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Fuzzy Logic Based Thresholding for Hyper Shrinkage

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Abstract

Signal denoising is the process of reducing the unwanted noise in order to restore the original signal. Donoho and Johnstone's denoising algorithm based on wavelet thresholding replace the small coefficients by zero and keep or shrink the coefficients with absolute value above the threshold. So the threshold selection becomes more important in signal denoising. In this paper the threshold selection based on Fuzzy Logic concepts for Hyper Shrinkage is developed. Fuzzy logic represents a good mathematical framework to deal with uncertainty of information. A fuzzy membership function (MF) is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. Wavelet Transform (WT) is useful for analyzing the non-stationary signal. The ElectroCardioGram (ECG) signal contains important information about the heart and here ECG signal is used to verify the proposed method. The Discrete Wavelet Transform (DWT) provides sufficient information both for analysis and synthesis of the original signal, with a significant reduction in the computation time. The Inverse Discrete Wavelet Transform (IDWT) provides the reconstruction of signals. The software used for the simulation is MATLAB.

Keywords: denoising, thresholding, Hyper shrinkage, Fuzzy logic, Membership function.

1. Introduction

Denoising is the process of reducing the noise from the signal and it attempts to remove whatever noise is present and retains the signal present regardless of the frequency content of the signal but it is different from smoothing, whereas, smoothing only removes high frequencies and retains low frequencies. Thresholding is a technique used for signal denoising. The discrete wavelet transform uses two types of filters: averaging filters and detail filters. When the signal is decomposed using the WT, a set of Wavelet coefficients that correlates to the high frequency subbands are obtained. These high frequency subbands consist of the details in the data set. If these details are small enough, they might be omitted without substantially affecting the main features of the data set. Additionally, these small details are often those associated with noise; therefore, by setting these coefficients to zero, noise is killed. This becomes the basic concept behind thresholding-set all frequency subbands coefficients that are less than a particular threshold to zero and use these coefficients in an inverse wavelet transformation to reconstruct the data set.

Donoho and Johnstone (1994) proposed a new method of wavelet denoising by *soft* thresholding which has been applied to both signal and image. They proved an optimal recovery is possible when compared to non-Wavelet methods. [4] *Soft* Thresholding in the iterative Wavelet domain (2001), here it is carried out in an iterative Wavelet and hence, is advantageous as compared to the previous technique. It increases the computational efficiency and provides better signal to noise ratio as well as data compression. [6] Shrinking a Wavelet coefficient towards zero to remove noise is named as Principle of Waveshrink. Threshold acts as an oracle between a significant and insignificant coefficients. Donoho and Johnstone [8] give two ways to set this threshold: Minimax threshold [17] which minimizes a bound on the asymptotic risk and the Universal threshold ensures that, asymptotically, all detail coefficients are annihilated (1995) .[2] In order to overcome the disadvantage of *Hard* shrinkage (uniformly smaller risk and less sensitive to small perturbations in the data) and soft shrinkage (uniformly smaller risk and L2 risk) Bruce and H. Y. Gao introduced a *Semisoft* (1995), [24] and *Firm* shrinkage (1997) .[25] The main disadvantage of firm shrinkage is it requires a two threshold value. Usually a data dependent wavelet threshold mainly concentrated only on magnitude but Todd Ogden and Emanuel Parzen Silverman worked with both position and magnitude of the coefficient. [13] The non-Garrote shrinkage function (1998) which provides a good compromise between the *Hard* and *Soft* but it's less sensitive than *Hard* and less biased than *Soft*. [16]

S. Poornachandra and N. Kumaravel (2005) introduced a new shrinkage function called *Hyper trim* shrinkage which has a continuous derivative and gives better mean square error. [10] They proposed a *Hyper* shrinkage (2007) which uses a Hyperbolic function, [9] shows an improvement in variance and bias estimation given by Andrew and Bruce (1996). [1] To recover a function of unknown smoothness from noisy sampled data David L. Donoho and Iain M. Johnstone (1994) introduces a *SURE* shrink which suppresses noise by thresholding an empirical wavelet coefficient and thresholding is done in an adaptive manner. [20] For eliminating noise from ECG signal α-trimmed thresholding is used in wavelet based adaptive filter model. In this, thresholding as well as adaption technique is carried out by S. Poornachandra and N. Kumaravel (2004). [22] The Subband dependent thresholding technique was proposed by S. Poornachandra and N. Kumaravel (2008) shows a better recovery of signal which was tested by a real time ECG signal. Application of this denoising method is used in biological and communication signals. [11] Wavelet gives better performance than other filters like Hanning, Low pass and Elliptic filters. [3] But it has a high computational cost. Although a wavelet denoising has more advantage, it also exhibits some disadvantage that is, discontinuities and exhibits a pseudo-Gibbs phenomenon. [12] For low and moderate noise signal reconstruction, an improvement of wavelet shrinkage used named as a Bayesian Wavelet shrinkage, by Greame K. Ambler and B.W. Silverman (2004). It gives a good Average Mean-Square Error (AMSE) and Signal to Noise Ratio (SNR), but disappoints for high noise levels. [5] G. Chen and T. Buy (2003) used a multiwavelets thresholding using neighboring coefficients which gives a better result than the single wavelet for denoising. [18]

