real power load versus minimum voltage and total reactive power load versus minimum voltage with DG for constant power load at different substation voltage. A, B and C indicate the critical loading point with DG beyond which a small increment of loading causes the voltage collapse.

## Conclusions

This paper presents an artificial bee colony algorithm to place the DG optimally in radial distribution system to improve the voltage stability. Using voltage stability index, it is possible to compute the voltage stability index at every node and identify the node at which the value of the voltage stability index is minimum and is most sensitive to voltage collapse. Effectiveness of the proposed method has been demonstrated through a 33-bus radial distribution network. Different load models, i.e., constant power, constant current, constant impedance and composite load modelling are considered for the purpose of voltage stability analysis. It was observed that before and after DG placement the critical loading for constant current load is maximum and constant power load is minimum. But a great improvement in voltage stability and critical loading conditions for all load models after DG placement.

## References

- [1] M. Z. El-Sadek, "Power System Voltage Stability and Power Quality", Mukhtar Press, Assuit, Egypt, 2002.
- [2] G. M. Huang and L. Zhao, "Measurement based voltage stability monitoring of power system", Available: [www.pserc.wisc.edu](http://www.pserc.wisc.edu)
- [3] M. Moghavvemi and M. O. Faruque, "Technique for assessment of voltage stability in ill-conditioned radial distribution network", IEEE Power Engineering Review, pp. 58-60, January 2001.
- [4] V. Ajjarapu, B. Lee: "Bibliography on voltage stability", IEEE Transaction on power systems, Vol. 13, No. 1, pp.115, February 1988.
- [5] G.B. Jasmon, L.H.C.C. Lee: "Distribution network reduction for voltage stability analysis and load flow calculation", International journal of electrical power and energy systems, Vol. 13, No. 1, pp. 9, February 1991.
- [6] F. Gubina, B. Strmcnik: "A simple approach to voltage stability assessment in radial networks", IEEE Trans. on power system, Vol. 12, No. 3, pp. 1121, August 1997.
- [7] M. Charkravorty and D. Das, "Voltage stability analysis of radial distribution networks", International Journal of Electrical Power & Energy Systems, Vol. 23, No. 2, pp. 129-135, 2001.
- [8] Dervis Karaboga and Bahriye Basturk, "Artificial Bee Colony (ABC) Optimization Algorithm for Solving Constrained Optimization Problems", Springer-Verlag, IFSA 2007, LNAI 4529, pp. 789–798, 2007.

- [9] Karaboga, D. and Basturk, B., “On the performance of artificial bee colony (ABC) algorithm”, Elsevier Applied Soft Computing, Vol. 8, pp. 687–697, 2007.
- [10] S.Hemamalini and Sishaj P Simon., “Economic load dispatch with valve-point effect using artificial bee colony algorithm”, xxxii national systems conference, NSC 2008, pp. 17-19, 2008.
- [11] Fahad S. Abu-Mouti and M. E. El-Hawary “Optimal Distributed Generation Allocation and Sizing in Distribution Systems via artificial bee Colony algorithm”, IEEE transactions on power delivery, Vol. 26, No. 4, 2011.
- [12] S.Ghosh and D.Das “Method for load-flow solution of radial distribution networks”, IEE, ZEE Proceedhgs Online No. 19990464, Vol.146, 1999.
- [13] T. N. Shukla, S.P. Singh, Srinivasarao and K. B. Naik “Optimal sizing of distributed generation placed on radial distribution systems”, Electric power components and systems, Vol. 38, pp. 260-274, 2010.

