

## **A simple method of tuning PID controller for Integrating First Order Plus time Delay Process**

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### **Abstract**

A simple method is proposed to design a PID controller for Integrating First Order Plus Time Delay system. Design is simple compared to the other tuning methods. It has been proposed for the pneumatic control system based on the method of gain scheduling. The performance of the controller is measured by the simulation and it is compared with the other two tuning methods which is Skogested [1] and Shinskey [2]. Simulation results shows that the proposed method has lesser error ISE and IAE than the other two methods. Disturbance rejection is also good in the proposed method.

### **INTRODUCTION**

PID controller design based on stability analysis, constant open loop transfer function, pole placement method, stable inverse of the model and direct synthesis method has been proposed. In all the above methods the design procedure is somewhat complicated. A simple method is proposed for First Order Plus Time Delay system by using the method of gain scheduling [3]. But PID controller for Integrating First Order Plus Time Delay (IFOPTD) system based on gain scheduling method has not been proposed yet. PID Controller design for IFOPTD process based on direct synthesis method is proposed. This method performs well for regulatory control but it is not satisfactory for servo control [4]. Time optimal plug and control based PID controller for IFOPTD

process has been proposed in the literature [5]. It is implemented in the industrial regulators. The proposed controller is robust with respect to the measurement noise but it has higher overshoot. Ingimundarson [6] compared the performance of PID controller with the dead time compensating controllers (DTC). The performance of a PID is better than a DTC but it is somewhat difficult to achieve this optimal performance with manual control. The objective of this paper gives the simple and efficient tuning formulas for IFOPTD process using gain scheduling method.

## DESIGN

Consider an integrating First Order Plus Time Delay System (IFOPTD)  $k_p \exp(-\tau_d s) / s(\tau s + 1)$  with '+' sign for stable systems and '-' sign for unstable systems. The control law for PID controller is given by,

$$\frac{u(s)}{e(s)} = k_c \left( 1 + \frac{1}{\tau_I s} + \tau_D s \right) \quad (1)$$

Where  $u(s)$  is the output variable and  $e(s)$  is the error signal. The closed loop transfer function of the output variable and the input variable is given by,

$$\frac{y(q)}{y_r(q)} = k_c \left( 1 + \frac{1}{\tau_I s} + \tau_D s \right) \frac{k_p}{s(\tau s + 1)} e^{-\tau_d s} \quad (2)$$

Multiply by  $s$ ,

$$\begin{aligned} \frac{y(q)}{y_r(q)} &= \frac{\left( k_c k_p s \tau + \frac{k_c k_p}{\tau_I} + \frac{k_c k_p \tau_D \tau s^2 \tau}{\tau} \right) e^{-\tau_d s \frac{\tau}{\tau}}}{s \tau (s \tau + 1)} \\ \frac{y(q)}{y_r(q)} &= \frac{(k_1 q + k_2 + k_3 q^2) e^{-\varepsilon q}}{q(q + 1)} \end{aligned} \quad (3)$$

Where,

$$k_1 = k_c k_p \quad (4)$$

$$k_2 = \frac{k_c k_p}{\tau_I} \quad (5)$$

$$k_3 = k_c k_p \tau_D \quad (6)$$

$$\varepsilon = \tau_d \quad (7)$$

$$q = s \tau \quad (8)$$

The closed loop transfer function is given by,

$$\frac{y(q)}{y_r(q)} = \frac{(k_1q + k_2 + k_3q^2)}{(q^2 + q) + (k_1q + k_2 + k_3q^2)}$$

The above equation can be written as,

$$\frac{y(q)}{y_r(q)} = \frac{(k_1q + k_2 + k_3q^2)e^{0.5\epsilon q}}{(q^2+q)e^{0.5\epsilon q} + e^{-0.5\epsilon q}(k_1q + k_2 + k_3q^2)} \quad (9)$$

