

## **Optimal Phasor Measurement Unit Placement in the Observability of Power System by using Spanning Tree Algorithm**

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### **Abstract**

Phasor Measurement Unit's (PMU) are power system devices which provide real time synchronized phasor measurements. Synchronization is achieved by same-time sampling of voltage and current waveforms by means of timing signals from the Global Positioning System Satellite (GPS). Synchronized phasor measurements make higher the standards of power system monitoring, control, and protection. Since PMU's are expensive they need to be placed in optimal way in power network in order to bring down the overall cost. This paper solves the optimal placement of PMU's problem in power network using spanning tree algorithm. Integer linear programming algorithm is used to determine the optimal number and location of PMUs needed to make the network power system completely observable. Conventional techniques assume PMU's as multichannel in case PMU's are single or two channel optimal PMU locations will change. The following analyses explain optimal

PMU placement in single channel, two channel and multichannel cases. Spanning tree algorithm has been applied for IEEE 14 bus system and IEEE 30 bus system for complete observability of these systems by considering and ignoring zero injection buses separately.

**Index Terms:** Benchmarking, exhaustive search, measurement redundancy, observability, optimal placement, phasor measurement units, state estimation. Spanning tree Algorithm, PMU, Optimal PMU placement and full system.

## I. INTRODUCTION

Power system plays an important role in electrifying our homes, Industries, offices etc. So the protection and control of power system is important to engineers for safe and reliable supply of power to consumers. Power system protection and control is usually achieved by estimating state of power system regularly. Previously SCADA has been used for state estimation and for monitoring power system. This disadvantages forced engineers to discover new technology device called PMU (phasor measurement unit). PMU eliminate this problem by measuring synchronous values of voltage and current phasors. In the proposed method we find the optimal placement of PMU in power system for its complete observability using Spanning tree algorithm.

PMU placement at all substations allows direct measurement of the state of the network. However PMU placement on each bus of a system is difficult to achieve either due to cost factor since PMU is a highly expensive device or due to non-existence limitation of communication facilities in some parts of substations. As a consequence of Ohm's law, when a PMU is placed at a bus neighboring buses also become observable. This implies that a system can be made observable with a lesser number of PMUs than total number of buses system to enable the power system complete observability of power system network.

## II. PHASOR MEASUREMENT UNIT

Phasor Measurement Unit (PMU) is a contemporary metering device with capability of measuring the positive sequence of voltage and current phasors. The active power flow in a given line in power system is proportional to the difference between the phase angles of terminals of that line. Therefore, measuring the phase angles difference across the power system transmission lines is important to power system engineers. One of the greatest advantages of PMUs is the fact that in case of unexpected incident, PMUs are capable of not only sending alerts to inform the internal power system operator, but to have the capability of sending warnings to the neighbouring network operator about the unexpected events as well. This can be

accomplished by implementing the early warning system on operator's desk that would trigger simple alarms. These alarms will be initiated in case of violation of power system operating condition. Since PMU's are expensive they need to be placed optimally in network to reduce overall cost. For this several different algorithms has been proposed. Some of them are

### **1. Matrix manipulation algorithm**

This method uses an exhaustive approach as in the binary search, but has some differences. The algorithm is developed Matrix Manipulation convergence criterion can therefore be simplified as to say that it compares past solution data to present data. This cycle is perpetuated until all the  $Z$  vectors or all solutions are generated. This method is used only on smaller power systems or large ones with little lines.

### **2. Binary particle swarm optimization**

This Algorithm is based on the natural phenomena of individuals co-operating with each other in a swarm to find the optimal location where the swarm needs to be at. Each trial position and velocity is continuously updated and compared to its past condition and also if it is better than the conditions adopted presently by the entire swarm. If it is better, then this is the new swarm position and velocity. This process is continued until the ideal location (optimal PMU placement solution) is found.

