Order Reduction of MIMO systems using Firefly Algorithm

Tejaswini Marella*

PG student, Department of Electrical and Electronics Engineering, P.V.P.Siddhartha Institute of Technology Kanuru, Krishna Dist. Vijayawada Andhra Pradesh-520 007, India <u>tejaswinipvp@gmail.com</u>

N.Vijaya Anand

Associate Professor, Department of Electrical and Electronics Engineering, P.V.P.Siddhartha Institute of Technology Kanuru, Krishna Dist. Vijayawada Andhra Pradesh-520 007, India Fax: 0866-2581184 <u>nidumoluvijay@gmail.com</u>

Dr.M.SivaKumar

Professor, Department of Electrical and Electronics Engineering, Gudlavalleru Engineering College Gudlavalleru, Krishna Dist. Andhra Pradesh-521356 India Fax: 08674-273957 <u>profsivakumar.m@gmail.com</u>

ABSTRACT

A new model reduction technique of linear multi variable system is proposed using the prominence of Firefly Algorithm (FA). The denominator polynomial of the lower order transfer function matrix is determined by Dominant pole retention method by retaining the dominant poles of original stable higher order system .While the numerator polynomial of the lower order transfer function matrix is determined by minimizing Integral Square Error (ISE) between the transient responses of original and lower order models using FA. The efficacy of the algorithm is tested through a 10th order model of a Single Machine Infinite Bus (SMIB) power system.

Keywords: Dominant pole retention method, Firefly Algorithm, Integral Square Error, Lower Order Model, Multi variable system.

1. INTRODUCTION

Analysis of a large interconnected power system is extremely time consuming and may even exceed the storage capacity of modern fast computers because of high order system matrix. The complexity often makes it difficult to obtain a good understanding behavior of a system. The proper analysis of high order systems is very tedious and costly. If a low order linear model is derived for a high order system then the preliminary design and optimization is achieved with very much ease. Hence, methods have to be developed to obtain simplified models of the systems by using different reduction techniques. Several methods are available in international literature for the model reduction of high-order systems such as dominant pole [1], Pade Approximation [2], Stability Equation method [3], and Routh Approximation method [4]. To improve the correlation between original higher order and lower order model some of the mixed methods were developed, where the denominator polynomial is determined by preserving the stability of higher order system while the numerator polynomial is obtained using mixed method with easy numerical implementation such as Routh-Pade [5], Dominant pole-Pade [6], Stability equation-Pade [7]. All these methods have their own merits and demerits when used on a particular system.

A large number of order reduction methods of multivariable systems based on step error minimization are also developed [8][9]. In these methods, the denominator polynomial of Lower Order Model (LOM) is chosen so as to preserve stability of higher order system such as dominant pole, Routh approximation methods, etc. and then the numerator polynomial of the LOM are determined by minimization of the ISE [10] to determine the transient responses of original and lower order models to obtain optimal value. Usage of optimization methods became familiar in most of applications in various disciplines. Several optimization techniques are proposed based on inspiration of nature known as evolutionary techniques. Genetic Algorithm (GA) [11], Particle Swarm Optimization (PSO)[12] etc., are some of the techniques based artificial intelligence.GA is based on survival of the fittest with crossover and mutation operations.PSO is based on bird flocking or fish schooling, which is similar to GA in initialization of population of random solutions and updating values to get optimal solution. PSO is popular as it has no evolution operators. One of the most promising advantage of PSO over GA is its simplicity, as it uses a few parameters and easy to implement.

In spite of various existing optimization methods, there is greater need for global optimization methods for MIMO systems. One of the most recent developments is based on inspiration of behavior of fireflies called Firefly algorithm [13], having few similarities of the most prominent algorithm PSO, which has been succesfully applied to various applications in power systems [14-17], digital image processing [18] etc.., and proved its fast convergence, easy implementation and wide exploration of search space than most of the existing popular techniques. The proposed method is based on developing a lower order model of a stable higher order MIMO system using firefly algorithm and Dominant pole retention method.Paper is organized as follows: In Section 2, nature inspired Firefly Algorithm is explained. In

426

Section 3, problem formulation. In Section 4, Comparison of results with other methods available in literature.