The exponential term  $e^{-\epsilon q}$  is negligible. Equation (9) is solved by using taylor series expansion,

$$\frac{y(q)}{y_r(q)} = \frac{(k_1q+k_2+k_3q^2)\left(1+0.5\epsilon q+\frac{0.25}{2}\epsilon^2q^2+\frac{0.125}{6}\epsilon^3q^3\right)}{(q^2+q)\left(1+0.5\epsilon q+\frac{0.25}{2}\epsilon^2q^2+\frac{0.125}{6}\epsilon^3q^3\right)+(k_1q+k_2+k_3q^2)\left(1-0.5\epsilon q+\frac{0.25}{2}\epsilon^2q^2-\frac{0.125}{6}\epsilon^3q^3\right)} \quad (10)$$

From equation (10), powers of numerator and denominator is same. On equating the co-efficient of q of the numerator and the denominator, we get

$$k_2 = \frac{1}{\epsilon} = \frac{\tau}{\tau_d} \quad (11)$$

On equating the co-efficient of  $q^2$  of the numerator and the denominator, we get

$$k_1 = \frac{1}{\epsilon} + 0.5 \quad (12)$$

On equating the co-efficient of  $q^3$  of the numerator and the denominator, we get

$$k_3 = 0.5 + \frac{0.25\epsilon}{2} - \frac{.125\epsilon}{3} \quad (13)$$

Equation (11), (12) and (13) is solved by using the equations (4), (5), (6), (7) and (8). The parameters of PID controller is given by,

$$k_c = \frac{1}{k_p} \left( \frac{\tau}{\tau_d} + 0.5 \right) \quad (14)$$

$$\tau_I = k_c k_p \tau_d \quad (15)$$

$$\tau_D = \frac{1}{k_c k_p} \left( 0.5\tau + \frac{0.25}{2}\tau_d - \frac{0.125}{3}\tau_d \right) \quad (16)$$

## RESULTS AND DISCUSSIONS

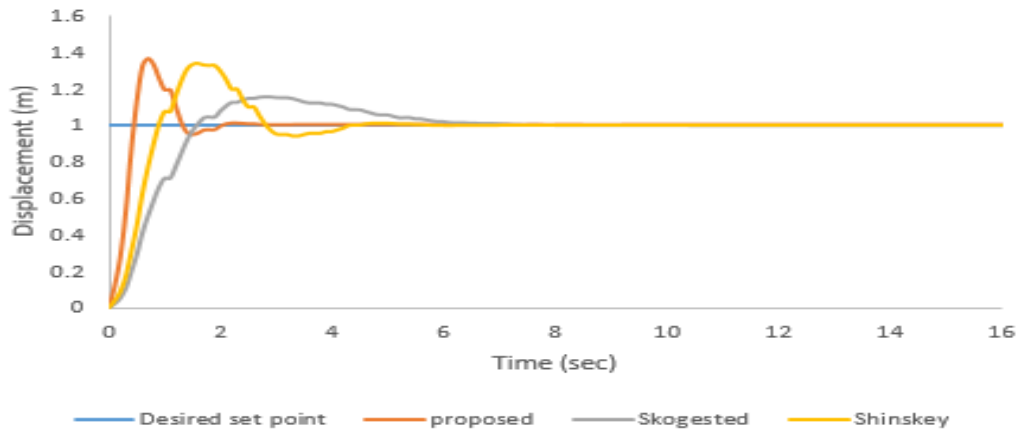
Consider the pneumatic control system for position control applications. The transfer function of pneumatic control system is given by,

$$G(s) = \frac{e^{-4s}}{s(s+1)} \quad (17)$$

The parameters of PID controller is derived by solving the equations (14), (15) and (16). We get,