The fitness function used to determine the fitness of all the individual particles is:

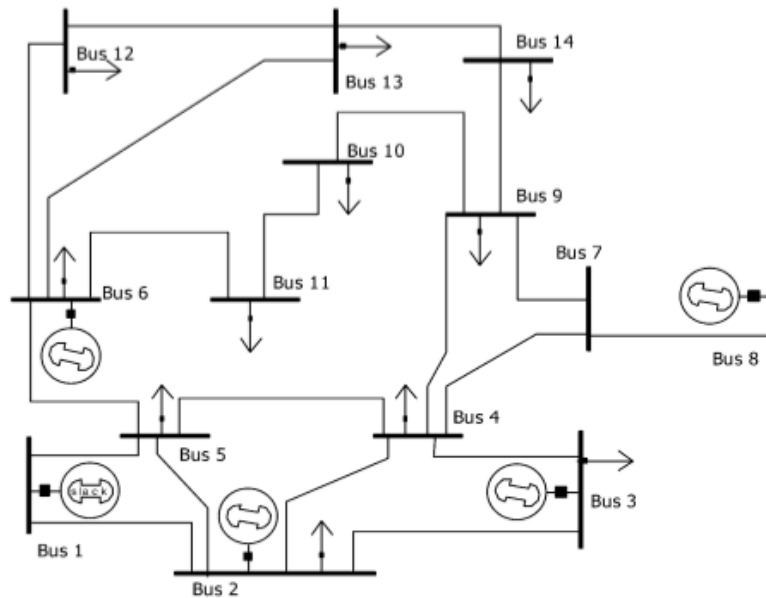
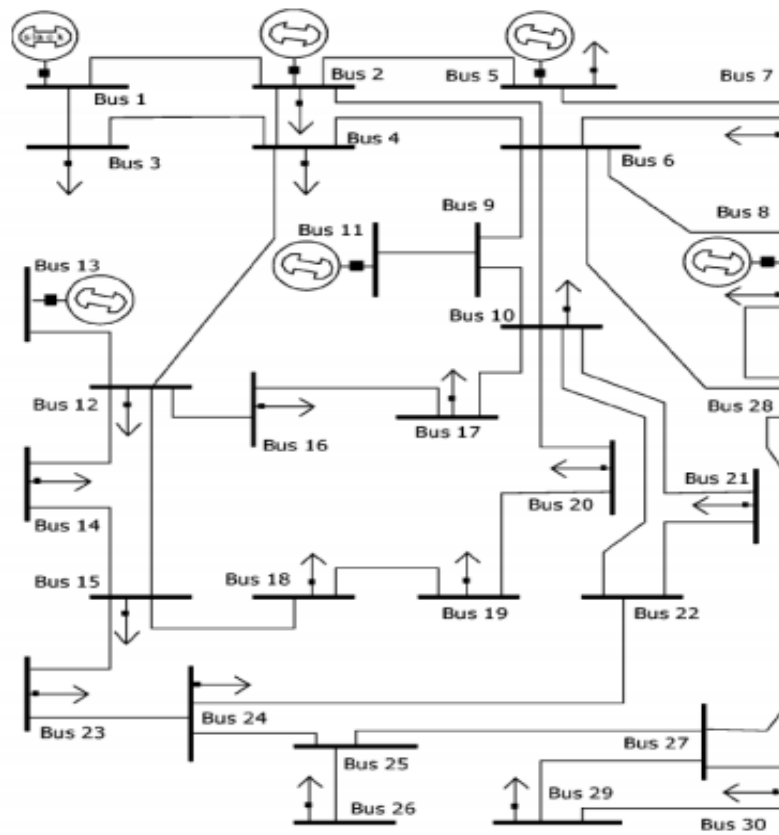
$$f = c1.NPMU + c2.Numobs$$

Where,

$NPMU$  and  $Numbs$  is the number of PMUs presently in the system and the number of unobserved buses, respectively. Binary particle swarm optimization doesn't consider computational time .Not user friendly and not adaptable. To overcome above disadvantages spanning tree algorithm for optimum PMU placement has been introduced which is user friendly, adaptable and suitable for large systems irrespective of number of connection lines.