2. FIREFLY ALGORITHM (FA)

The Firefly Algorithm (FA) is a recently developed nature inspired algorithm by Xin-She Yang [13].Based on the flashing of fireflies in the summer in the tropical temperature regions. Fireflies are of different species, to attract a prey each of them produces their own pattern. Based on intensity of brightness it can also communicate to other fireflies. This communication decreases as the distance between fireflies increases. The behavior of fireflies such as their attractiveness and communication leads to the inspiration for FA. The main advantage of Firefly is its fast convergence with global communication among all the fireflies, it is more effective in various optimization problems. The flashing of fireflies is associated with objective function to solve more optimization problems. For simplicity in describing our new FA the following three idealized rules are considered.

- All fireflies are unisex, and they will move towards more attractive and brighter ones.
- Degree of attractiveness of a firefly is proportional to its brightness.
- Brightness of a firefly is determined by the value of the objective function of a given problem.

In FA method, assume that there exists a swarm of pop (fireflies) solving the order reduction problem iteratively and X (i,k) represents a solution for a firefly 'i'in algorithm iteration k. Initially all fireflies are in random manner. Each firefly has its own attractiveness which shows how strong it attracts other members of the swarm. Firefly attractiveness is determined by

where r_{ij} is Cartesian distance between two fireflies, β_0 and γ are maximum attractiveness and absorption coefficient values respectively which are input parameters to this method. To explore search space effectively firefly i is changing its position iteratively based on attractiveness of other swarm members with higher light intensity i.e. $i_j > i_i$; for all j = 1... m and $j \neq i$ which is varying across distance and a fixed random step vector u_i for each search space dimension k is

$$u(i,k) = \propto * (rand - 0.5)$$
 ... 2.2

Where α being the randomization parameter and rand is a random number between 0 and 1.

The movement of a firefly i,attracted to another brighter firefly j, is expressed as

$$X(i,k) = X(i,k) + \beta_0 * exp(-\gamma r_{ij}^2 * (x_j - x_i) + \alpha * (rand - 0.5)$$
 ... 2.3

terminate the method if the desired solution is obtained or maximum no. of iterations are reached.

b. FLOWCHART:



Fig 2.1 Flowchart of Firefly Algorithm

3. PROBLEM FORMULATION

A synchronous machine supplying power through a step-up transformer and a high-voltage transmission line to an infinite grid is considered (SMIB) as shown in Fig 3.1. In this system under study, X_t and X_l represents the reactance of the transformer and the transmission line respectively; V_t and E_b are the generator terminal and infinite bus voltage, respectively.



Fig 3.1 A simple SMIB power system

The linearised model of system consisting of a three-phase synchronous machine with a standard IEEE Type-I exciter with Rate Feedback (RF) and Power System Stabilizer (PSS) as shown in Fig 3.2 is expressed interms of Heffron and Phillips constants.



Fig 3.2 Block diagram representation of Heffron- Phillips model of SMIB power system

In general a model can be mathematically represented in state-space form as:

$$\dot{X}$$
=AX+BU and Y = CX + DU ... 3.1

$$X^{T} = \left[E_{q} \omega \delta V_{1} V_{2} V_{3} V_{4} V_{5} V_{R} E_{FD} \right] (State - variables)$$

$$U^{T} = \left[\Delta V_{Ref} \Delta T_{m} \right] (Inputs)$$

 $Y^{T} = [\delta V_{t}]$ (Outputs) with their numerical values and operating point of the system in Appendix-A.

Based on the numerical values of parameters and operating point of system, required matrixes can be obtained.

An n^{th} order multi-input multi-output (MIMO) linear time invariant Higher Order System (HOS) in general can be described by a transfer matrix with 'j' inputs and 'i' outputs.

where k=1,2,...i and l=1,2,...j. The general form of $[G_{kl}]$ is considered as

$$G_{kl}(S) = \frac{b_{kl}(s)}{D(s)} = \frac{b_n s^{n-1} + \dots + b_1}{a_n s^n + a_{n-1} s^{n-1} \dots + a_0} \qquad \dots \qquad 3.3$$

Where $b_{kl}(s)$ and D(s) are the different numerator polynomials in transfer function matrix and denominator polynomial of the HOS which are the Laplace transforms of the output variable b_{kl} (t) and the input variable D (t) respectively. It is required to obtain the mth (m<n) LOM and it is defined as:

In general form $[r_{kl}(s)]$ is considered as

$$r_{kl}(s) = \frac{d_{kl}(s)}{\tilde{D}(s)} = \frac{d_m s^{m-1} + \dots + d_1}{c_m s^m + c_{m-1} s^{m-1} \dots + c_0} \qquad \dots \qquad 3.5$$

Dominant pole retention method is one of the most familiar order reduction techniques available in literature. It provides a stable lower order model provided the Original higher order system is stable. This method has been mixed with several methods to improve its effectiveness. The denominator polynomial of lower order transfer function is obtained by using Dominant pole retention method

Consider D(s) which is denominator polynomial of HOS

$$D(s) = a_n s^n + a_{n-1} s^{n-1} \dots + a_0 \qquad \dots \qquad 3.6$$

430

Compute the poles of D(s) and retain the most dominat poles based on the required system dynamic performance.

Formulating the reduced order denominator polynomial $\widetilde{D}(s)$ by retaining poles of original HOS.

Numerator polynomials of proposed method are determined using FA by minimizing step ISE between HOS and LOM.

$$\min J(d_{1}, \dots, d_{m-1}) = \int_0^\infty [e_m(t) - e(t)]^2 dt \qquad \dots \qquad 3.7$$

Where $d_1...d_{m-1}$ are the numerator coefficients of lower order transfer function matrix, $e_m(t)$ the unit step response of the given lower order system at time 't' and e(t) is the unit step response of the higher order system while also satisfying the steady state constraint.

To match the steady state values between the HOS and LOM.

$$d_1 = \frac{b_1 * c_0}{a_0} \qquad \dots \qquad 3.8$$

Using the recursive algorithm introduced by Astrom [10], the integral in equation 3.7 is determined in terms of γ_i and δ_i co-efficients of error function

$$R(s) = e_m(s) - e(s) = \frac{1}{s} (r_{kl}(s) - G_{kl}(s)) \qquad \dots \qquad 3.9$$

Thus, required LOM is obtained from the original HOS by minimizing ISE between them with no steady state response error.

4. RESULTS AND ANALYSIS

This linear time invariant multivariable 10th order practical system [12] under study is given by

The transfer function matrix of the power system under study is given by

$$[G(s)] = \frac{1}{D(s)} \begin{bmatrix} b_{11}(s) & b_{12}(s) \\ b_{21}(s) & b_{22}(s) \end{bmatrix} \qquad \dots \qquad 4.1$$

Where

 $D(s) = s^{10} + 64.21 s^9 + 1596 s^8 + 1.947e004 s^7 + 1.268e005 s^6 + 5.034e005 s^5 + 1.568e006 s^4 + 3.236e006 s^3 + 4.055e006 s^2 + 2.902e006 s + 2.528e005.$

Poles of D(s) are -19.0451 + i2.4859,-19.0451 - i2.4859,-11.9632, -9.6454,-0.2392 + i3.2348, -0.2392 - i3.2348, -2.1375,-0.8977 + i1.3552, -0.8977 - i1.3552, -0.1001.

and

$$\begin{split} b_{11}(s) &= -2300\,s^5 - 9.853e004\,s^4 - 1.378e006\,s^3 - 6.843e006\,s^2 - 6.105e006\,s - 5.435e005.\\ b_{12}(s) &= 29.09\,s^8 + 1868\,s^7 + 4.609e004\,s^6 + 5.459e005\,s^5 + 3.185e006\,s^4 + 8.701e006\,s^3 + 1.206e007\,s^2 + 7.603e006\,s + 6.481e005.\\ b_{21}(s) &= 85.23\,s^7 + 3651\,s^6 + 5.208e004\,s^5 + 2.98e005\,s^4 + 8.467e005\,s^3 + 3.102e006\,s^2 + 2.75e006\,s + 2.448e005.\\ b_{22}(s) &= -1.26\,s^8 - 85.17\,s^7 - 2089\,s^6 - 2.568e004\,s^5 - 1.936e004. \end{split}$$

The LOMs are obtained for the above linear multi variable HOS by using proposed algorithm.