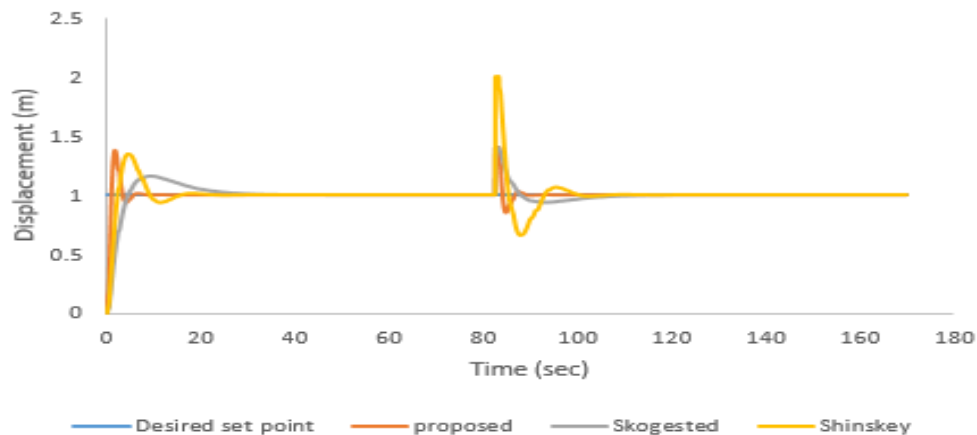
$$\begin{aligned}k_C &= 0.75 \\k_I &= 0.25 \\k_d &= 0.833\end{aligned}$$

These values are put it on the MATLAB- Simulink software and the results are taken. The controller parameters is setting by Skogested [1] are  $k_C = 0.125$ ,  $k_I = 0.0039$  and  $k_D = 0.125$ . The controller parameters is setting by Shinsky [2] are  $k_C = 0.232$ ,  $k_I = 0.0336$ ,  $k_D = 0.638$ . Figure 1 shows the comparison of the servo response of the present method with Skogested [1] and Shinsky [2].



**Figure 1.** Servo response of PID controller for pneumatic control applications

Figure 2 shows the comparison of the regulatory response of the present method with Skogested [1] and Shinsky [2].



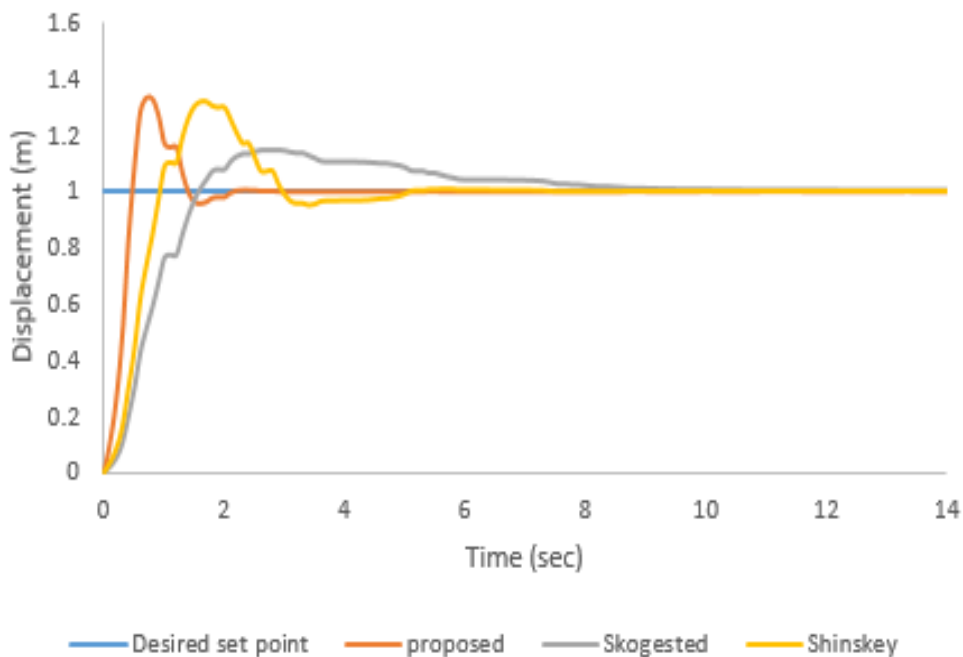
**Figure 2.** Regulatory response of PID controller for pneumatic control applications

Table 1 shows the performance of PID controller of pneumatic control for position control applications.