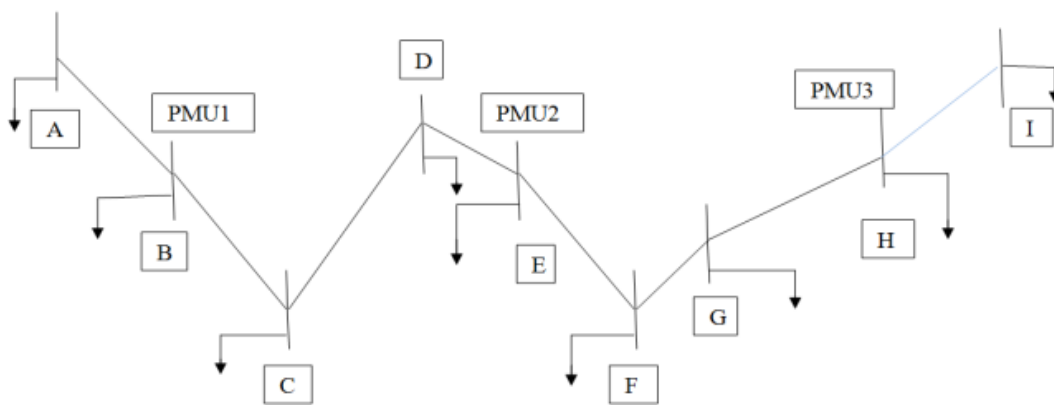
## **III. SPANNING TREE APPROACH**

The PMU placement technique (or alternatively the tree search) is illustrated first before the formal algorithm is presented. Consider the spanning tree in Fig 1(a) that is composed of 14 nodes and with 20 branches.

**Fig.1 (a): IEEE 14 bus system****Fig.1 (b): IEEE 30 bus system**

### A. OBSERVABILITY

System is said to be completely observable if voltage and current phasor of every bus in network is known. Fig.1 (b) shows a completely observed system. The voltage at buses B, E and H are directly measured buses by PMU-1, PMU-2 and PMU-3 respectively, while voltages at buses A, C, D, F, G and I can be calculated using the measured buses. Where bus voltages and line currents are directly. We define buses B and F as PMU buses where bus voltages and line currents are directly measured. We define buses A, C, D, F, G and I as calculated buses because their voltages are calculated from the PMU measurements of the buses linked to them.



