International Journal of Electrical Engineering.
ISSN 0974-2158 Volume 6, Number 1 (2013), pp. 89-97

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# General Robust Stability Analysis of Uncertain system

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## Abstract

The main objective of this paper is to investigate the robust control aspect of chaotic and uncertain system. The robust stability analysis of closed loop attitude control system of an aircraft is presented using the Kharitonov theorem. The proposed technique for examining the robust stability of chaotic polynomial is efficient and expeditious one as the onerous application of Routh stability criterion for infinite no of interval polynomials of the perturbed system can be completely dispensed with.

**Keywords-** Robust control, Kharitonov's theorem, Routh criterian, Interval systems

## Introduction

Study and design of robust system has been a matter of concern and drawn the attention of many authors in recent years [1-3]. The designed system is said to be robust if it remains insensitive to the presence of parametric uncertainties. Incorporation of robust features in the modern control design is indispensably necessary. Here, in this paper we take into consideration the example of the precise attitude control of an aircraft which is variedly used in aerospace application. For the precise attitude control of an aircraft we need to control the positions of the fins of a modern aircraft. Fins are mostly the flattened part that projects from an aircraft for providing stability. Owing to the requirement of improved response and reliability, modern aircraft are controlled by electric actuators and electronic control [1]. Initially in order to check the stability of characteristic polynomial by Routh criterion it was assumed that the coefficient of characteristic polynomial are constant but practically it does not happen. The coefficients are bound to change under extraneous circumstances in certain range and thus lead to the formation of interval polynomial.

The stability of such kind of polynomials where coefficients are varying in nature has attracted the attention of many researchers. These systems are not immune and insensitive to parameter variation. When we envisage the design of attitude control aircraft under parametric variation, the robust stability analysis seems to be extremely important [7, 8]. This analysis is done using Kharitonov theorem which entails formations of four polynomials (from the parameter variation within some extremes) and application of Routh criterion to these four polynomials only to determine the system stability under perturbation [2]. Earlier, in [3-6], [9, 10], the robust stability analysis was carried out by determining all the uncertain values within some defined extremes as dictated by Kharitonov's theorem. Here we have proposed the method shown by [2] for the robust stability analysis of attitude control system of an aircraft. The main objective of this paper is to present an analysis of the robust stability of attitude control of an aircraft under variations in circuit parameters. The proposed method is found to be efficient and computationally simpler for an infinite no of interval polynomials of the perturbed system.

## Mathematical Preliminaries

In classical control the coefficient of the characteristic equation was assumed to be constant. Practically the coefficients are the functions of energy storing or dissipating elements which may change its value due to external conditions. So the same characteristic equation will now represent an infinite family of characteristic polynomial when the components are subjected to uncertainties. Robust control design necessitates the study and formulation of interval polynomial. The concept of interval analysis is useful in the study of robust control of systems with parametric uncertainty within known bounds. The uncertain parameter is incorporated in terms of interval entity and the robust stability is analysed here by Kharitonov's theorem.

### Kharitonov's Theorem [2]

When we deal with the robust stability of interval polynomial i.e., when the coefficients of characteristics equation are no longer constant, Kharitonov theorem come to a great rescue. Suppose that a family of polynomials is given by

$$s^n + a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_{n-1} s + a_n \tag{1}$$

where $\alpha_i \leq a_i \leq \beta_i$ , $1 \leq i \leq n$.

According to Kharitonov's theorem, the family of polynomials (1) are said to be stable if and only if the following four polynomials are stable:

$$s^n + \alpha_1 s^{n-1} + \alpha_2 s^{n-2} + \beta_3 s^{n-3} + \beta_4 s^{n-4} + \cdots \tag{2}$$

$s^n + \alpha_1 s^{n-1} + \beta_2 s^{n-2} + \beta_3 s^{n-3} + \alpha_4 s^{n-4} + \cdots$ (3)

$s^n + \beta_1 s^{n-1} + \beta_2 s^{n-2} + \alpha_3 s^{n-3} + \alpha_4 s^{n-4} + \cdots$(4)

$s^n + \beta_1 s^{n-1} + \alpha_2 s^{n-2} + \alpha_3 s^{n-3} + \beta_4 s^{n-4} + \cdots$ (5)

## Attitude Control System Of An Aircraft

In order to have the precise attitude control of an aircraft one need to control the positions of the fins of a modern aircraft. Fins are mostly the flattened part that projects from an aircraft for providing stability. Owing to the requirement of improved response and reliability, modern aircraft are controlled by electric actuators and electronic control.

Figure 1 and figure 2 shows the controlled surfaces and simplified block diagram of one axis of such a position control or attitude control system. Figure 3 shows the analytical block diagram of the same system using expanded model of dc motor. The system considered here is simplified to the extent that saturation of the amplifier gain and gear backlash etc has been neglected. $\Theta_r$ is the desired output (desired position of control surface) and $\Theta_y$ is the actual output (actual position of control surface) to the system.