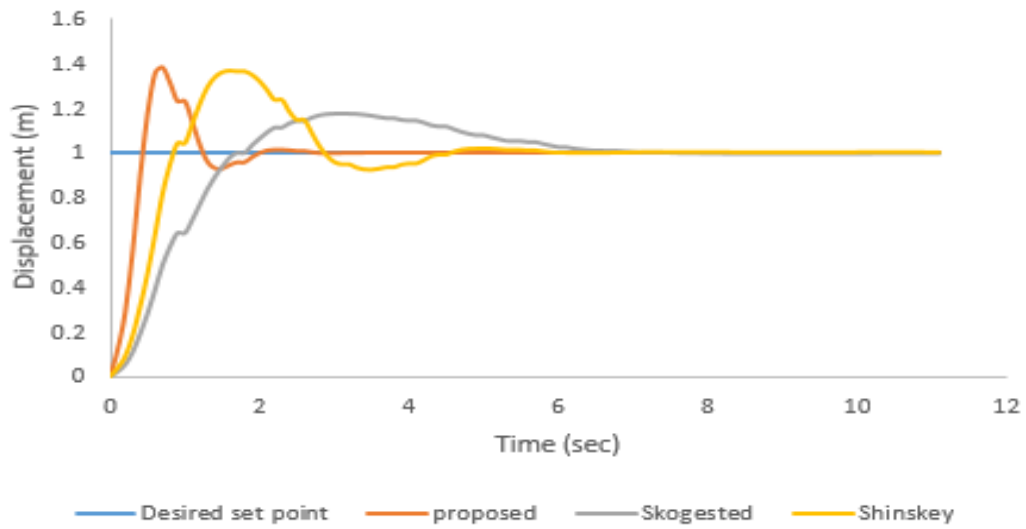
**Table 1.** ISE, IAE values for servo and regulatory response of PID controller

Performance measures	Servo response			Regulatory response		
	Skogested (2003)	Shinsky (1994)	Proposed method	Skogested (2003)	Shinsky (1994)	Proposed method
ISE	4.18e-009	8.213e-012	3.312e-013	1.64e-013	3.403e-017	1.365e-031
IAE	6.465e-005	5.755e-001	2.866e-006	4.05e-007	5.833e-009	3.694e-016

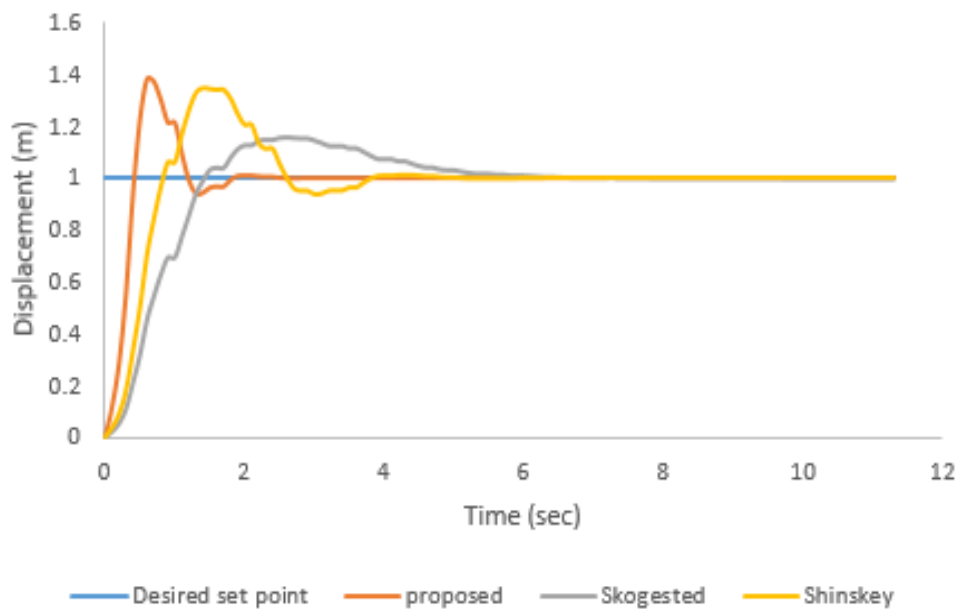
Table 1 gives the comparison values of ISE and IAE for these three methods. Results shows that proposed method has better performance than the other two methods. Proposed method has less ISE and IAE for both servo and regulatory response. Figure 3, 4 shows the performance of servo problem with process gain uncertainty ( $\pm 20\%k_p$ ). Figure 5, 6 shows the performance of servo problem with time constant uncertainty ( $\pm 20\%\tau_m$ ).