**Fig 2:** Complete Observability using PMU's

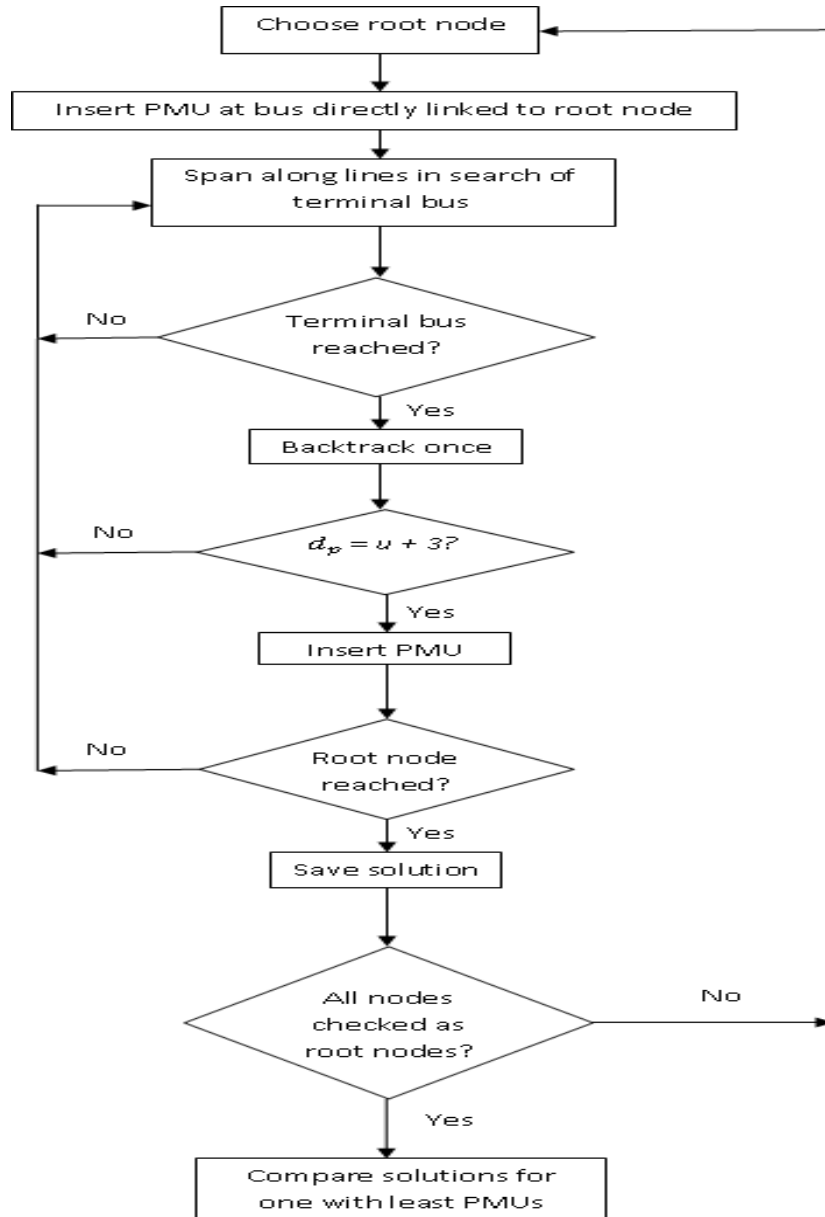
### B. TREE SEARCH PLACEMENT TECHNIQUE

The objective is to place PMUs such that the entire system is observable. The envisioned technique consists of a series of “walks” along branches of a spanning tree and queries are made on each node if a PMU placement is possible. The search procedure starts at a root node and goes down the tree until it reaches a terminal node. At this point, it backtracks and searches for another route. Identify a walk by a bus pair from bus-to bus. A root node is specified arbitrarily; the search for PMU locations is terminated when the procedure returns to this root node. At this time, the spanning tree has been fully searched. The resulting PMU placement strategy guarantees the existence of a completely observable condition for the spanning tree. It likewise guarantees the complete observability for the parent graph.

### C. ILLUSTRATION

Consider the spanning tree in Fig 1 (a) that is composed of 14 nodes and with 20 branches. Jump-start the placement process by arbitrarily designating Node 12 as the root node. The logical first PMU placement should obviously be one bus away from

the Node 12 as the root node.



**Fig.3:** Flow chart of Spanning Tree Algorithm

Then, we take a series of forward moves along a chosen path defined by the nodal sequence 6-11-10-9-7-8 each time querying for possible PMU placement. The next logical placement will be at Node 9, which makes nodes 10,7 observable with depth-of-zero unobservability. Note that the PMUs are physically separated 3 buses from each other along the chosen path.

#### D. THEORETICAL FORMULATION

We can now pose the following rule that: given a desired depth of unobservability, the next candidate PMU placement node must be of distance.

$$D_p = U + 3$$

Where,  $-D_p$  is the number of buses away from the current PMU placement the next one will be;

$-U$  is the desired level of unobservability (For complete observability  $U=0$ ).

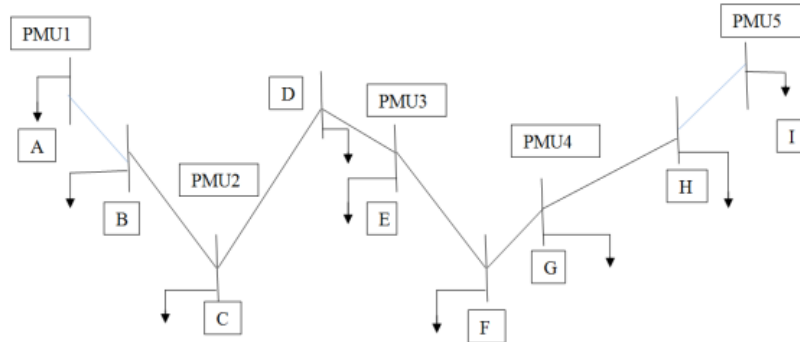
1. The next move is to the terminal node 8 but PMU at this place observes only one bus so the next PMU location will be at location 7.
2. By placing PMU's 6,9,7 the entire span 5,6,11,10,9,7,8 observable and also the nodes connected to the PMU buses(6,9,7) i.e.,12,13,14,4.is also observable. We backtrack until we reach a node where an unobservable path can be taken.
3. Then move to nodes 1, 2, 3 where here 1 act as sub root node and we again apply spanning tree algorithm by placing PMU at node directly connected to root node i.e., 2 which makes unobservable path 1, 2, 3 observable.
4. So by placing PMU'S at 2,6,7,9 entire networks is observable.
5. To ensure minimum number of PMU placements, it is necessary to perform another search from a different root node from 1 to 14.
6. Compare all solutions for one with least number of PMU's. The same approach is applied to IEEE 30 bus system also.
- 7.