Denominator polynomial obtained using Dominant pole retention Method is

$$\widetilde{D}(s) = s^3 + 0.5785s^2 + 10.5690s + 1.0532$$

Poles -0.1001, -0.2392 -i3.2348 and -0.2392 +i3.2348 of HOS are retained in order mimic dynamic characteristics of the original HOS. As the retained ploes are living on left half of S-plane represents it is a stable lower order denominator obtained form a stable HOS.

Numerator polynomials of LOM are obtained by FA.Performance of the algorithm depends on input parameters which should be considered carefully to achieve the best optimal value. Parameters $\alpha = 0.95$, $\gamma = 0.8$, $\beta_0 = 1.0$, population size =20 and maximum no. of iterations 500 are considered in the proposed algorithm and implemented in MATLAB.

The general form of 3rd order lower transfer function matrix is

$$[r(s)] = \frac{1}{\tilde{D}(s)} \begin{bmatrix} d_{11}(s) & d_{12}(s) \\ d_{21}(s) & d_{22}(s) \end{bmatrix} \qquad \dots \qquad 4.2$$

where

$$\begin{split} \widetilde{D}(s) &= s^3 + 0.5785s^2 + 10.5690 \, s + 1.0532 \text{ and} \\ d_{11}(s) &= 3.1645s^2 - 16.6006s - 2.2595 \\ d_{12}(s) &= -0.8752s^2 + 29.2024s + 2.6977 \\ d_{21}(s) &= -0.6040 \, s^2 + 8.0311s + 1.0195 \\ d_{22}(s) &= 0.5170 \, s^2 - 1.5679s - 0.0808 \end{split}$$

The resonabilness of the 3rd order lower models acquired above is assessed by measuring similarity between the time responses of outputs of the original HOS and LOM,concerned to the same input step variation. These responses are shown in Fig 4.1-4.6 which are also compared with lower order models obtained by [12],with three definite input changes.

- When $\Delta V_{\text{Ref}}(s) = 0.05 \text{ p.u. and } \Delta T_{\text{m}}(s) = 0.$
- When $\Delta V_{\text{Ref}}(s) = 0$ and $\Delta T_{\text{m}}(s) = 0.05$ p.u.
- When $\Delta V_{\text{Ref}}(s) = 0.05 \text{ p.u}$ and $\Delta T_{\text{m}}(s) = 0.05 \text{ p.u}$.



Fig 4.1 Comparison of Step responses of δ When $\Delta V_{Ref}(s)=0.05$ p.u. and $\Delta T_m(s)=0.$



Fig 4.2 Comparison of Step responses of V_t When $\Delta V_{Ref}(s) = 0.05$ p.u. and $\Delta T_m(s) = 0$.



Fig 4.3 Comparison of Step responses of δ When $\Delta V_{Ref}(s)=0$ and $\Delta T_m(s)=0.05$ p.u.



Fig4.4 Comparison of Step responses of V_t When $\Delta V_{Ref}(s) = 0$ and $\Delta T_m(s) = 0.05$ p.u.



Fig 4.5 Comparison of Step responses of δ When $\Delta V_{Ref}(s)=0.05$ p.u and $\Delta T_m(s)=0.05$ p.u.



Fig4.6ComparisonofStepresponsesof V_t When $\Delta V_{Ref}(s) = 0.05$ p.uand $\Delta T_m(s) = 0.05$ p.u. V_t V_t V_t

From the responses obtained through the simulation it is clear that proposed method is closely matching with original system performance and is having a less settling time with stability being preserved for step variations in the input.

Considered	Proposed	Execution time of	G.Parmar	Execution
transfer	method	Proposed Method (sec)	[12]	time
function.	(ISE)		(ISE)	of [12] (sec)
$b_{11}(s)/D(s)$	8.7174	34.7905	8.96213	66.3999
b ₁₂ (s)/D(s)	0.5527	34.0691	3.03754	66.8405
b ₂₁ (s)/D(s)	1.0754	34.6958	1.19436	66.7224
$b_{22}(s)/D(s)$	0.0448	35.3835	0.06474	66.1706

Table 4.1 Comparison of ISE of various reduced 3rd order models with their execution time.

From the Table 4.1 it is clear that the ISE values of the LOMs obtained by the proposed method is minimized in comparison with other methods available in the literature and it has shown its fast convergence .