Donoho has initially proposed the fixed thresholding based denoising of signals. Here, the value of the threshold is computed as:

$$\lambda = \sigma\sqrt{2\log(n)} \tag{1}$$

Where $\sigma = \frac{MAD}{0.6745}$, MAD is the median of wavelet coefficients and n is the total number of wavelet coefficients. Shrinkage function determines how the threshold is applied to the data. The Wavelet shrinkage functions proposed by Donoho and Johnstone's are the *Hard* and *Soft* shrinkage function which is the basis for all shrinkage functions.

2. Wavelet Transform

Wavelet means a 'small wave'. So wavelet analysis is about analyzing signal with short duration finite energy functions. They transform the signal under investigation into another representation which represents the signal in a more useful form. This transformation of the signal is called WT. [23] The WT provides a time-frequency representation of the signal. WT describes a signal by using the correlation with the translation and dilation of a function called mother wavelet. The translation operation allows signal features to be isolated in time, while the dilation operation allows features existing at different scales to be identified. In this way, the WT represents a signal as a sum of wavelets with different locations and scales. The most basic wavelet transform is the Haar developed by Alfred Haar in 1910 and it works well for signal that are approximately piecewise-smooth constant. [21] For more piecewise-smooth signals Daubechies WT are used and it compactly supports the orthonormal wavelet. [15] There is a wide range of application for WTs they are applied in different fields ranging from signal processing to biometrics, and the list is still growing.

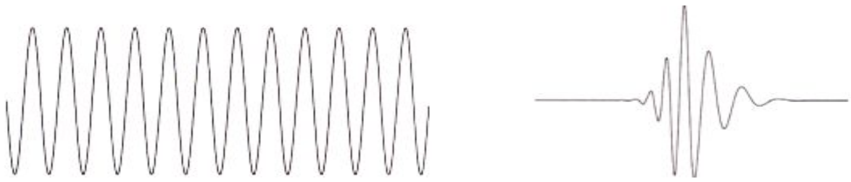


Figure 1: Wave and Wavelet.

3. Wavelet Shrinkage techniques

In *Hard* shrinkage the Wavelet coefficients are compared with the threshold value, if the Wavelet coefficients below the threshold are made zero and coefficients above are not changed. The *Hard* shrinkage function is given in equation (2) as

$$\delta_{\lambda}^{H}(x)=\begin{cases} \mathrm{x}, & \text{if } |\mathrm{x}| \leq \lambda \\ 0, & \text{if } \mathrm{x} > \lambda \end{cases} \tag{2}$$

Where λ ϵ [0, α] is the threshold.

In *Soft* shrinkage the Wavelet coefficients are shrunk towards zero [4]. The *Soft* shrinkage function is not continuous. Due to the discontinuity of shrinkage function, *Hard* shrinkage estimates tend to have a bigger variance and can be unstable and sensitive to small changes in the data. The *Soft* shrinkage estimates bigger bias due to the shrinkage of large coefficients. The *Soft* shrinkage function is given in equation (3) as

$$\delta_{\lambda}^{s}(x)=\begin{cases} 0, & \text{if } |\mathrm{x}| \leq \lambda \\ \mathrm{x}-\lambda, & \text{if } \mathrm{x} > \lambda \\ x+\lambda, & \text{if } \mathrm{x} < -\lambda \end{cases} \tag{3}$$

Where λ ϵ [0, α] is the threshold.

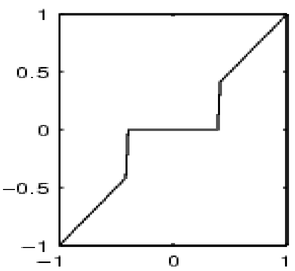


Figure 2: Hard shrinkage

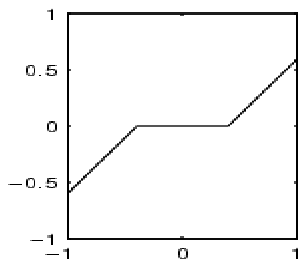


Figure 3: Soft shrinkage

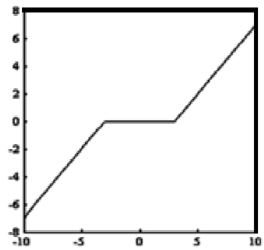


Figure 4: Hyper shrinkage.