#### III. MINIMUM CHANNEL PMU

PMU's are different types such as

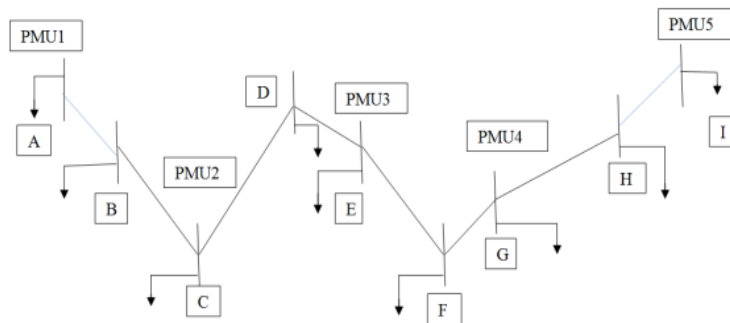
1. Single channel PMU
2. Two channel PMU

In case there is only single channel the depth of observability condition  $D_p=U+3$  changes to  $D_p=U+2$  as shown in .Moreover, for single channel placing PMU starts from root node itself as it provide less number of PMUs than placing at one bus away from root node. If there is only single channel PMU, observable buses in this condition is just the entire span of traverse, but not the other buses that also connected to PMU buses as that in case of multichannel.



**Fig.4.** PMU placement for single channel PMU

In case there is only two channel the depth of observability condition  $D_p=U+3$  is same. If there is only two channels PMU, observable buses in this condition is just the entire span of traverse, but not the other buses that also connected to PMU buses as that in case of multichannel.



**Fig.5:** PMU placement for two channel PMU

#### IV. ILLUSTRATION OF SPANNING TREE APPROACH TO SINGLE CHANNEL PMU

Consider the spanning tree in Fig 1 that is composed of 14 nodes and with 20 branches.

We can now pose the following rule that known a desired intensity of unobservability, the after that candidate PMU placement node must be of distance.

$$D_p = U + 2$$

Where,

- $D_p$  is the number of buses away from the current PMU placement the next one will be;
- $U$  is the desired level of unobservability (For complete observability  $U=0$ ).



The next move is to place PMU at bus 14 according to  $D_p=U+2$  rule and next will be 7 by applying same rule. Note that PMU's are separated by two buses.

By placing PMU's 12,14,7 the entire span 12, 13, 14,9,7,8 observable. Backtrack until we reach a node where an unobservable path can be taken.

Then move to nodes 1,2,3,4,5,6,11,10 where here 1 act as sub root node and we again apply spanning tree algorithm by placing PMU at sub root node and applying above rule PMU placement will be at 3,5,11.

So by placing PMU's at 1,3,5,7,11,12,14 entire networks is observable.

To ensure minimum number of PMU placements, it is necessary to perform another search from a different root node from 1 to 14.

Compare all solutions for one with least number of PMU's.

For two channel PMU placement will be at locations as given by  $D_p=u+3$ . Only buses present in span which has been traversed is taken as observable buses but not buses linked to PMU buses as that in case of multichannel.

## V. SIMULATION RESULTS

It is shown that spanning tree algorithm has less average compared to other algorithms. Algorithm is said to be better if its average is less.