5. CONCLUSION

The method retains dominant pole which always generates a stable LOM for a stable HOS and also retains the dynamic performance. This method allows the numerator coefficients of the HOS as free parameter in the process of order reduction. Numerator polynomials are obtained by minimsing ISE and matching the steady state response using FA having a dominance facet of fast convergence and easy pursuit. Each element of the transfer function matrix of the HOS is assessed individually. This method is simple and cogent. The cognency of the proposed method is tested on a practical 10th order SMIB system. The step response of the original system and lower order model is almost allied.

LIST OF SYMBOLS AND ABBREVIATIONS

P, Q	Synchronous	machine	(sync.m)	activepower,	
	reactivepower				
$\boldsymbol{\delta}, \boldsymbol{\omega}, \mathbf{V}_{t}$	Sync.mtorqueang	Sync.mtorqueangle, speed, terminal voltage.			
$K_1, K_2, K_3, K_4, K_5, K_6$	Heffron-Phillips	Heffron-Phillips constants.			
T_a, T_e, T_m	Sync.m accelerating, electrical and mechanical torque.				
Н	Sync.m inertia co	onstant			
R _e , X _e	External system e	equivalent i	resistance and	d reactance.	
$\dot{E}_{q}E_{fd}, \dot{T}_{do}$	Voltage proporti	ional to c	l-axis flux	linkages, field	
	voltage and				
	Open circuit time	constant			

436	Tejaswini Marella et al				
K_E, S_E, T_E	Self-excited field constant, Saturation function and exciter time constant.				
K_A, T_A, V_R	Regulator gain, time constant and output voltage.				
K _f ,T _f	RF gain and time constant				
K_R, T_R	Filter gain and time constant				
$K_{0,}T_{0},V_{s}$	Speed gain, reset time-lag constant and voltage output of PSS				
T_1, T_2, T_3, T_4	Lead-Lag time constants of PSS				
Δ	Step change of input				
FA	Firefly Algorithm				
HOS	Higher Order System				
ISE	Integral Square Error				
LOM	Lower Order Model				
MIMO	Multi-Input Multi-Output				
PSS	Power System Stabilizer				
RF	Rate Feedback				
SMIB	Single Machine Infinite Bus				

Appendix-A

Synchronous machine: 3-phase, 160 MVA, pf = 0.894, $x_d = 1.7$, $x_q = 1.6$, $x_d' = 0.245 \text{ p.u.}$, $\tau_{do} ' = 5.9$, H = 5.4s, $\omega_r = 314$ rad s=1. Type-I exciter: $K_A = 50$, $K_E = -0.17$, $S_E = 0.95$, $K_F = 0.04$, $K_R = 1$, $K_0 = 1$, $\tau_A = 0.05$, $\tau_E = 0.95$, $\tau_F = 1.0$, $\tau_R = 0.05$, $\tau_0 = 10.0$, $\tau_1 = \tau_3 = 0.440$ s, $\tau_2 = \tau_4 = 0.092$ s. External System: Re = 0.02, Xe = 0.40 p.u. (on 160 MVA base). Operating point: $P_0 = 1.0$, $Q_0 = 0.5$, $E_{FD0} = 2.5128$, $E_{q0} = 0.9986$, $V_{t0} = 1.0$, $T_{mo} = 1.0$ p.u., $\delta_0 = 1.1966$ rad, $K_1 = 1.1330$, $K_2 = 1.3295$, $K_3 = 0.3072$, $K_4 = 1.8235$, $K_5 = -0.0433$, $K_6 = 0.4777$.

REFERENCES

- [1] M.F.Hutton and B.Friedland," Dominant poles for Reducing Order of Linear Time-invariant Systems," *IEEE Transactions on Automatic Control, vol 25, No 3, pp 329-337,1975.*
- [2] Y.Shamash," Stable reduced order models using Pade type approximation,"*IEEE Trans.On Automat.Contr.*, *Vol.AC-19*, *pp.615-616*, *1974*.
- [3] T. C. Chen, C.Y.Chang and K.W.Han," Reduction of transfer functions by the stability equation method," *Journal of Franklin Institute*, *Vol. 308,No.4, pp* 389-404,1979.