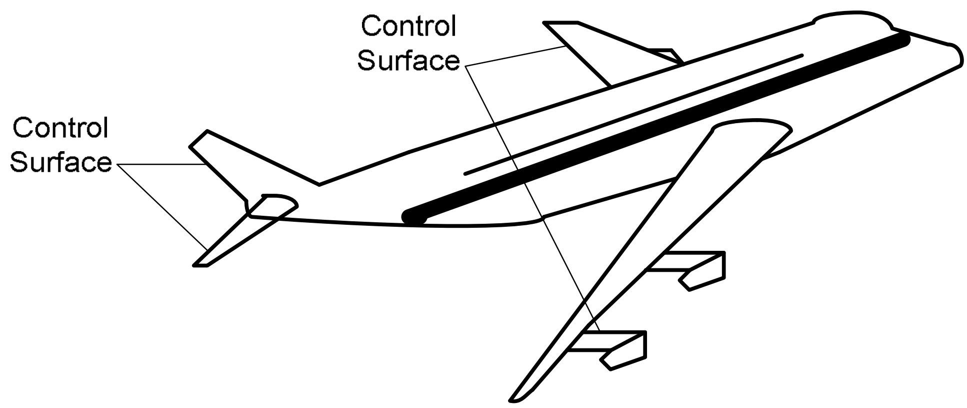


Fig.1 Schematic diagram of control surfaces of Aircraft

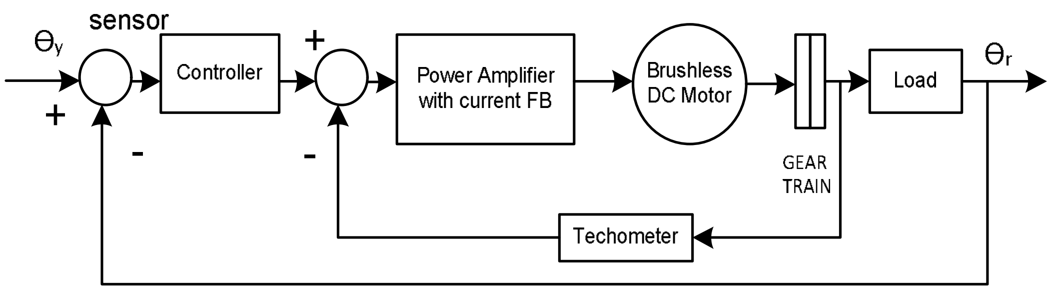


Fig.2 Block diagram of an attitude control system of an aircraft

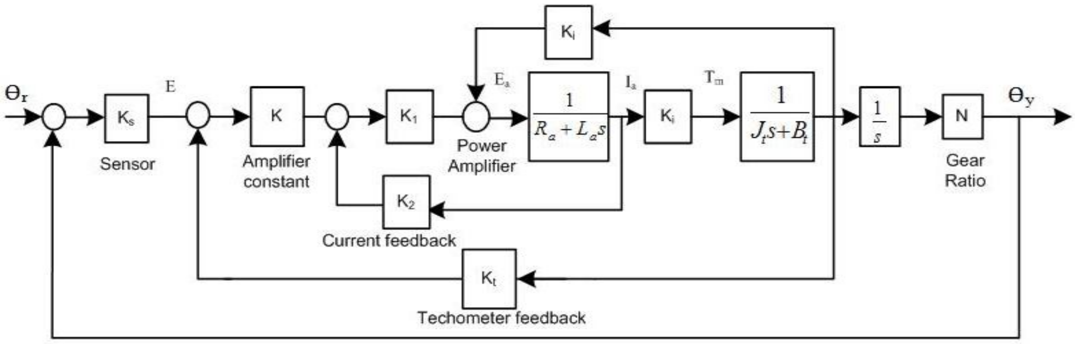


Fig.3 Analytical block diagram of an attitude control system using expanded model of DC motor.

The open-loop transfer function of an attitude control system of aircraft is given as [1]

$$G(s) = \frac{K_S K_1 K_i K N}{s[L_a J_t s^2 + (R_a J_t + L_a B_t + K_1 K_2 J_t)s + R_a B_t + K_1 K_2 B_t + K_i K_b + K K_1 K_t K_i]} \quad (6)$$

where $J_t = J_m + N^2 J_L$ and $B_t = B_m + N^2 B_L$. The implications of other symbols used in (6) are given and illustrated in appendix. In this section we presume that the components and parameters used in (6) are constant and in the next section, realistic uncertainty in parameter variation is taken into consideration for analysis of robust aspect of control design.