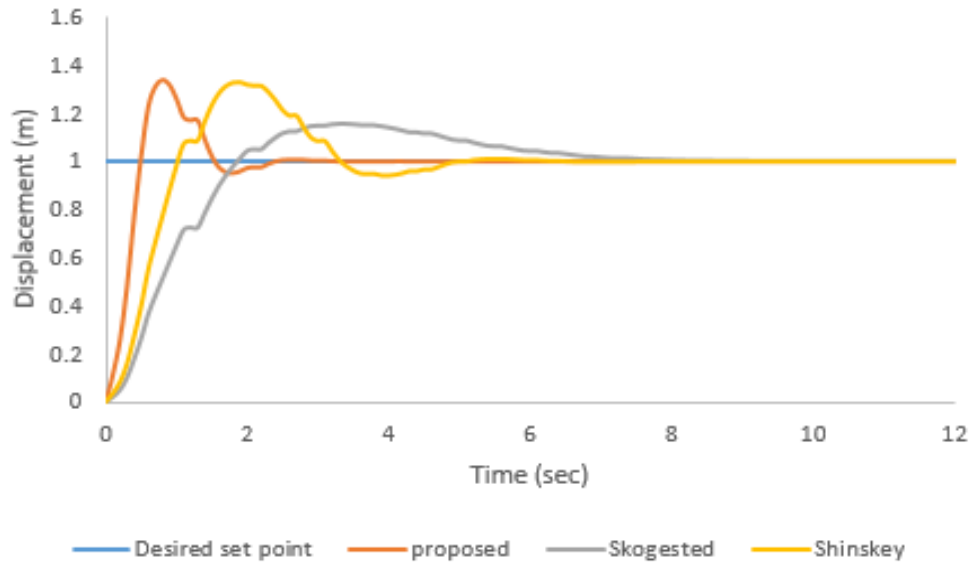
**Figure 3.** Servo response of PID controller with uncertainty  $+20\%k_p$