For example: For spanning tree optimum locations is 1,7,10,12,15,19,30 means it require 2,2,6,5,4,2,2 channel PMU's and for ILP optimum locations is 1,2,10,12,15,18,25 Which require 2,4,6,5,4,2,3 channel PMUs. As cost of PMUs is in order  $6>5>4>3>2>1$  lesser the channel of PMU required lesser will be overall cost. So spanning tree algorithm reduces total overall cost more efficiently than other algorithms. Spanning tree algorithm has been applied to IEEE 14 and IEEE 30 bus system and optimal PMU placement for 14 bus system is 2,6,7,9 and for 30 bus system it is 1,7,9,10,12,15,19,25,28,30 and if zero injection buses is considered it is 2,6,9 and 1,7,10,12,15,19,30 for 14 and 30 bus systems respectively. In case there is only single channel or two channels PMU'S the possible locations for optimal PMU placement is found as shown in below tables.

Root node	PMU bus	Observability
1	1,3,5,7,9,11,12,14	Complete
2	1,2,4,6,7,10,12,14	Complete
3	1,3,5,7,9,11,12,14	Complete
4	1,3,4,6,7,10,12,14	Complete
5	1,3,5,7,9,11,12,14	Complete
6	1,3,5,6,7,10,12,14	Complete
<b>7</b>	<b>1,3,5,7,9,11,13</b>	<b>Complete</b>
<b>8</b>	<b>1,3,5,7,9,11,13</b>	<b>Complete</b>
9	1,3,5,7,9,11,12,14	Complete
<b>10</b>	<b>1,3,5,7,10,11,13</b>	<b>Complete</b>
11	1,3,5,7,9,11,12,14	Complete
<b>12</b>	<b>1,3,5,7,11,12,14</b>	<b>Complete</b>
13	1,3,5,7,9,11,12,13	Complete
<b>14</b>	<b>1,3,5,7,11,12,14</b>	<b>Complete</b>

Results for Minimum Channel PMU for IEEE 14 bus System (1 channel)

Root node	PMU bus	Observability
<b>1</b>	<b>2,5,7,10,13</b>	<b>Complete</b>
2	1,3,6,7,9,13	Complete
<b>3</b>	<b>2,4,7,11,13</b>	<b>Complete</b>
<b>4</b>	<b>2,5,7,10,13</b>	<b>Complete</b>
<b>5</b>	<b>2,4,6,9,13</b>	<b>Complete</b>
<b>6</b>	<b>2,5,7,11,13</b>	<b>Complete</b>
<b>7</b>	<b>2,5,7,10,13</b>	<b>Complete</b>
<b>8</b>	<b>2,5,7,10,13</b>	<b>Complete</b>
9	2,5,7,10,14	Complete
10	2,5,7,9,12,14	Complete
<b>11</b>	<b>2,5,7,10,13</b>	<b>Complete</b>
<b>12</b>	<b>2,5,7,10,13</b>	<b>Complete</b>
<b>13</b>	<b>2,5,7,10,14</b>	<b>Complete</b>
14	2,5,7,9,10,13	Complete

Results for Minimum Channel PMU for IEEE 14 bus System (2 channel)