Another method which overcomes the disadvantages of the above is *Hyper* shrinkage which uses the hyperbolic functions, which is nonlinear model. Unlike a *Hard* shrinkage the *Hyper* shrinkage model is continuously differentiable. While comparing the point wise distribution of *Hyper* shrinkage with *Soft* shrinkage is almost the same and thus *Hyper* shrinkage retains the stability of the shrinkage model. [9] The *Hyper* shrinkage model is given in equation (4) as

$$\delta_{\lambda}^{hyp}(x)=\tanh(\rho * x)(|x|-\lambda)_{+}, \quad 5 \geq \rho \geq 0 \tag{4}$$

Where, ρ is the boundary contraction parameter. The condition developed to make all coefficient fall within the curve is:

$$\rho = \frac{\Delta}{\max|x|} \tag{5}$$

Where, Δ is the exponent region.
The three main steps of denoising using wavelet coefficient shrinkage technique are as follows:
Step 1: Calculate the Wavelet coefficient matrix by applying a WT of the data.
Step 2: Modify the detail coefficients (Wavelet coefficients) using the threshold technique.
Step 3: Inverse transforms applied to the modified detail coefficients to obtain the denoised coefficients.

4. Fuzzy logic

Fuzzy logic has been applied to many fields, from control theory to intelligent. Fuzzy logic is a form of many valued logics or probalistics logic; it deals with reasoning that is approximate rather than fixed and exact. A fuzzy MF is a curve that defines how each point in the input space is mapped to a membership value (or degree of membership) between 0 and 1. [14] In this paper the MFs are used to choose the optimal threshold value for signal denoising. Five MFs are used Triangle, Trapezoid, Sigmoidal, Gaussian, and Bell.
When the DWT is applied to the ECG signal, we get the coefficients (approximate and detail) of the signal. The DWT provides sufficient information both for analysis and synthesis of the original signal, with a significant reduction in the computation time. The DWT is considerably easier to implement when compared to the continuous wavelet transform. The Daubechies wavelets are widely used in solving a broad range of problems, eg. self similarity properties of a signal or fractal problems, signal discontinuities, etc., and in this paper Daubechies wavelets are used for transforming. Then the threshold values are estimated and applied to a shrinkage function. Tuning of the threshold value with a fuzzy MF is done. This threshold value acts as an oracle, which distinguishes between the significant and insignificant coefficients. The IDWT is used to reconstruct the signal.

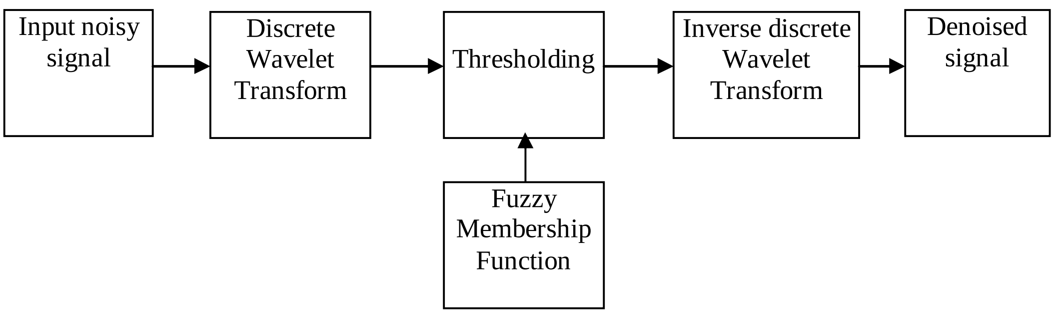


Figure 5: Block Diagram Of Proposed Denoised Model.

The quality of a signal is often expressed quantitatively as the *signal to noise ratio* of the true signal amplitude (average amplitude) to the standard deviation of the noise. For evaluating the compression, *percentage root mean square difference* we use the expression.

$$PRD = \sqrt{\frac{\sum_{n=0}^{N} (V(n) - V_R(n))^2}{\sum_{n=0}^{N} V^2(n)}} * 100\ \% \tag{6}$$

Where V (n) is the original signal and V_R (n) is reconstructed signal. *Mean square error* measures the average of the square of the "error," with the error being the amount by which the estimator differs from the quantity to be estimated.

$$MSE(\hat{f}, f) = \frac{1}{n} \sum E\{f(x_i) - f(x_i)\}^2 \tag{7}$$

Where $\hat{f}$ is estimated value and *f* is actual value.