**Figure 4.** Servo response of PID controller with uncertainty  $-20\% k_p$

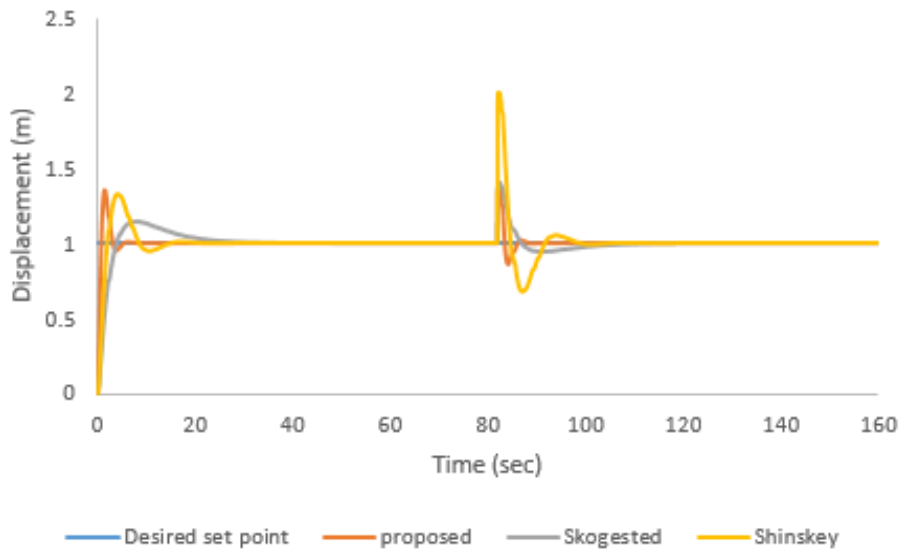


**Figure 5.** Servo response of PID controller with uncertainty  $+20\% \tau_m$

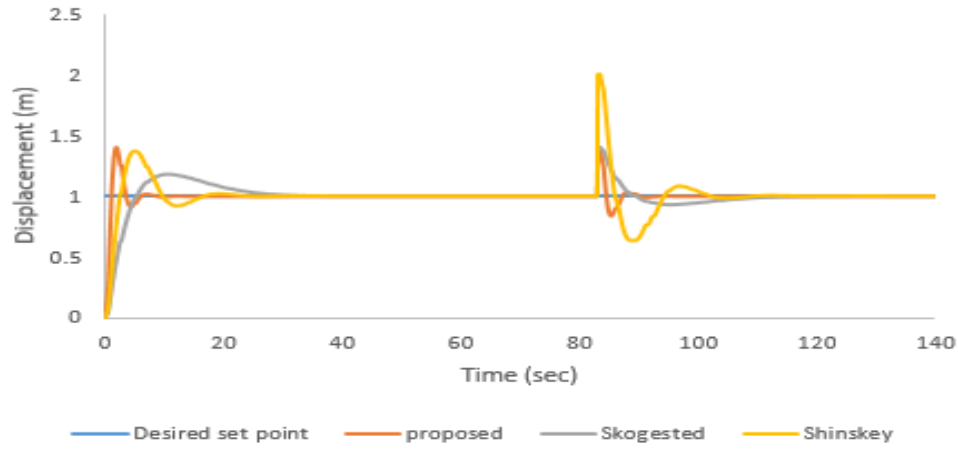


**Figure 6.** Servo response of PID controller with uncertainty  $-20\% \tau_m$

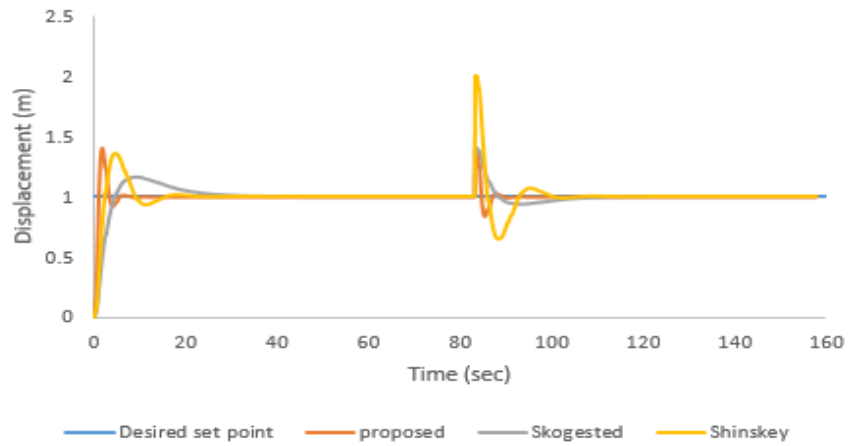
Figure 7 and 8 shows the performance of regulatory problem with process gain uncertainty ( $\pm 20\% k_p$ ). Figure 9 and 10 shows the performance of regulatory problem with time constant uncertainty ( $\pm 20\% \tau_m$ )