- [4] Mukherjee S and Mishra R.N. ," Order reduction of linear systems using an error minimization technique," *Journal of Franklin Institute*, *Vol. 323*, *No. 1*, *pp.23–32,1987*.
- [5] Y.Shamash, "Model Reduction using the Routh Stability Criterion and Pade approximation technique," *Int.J.Control. Vol.21, pp 475-484,1975.*
- [6] Y.Shamash," Linear system reduction using Pade approximation to allow retention of dominant modes," *Int.J.Control. Vol.21, pp 257-272,1975.*
- [7] T. C. Chen, C.Y.Chang and K.W.Han," Model Reduction Using the Stability Equation Method and the Pade Approximation Method," *Journal of Franklin Institute, Vol. 309, no.6,pp 473-490,1980.*
- [8] Lamba, S.S., Gorez, R. and Bandyopadhyay, B., "New reduction technique by step error minimization for multivariable systems," *International Journal of Systems Science, Vol. 19, No. 6, pp.999–1009,1988.*
- [9] Mukherjee S. and Mishra R.N," Reduced order modeling of linear multivariable systems using an error minimization technique," *Journal of Franklin Institute*, Vol. 325, No. 2,pp.235–245,1988.
- [10] Astrom,K.J., "Introduction to Stochastic Control Theory ",Academic press Newyork.
- [11] Parmar G, Prasad R. & Mukherjee S," Order reduction of linear dynamic systems using stability equation method and GA," *International Journal of Computer and Information Engineering, Vol. 1, No. 1,pp.26–32,2007.*
- [12] Parmar G, Mukherjee S and Prasad R," Reduced order modelling of linear multivariable systems using particle swarm optimisation technique," *Int. J. Innovative Computing and Applications, Vol. 1, No. 2,pp.128–137,2007.*
- [13] X. S. Yang," Nature-Inspired Metaheuristic Algorithms," *Luniver Press* second Edition, UK, (2010).
- [14] Mandeep Loona, Shivani Mehta,Sushil Prashar," A Hybrid Firefly-DE Algorithm For Economic Load Dispatch," *International Journal of Research in Advent Technology*, vol.2, *Issue.8, August 2014 E-ISSN: 2321-9637.*
- [15] J.Merlin, R.S.Nagajothi," Economic Dispatch Using Firefly Algorithm," International Journal of Innovative Research in Science, Engineering and Technology, vol.3, Special Issue 3, March 2014.
- [16] R.Subramanian, K.Thanushkodi, "An Efficient Firefly Algorithm to Solve Economic Dispatch Problems," *International Journal of Soft Computing and Engineering (IJSCE) ISSN: 2231-2307*, Vol.2, *Issue-1, March 2013*.
- [17] Asokan K.Aand Ashok Kumar R.A, " Application of Firefly algorithm for solving Strategic bidding to maximize the Profit of IPPs in Electricity Market with Risk constraints," *INPRESSCO*, vol.4, No.1 2014.
- [18] M..H. Horng," Vector quantization using the firefly algorithm for image compression," *Expert Systems with Applications, Vol.39, pp. 1078-1091 ,2012.*

BIBLIOGRAPHY OF AUTHORS



Tejaswini Marella was born in Visakhapatnam, A.P, India in the year 1987.she received bachelor's degree in Electrical & Electronics Engineering from JNUH in year 2008. Presently she is PG student in P.V.P.Siddhartha Institute of Technology, Kanuru, Krishna Dist., Vijayawada, A.P, India.



Vijaya Anand Nidumolu was born in Eluru,A.P,India in the year 1977.He received his B. E in Electrical ElectronicsEngineering from Dr.B.A.Marthwada university in year 1999. He obtained his M.E in Power Systems & Automation from Andhra university in year 2002. His area of interest are modeling of large scale systems and power system operation & control.Presently he is working as Associate Professor in P.V.P.Siddhartha Institute of Technology, Kanuru, Krishna Dist.,Vijayawada, A.P, India.



Dr Mangipudi Siva Kumar was born in Amalapuram, E. G. Dist, AP, India, in 1971. He received bachelor's degree in Electrical & Electronics Engineering from JNTU College of Engineering, Kakinada and M.E and Ph.D degree in control systems from Andhra University College of Engineering, Visakhapatnam, in 2002 and 2010 respectively. His research interests include model order reduction, interval system analysis, design of PI/PID controllers for Interval systems, sliding mode control, Power system protection and control. Presently he is working as Professor & H.O.D of EEE, Gudlavalleru Engineering College, Gudlavalleru, A.P, India.