If $K_s = 1$, $K$ =adjustible, $K_1 = 10$, $K_2 = 0.5$, $K_t = 0$, $R_a = 5$, $L_a = 0.003$, $K_i = 9$, $K_b = 0.0636$, $J_m = 0.0001$, $J_L = 0.0.01$, $B_m = 0.005$, $B_L = 1$, $N = 0.1$

Using (6), the open loop transfer function of attitude control system becomes:

$$G(s) = \frac{1.5\times10^7 K}{s(s^2+3408.3s+1204000)} = \frac{1.5\times10^7 K}{s(s+400.26)(s+3008)} \quad (7)$$

The closed loop transfer function is

$$\frac{\Theta_y(s)}{\Theta_r(s)} = \frac{1.5\times10^7 K}{s^3+3408.3s^2+1204000s+1.5\times10^7 K} \quad (8)$$

**Robust Stability Of Attitude Control System**

In terms of the components and parameters of attitude control system in fig. 2, the characteristic polynomial of the closed loop system is given as

$$s^3 + \frac{R_aJ_t+L_aB_t+K_1K_2J_t}{L_aJ_t}s^2 + \frac{R_aB_t+K_1K_2B_t+K_iK_b+KK_1K_tK_i}{L_aJ_t}s + \frac{K_SK_1K_iKN}{L_aJ_t} \tag{9}$$

This is nothing but a polynomial having variable coefficients. The stability of such kind of polynomials where coefficients are varying in nature is ubiquitously present and has attracted the attention of many researchers. These systems are not immune and insensitive to parameter variation. Hence robust stability analyses of such systems seem to be indispensable one because these are variedly used in aerospace application. In practice the parameters of (9) are not constant, because armature resistance of motor $R_a$ and armature inductance of motor $L_a$, change with time because of aging effects. In a similar fashion, other parameter such as load inertia $J_L$, inertia of motor rotor $J_m$, back emf constant of motor $K_b$, gain of preamplifier $K$, etc are also not constant and may vary. Therefore presumably it is specified that the different components and feedback gains are subject to the following ranges of uncertainty.

$$K_s = 1 \pm 0.5\frac{V}{rad}, K = 100 \pm 50, K_1 = 10 \pm 4\frac{V}{V}, K_2 = 0.5 \pm \frac{0.2V}{A}, K_t = 0, R_a = 5.0 \pm 2.0\Omega, L_a = 0.003 \pm .001H, K_i = 9 \pm 4, K_b = 0.0636 \pm 0.0100V/rad/sec, J_m = 0.0001 \pm 0.00005, J_L = 0.01 \pm 0.005, B_m = 0.005 \pm 0.001, B_L = 1.0 \pm 0.5, N = 0.1 \tag{10}$$

After taking all these realistic uncertainties into account we examine the robust stability of attitude control system of an aircraft. Robust stability analysis of state feedback control of PWM DC-DC Push-Pull converter using Kharitonov's theorem has been presented in [4]. Here we will apply Kharitonov's theorem to our system.

### *A. Robust Stability Using Kharitonov's Theorem*

Here the stability of (9) is investigated by testing the stability of a large set of polynomials using Kharitonov's theorem [2]. Characteristics polynomial (9) may be written in the form given below

$$P(s) = s^3 + a_1s^2 + a_2s + a_3 \tag{11}$$

In which the ranges of coefficients between two extremes are $a_1 \in [\alpha_1, \beta_1]$, $a_2 \in [\alpha_2, \beta_2]$ and $a_3 \in [\alpha_3, \beta_3]$. Here we intend to do the robust analysis of characteristic interval polynomial (11) in the following three steps.

*Step 1:* Evaluation of the extremal range of coefficients, i.e.

$$\alpha_1 = \min_q \left\{\frac{R_a J_t + L_a B_t + K_1 K_2 J_t}{L_a J_t}\right\} = 2.4 \times 10^3$$

$$\beta_1 = \max_q \left\{\frac{R_a J_t + L_a B_t + K_1 K_2 J_t}{L_a J_t}\right\} = 4.3 \times 10^3$$

$$\alpha_2 = \max_q \left\{\frac{R_a B_t + K_1 K_2 B_t + K_i K_b + K K_1 K_t K_i}{L_a J_t}\right\} = 10.9 \times 10^5 \qquad (12)$$

$$\beta_2 = \min_q \left\{\frac{R_a B_t + K_1 K_2 B_t + K_i K_b + K K_1 K_t K_i}{L_a J_t}\right\} = 15.1 \times 10^5$$

$$\alpha_3 = \min_q \left\{\frac{K_S K_1 K_i K N}{L_a J_t}\right\} = 3.7 \times 10^7$$

$$\beta_3 = \max_q \left\{\frac{K_S K_1 K_i K N}{L_a J_t}\right\} = 3.4 \times 10^9$$

The vector $q$ represents the vector of the uncertain circuit components $K_s, K, K_1, K_2, K_t, R_a, L_a, K_i, K_b, J_m, J_L, B_m, B_L, N$.In (12) the min (minimum) and max (maximum) of the coefficient of the characteristic polynomial (9) are computed when the components are subjected to the uncertainties as represented in (10). Now we apply the Kharitonov's theorem [2] to (11) for testing the robust stability as the family of polynomials represented by (11) has infinite coefficients. The four Kharitonov polynomials corresponding to (9) are given in step 2 ahead.