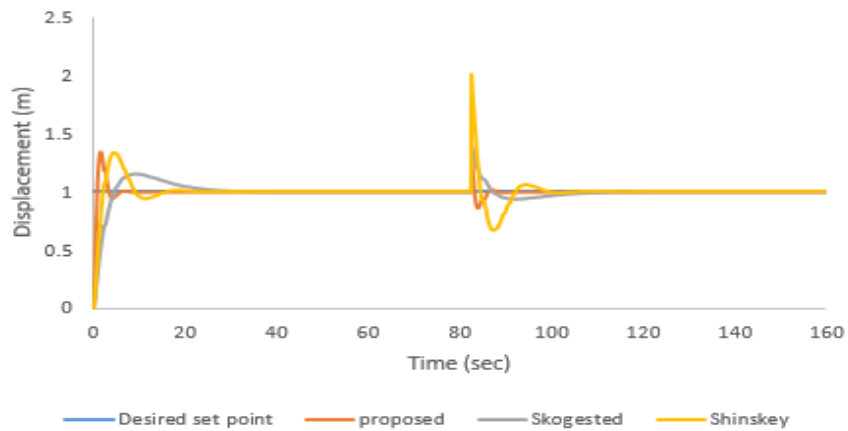
**Figure 7.** Regulatory response of PID controller with uncertainty  $+20\% k_p$



**Figure 8.** Regulatory response of PID controller with uncertainty  $-20\%k_p$



**Figure 9.** Regulatory response of PID controller with uncertainty  $+20\%\tau_m$



**Figure 10.** Regulatory response of PID controller with uncertainty  $-20\%\tau_m$



Table 2 and table 3 gives the ISE and IAE values for PID controller with parameter uncertainty in process gain and time constant.

**Table 2.** ISE values for PID controller with parameter uncertainty

Method	Servo				Regulatory			
	+20%kp	-20%kp	+20%τm	-20%τm	+20%kp	-20%kp	+20%τm	-20%τm
Proposed	1.386e-012	2.276e-026	2.53e-011	3.331e-013	3.462e-033	1.499e-031	9.538e-032	7.974e-032
Skogested (2003)	2.231e-006	3.248e-006	3.279e-010	4.745e-008	2.216e-011	5.043e-011	8.878e-014	2.355e-013
Shinsky (1994)	4.266e-009	5.573e-008	1.225e-011	2.877e-012	2.404e-021	5.647e-016	2.288e-017	2.854e-017

**Table 3.** IAE values for PID controller with parameter uncertainty

Method	Servo				Regulatory			
	+20%kp	-20%kp	+20%τm	-20%τm	+20%kp	-20%kp	+20%τm	-20%τm
Proposed	1.177e-006	1.509e-013	5.03e-006	5.771e-007	5.884e-017	3.871e-016	3.088e-016	2.824e-066
Skogested (2003)	0.00149	0.001802	1.811e-005	0.0002178	4.708e-006	7.101e-006	2.98e-007	4.853e-007
Shinsky (1994)	6.531e-005	0.0002361	3.501e-006	1.696e-006	4.903e-011	2.376e-008	4.784e-009	5.342e-009

From table 2 and 3, proposed method has lesser error in both servo and regulatory responses compared to other two methods also derivation of controller settings is simple in this proposed method.

**CONCLUSIONS**

Tuning of PID controller for IFOPTD process has been proposed based on gain scheduling method. This method is simple. It is applied for the position control application of pneumatic actuator. Simulations are carried out to evaluate the two control strategies in the pneumatic control system for servo tracking and disturbance rejection tracking. Proposed method for IFOPTD system is compared with Skogested [1] and Shinsky [2]. The proposed method has lesser ISE and IAE than the other two methods for both servo and regulatory response. Simulation results shows that proposed method outperformed Skogested and Shinsky methods.

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