5. Results and Discussion

In this proposed work new method for tuning the threshold value is a fuzzy MF. A fuzzy MF provides a measure of the degree of similarity of an element to a fuzzy set. In this project an ECG signal is tested with different MF. A normalized ECG signal is added with a normalized additive white Gaussian noise. The noise added to the input ECG signal is incremented at a level that is 0 to 100% at the interval 10.

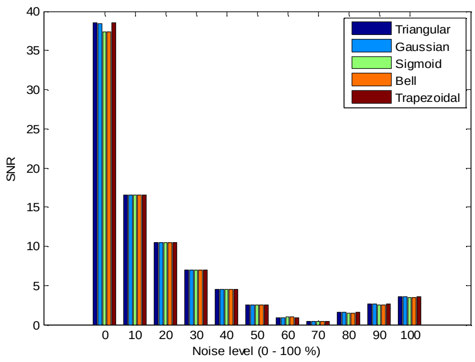
DWT is applied to the noise signal. Donoho’s universal threshold is used for calculating the threshold value and this value is used in *Hyper* shrinkage for shrinking the detail coefficients. Fuzzification process is carried out here by applying the membership function. The threshold values are mapped in the membership function. Here, the threshold limits are assumed, within the limit, every point of the threshold value is applied and simultaneously its SNR value is calculated.

The performance of different Fuzzy MFs is plotted as a bar graph. The estimated parameters in this work are SNR, PRD and MSE.

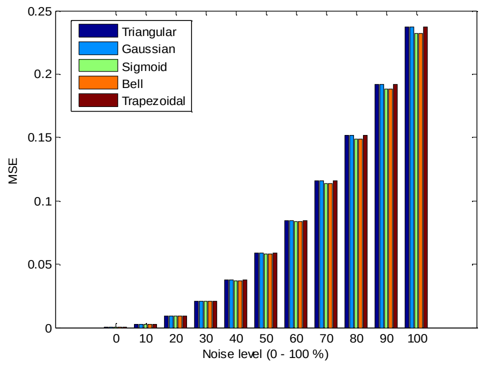
Figure 6a shows a different MFs for SNR at different noise levels, in which there is no variation up to 60% and over 60% the MFs are varied due to increase in noise than the original signal so the original signal is buried inside the noise. At 70% all the MFs are equal. At 80% and above Triangular, Gaussian and Trapezoidal MFs are increased, but Sigmoid and the Bell are decreased. In general, the higher the SNR value,the lower noise level.

Figure 6b shows the different MFs for Mean Square Error (MSE) at different noise levels, in which the sigmoid and bell MFs show the best results after 40% and above. Considering the MSE, lower value shows a better recovery of the signal.

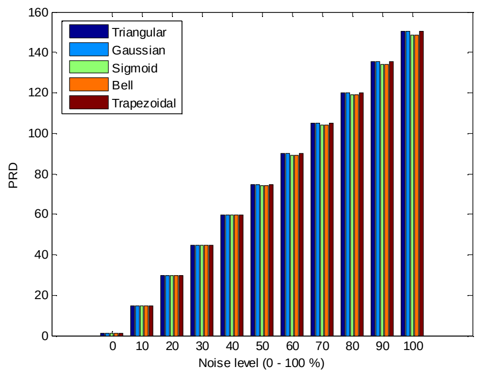
Figure 6c shows the different MFs for Percentage Root mean square Difference (PRD) at different noise levels in which the Sigmoid and Bell MFs shows a better result over 60% of noise level. Generally PRD lower value is a better recovery of the signal.
The objective is to find optimal threshold value and it is determined based on highest SNR value. By this method, different MFs (Triangular, Gaussian, Sigmoidal, Bell and Trapezoidal) are determined. Simulation result of different MFs is shown in the Figure 6 individually for SNR, MSE and PRD. The SNR value of the Triangle, Gaussian and Trapezoidal MFs show a better result, but considering PRD, MSE Sigmoidal and Bell MFs show a better result.



(a) MF comparison-SNR



(b) MF comparison-MSE



(c) MF comparison-PRD

Figure 6

Conclusion

In this paper a threshold selection scheme based on fuzzy membership function which chooses the optimal threshold value is used. Simulation of this project is done using the ECG signal, with this additive white Gaussian noise is added. At the different noise

level the simulation was carried out. The membership function chooses the threshold value based on the maximum SNR, since higher value of SNR indicates low noise.
Among these membership functions, observed that Triangle, Gaussian and Trapezoidal, SNR values are better than other membership function. But while comparing other estimation parameter PRD and MSE of Sigmoidal and Bell shows a better result. Visual results obtained from MATLAB simulation, the Triangular membership function shows better denoising of signal. MATLAB is a high-level language and interactive environment for computation, visualization, and programming. [7]

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