*Step 2:* Formulation of all the four Kharitonov polynomials

$$s^3 + 2.4 \times 10^3 s^2 + 10.9 \times 10^5 s + 3.4 \times 10^9 \qquad (13)$$

$$s^3 + 2.4 \times 10^3 s^2 + 15.1 \times 10^5 s + 3.4 \times 10^9 \qquad (14)$$

$$s^3 + 4.3 \times 10^3 s^2 + 15.1 \times 10^5 s + 3.7 \times 10^7 \qquad (15)$$

$$s^3 + 4.3 \times 10^3 s^2 + 10.9 \times 10^5 s + 3.7 \times 10^7 \qquad (16)$$

*Step 3:* Checking for robust stability

Now applying the Routh criterion to (13) through (16) it is found that all the four Kharitonov polynomials satisfy the Hurwitz stability conditions. Hence the chaotic attitude control system represented by (9) is stable.

## Conclusions

In this paper we have proposed Kharitonov's theorem in conjunction with Routh stability criterion for the robust stability analysis of attitude control of an aircraft. Based on this theorem, arrangement of four Kharitonov's polynomial is presented using upper and lower bounds. Here it is observed that the perturbed system is robust stable in the specified range of all the component values. Thus the computational cost can be reduced to a great extent for numerous interval polynomials. Moreover in future similar application of Kharitonov theorem can be developed for the robust stability of digital control system.

APPENDIX

**Nomenclatures and Abbreviations':**

$K_S$ = Gain of encoder
$K$ = Gain of the preamplifier
$K_1$ = Gain of power amplifier
$K_2$ = Gain of current feedback
$K_t$ = Gain of tachometer feedback
$R_a$ = Armature resistance of motor
$L_a$ = Armature inductance of motor
$K_i$ = Torque constant of motor
$K_b$ = Back-emf constant of motor
$J_m$ = Inertia of motor rotor
$J_L$ = Inertia of load
$B_m$ = Viscous-friction coefficient of motor
$B_L$ = Viscous-friction coefficient of load
$N$ = Gear-train ratio between motor and load
$\Theta_y$ = Position of control surface
$\Theta_r$ = Reference or desired Position of control surface

## References

[1] B. C. Kuo, F. Golnaraghi, "Automatic Control System," John Wiley & Sons, Eighth Edition, 2003.

[2] V. L. Kharinotov, "Asymptotic stability of an equilibrium position of a family of systems of linear differential equations," *Differ. Equa.*, vol. 14, pp. 2086-2088, 1978.

[3] Yogesh V. Hote, D. Roy Choudhary, and J.R.P. Gupta. "On Robust Stability of the Systems," *International Journal of Control, Automation, and Systems*, vol. 7, no.3 .pp. 469-474, Aug. 2009.

[4] D. Czarkowski, L. R. Pujaria, and M. K. Kazimierczuk, "Robust stability of state-feedback control of PWM DC-DC push-pull converter," *IEEE Trans.*

*IND. Electron.*, vol.42, no.1, pp. 108-111, Feb. 1995.

[5] L. R. Pujaria, “Robust stability of PWM Buck DC-DC converter,” in Proc. 1996 *IEEE INT. Conf. Control Appl.*, Dearbon, MI, pp.632-636.

[6] L. R. Pujaria, “On the stability of uncertain polynomials with dependent coefficients,” *IEEE Trans. Autom. Control*, vol. 35, no. 6, pp.756-759, Jun 1990.

[7] B. L. Stevens and F. L. Lewis, “Aircraft Control and Simulation,” Hoboken, New Jersey: John Wiley & Sons, Inc., 2nd ed., 2003.

[8] David G. Hull, “Fundamentals of Airplane Flight Mechanics,” Springer-Verlog London Limited, 2009.

[9] S. P. Bhattacharyya, Aniruddha Datta, and L. H. Keel, “Linear Control Theory: Structure, Robustness and Optimization,” CRC Press, Taylor and Francis Group, New York, 2009.

[10] T. Meressi, D. Chen, and B. Paden, “Application of Kharitonov’s theorem to Mechanical Systems,” *IEEE Trans. on Automatic Control,* vol. 38, no.3, pp. 488-491, 1993.

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