Root node	PMU bus	Observability
1	1,4,5,6,9,12,14,15,17,19,21,22, 23,25,28,30	Complete
2	2,3,5,6,9,12,14,15,17,19,21,22, 23,25,28,30	Complete
3	2,3,5,6,9,12,14,15,17,19,21,22, 23,25,28,30	Complete
4	1,4,5,6,9,12,14,15,17,19,21,22, 23,25,28,30	Complete
5	1,4,5,7,9,12,14,15,17,19,21,22, 23,25,28,30	Complete
6	1,4,5,6,7,8,9,10,13,15,16,17,19,20,21,22,23,24,25	Complete
7	2,3,6,7,9,12,14,15,17,19,21,22, 23,25,28,30	Complete
8	2,3,5,7,8,9,10,12,14,15,16,17,19,20,21,22,23,24,25	Complete
9	2,3,5,7,8,9,10,12,14,15,16,17,19,20,21,22,23,24,25	Complete
10	2,3,5,7,8,9,10,12,14,15,16,17,20,21,22,23,24,25,27,28,29,30	Complete
11	1,4,5,6,7,8,9,10,11,13,15,16,17,19,20,21,22,23,24,25	Complete
12	2,3,7,8,9,10,11,12,13,14,15,16,19,21,23,24,25,27,29,30	Complete
13	1,4,5,6,9,13,14,15,17,20,21,22,23,25,28,29,30	Complete
14	1,4,5,6,9,13,14,15,17,20,21,22,23,25,28,29,30	Complete
15	1,4,5,6,9,13,14,15,17,20,21,22,23,25,28,29,30	Complete
16	1,4,5,6,9,13,14,15,16,17,20,21,22,23,25,28,29,30	Complete
17	1,4,5,6,7,8,9,13,15,16,17,19,20,22,25,27,28,29,30	Complete
18	1,2,3,4,5,6,7,8,9,10,12,13,14,15,16,17,19,20,21,22,23,24,25,27,28,29,30	Complete
19	1,2,3,4,5,6,7,8,9,10,12,13,14,15,16,17,19,20,21,22,23,24,25,27,28,29,30	Complete
20	1,4,5,6,7,8,9,13,15,16,17,19,20,22,25,27,28,29,30	Complete
21	1,4,5,6,7,8,9,13,15,16,17,20,21,22,23,24,25,27,28,29,30	Complete
22	1,4,5,6,7,8,9,13,15,16,17,20,21,22,23,24,25,27,28,29,30	Complete
23	2,3,7,8,9,10,11,12,13,14,15,16,19,21,23,24,25,27,29,30	Complete
24	2,3,5,7,8,9,10,12,14,15,16,17,20,21,23,24,25,27,28,29,30	Complete
25	1,4,5,6,7,8,9,13,15,16,17,20,21,22,23,25,27,28,29,30	Complete
26	2,3,5,7,8,9,10,12,14,15,16,17,19,20,21,23,24,25,26,27,28,29,30	Complete
27	2,3,5,7,8,9,10,12,14,15,16,17,20,21,23,24,25,27,28,29,30	Complete
28	2,3,5,7,8,9,10,12,14,15,16,17,19,20,21,22,23,24,25,28,29	Complete
29	1,4,5,6,7,8,9,13,15,16,17,20,21,22,23,25,28,29,30	Complete
30	1,4,5,6,7,8,9,13,15,16,17,20,21,22,23,25,28,29,30	Complete

Results for Minimum Channel PMU for IEEE 30 bus System (1 channel)

Root node	PMU bus	Observability
1	2,6,8,9,10,13,15,17,20,21,23,25,30	Complete
2	1,6,8,9,10,13,15,17,20,21,23,25,30	Complete
3	1,6,8,9,10,13,15,17,20,21,23,25,30	Complete
4	2,5,8,9,10,13,15,17,20,21,23,25,30	Complete
5	2,4,8,9,10,13,15,17,20,21,23,25,30	Complete
6	2,4,5,7,8,9,10,14,15,16,17,19,20,21,22,23,24,25	Complete
7	3,5,8,9,10,13,15,17,20,21,23,25,30	Complete
8	3,5,6,7,9,10,13,15,16,17,19,20,21,22,23,24,25,27	Complete
9	3,5,6,7,8,9,10,13,15,17,19,20,21,22,23,24,25	Complete
10	3,5,6,7,8,9,13,15,16,17,20,21,22,23,24,25,27,28,29,30	Complete
11	1,5,7,8,9,10,12,13,16,17,19,20,21,22,23,24,25	Complete
12	3,4,6,9,11,13,14,15,20,21,22,23,25,27,29,30	Complete
13	1,7,9,12,14,15,16,21,23,24,25,28,29,30	Complete
14	1,7,9,12,13,15,16,21,23,24,25,28,29,30	Complete
15	1,7,9,12,13,14,15,16,21,23,24,25,28,29,30	Complete
16	1,7,9,12,13,14,15,20,21,23,24,25,28,29,30	Complete
17	1,5,7,8,9,10,12,15,16,19,20,22,25,27,28,29,30	Complete
18	1,2,3,4,5,6,7,8,9,10,12,13,14,15,16,17,19,20,21,22,23,24,25,27,28,29,30	Complete
19	1,2,3,4,5,6,7,8,9,10,12,13,14,15,16,17,20,21,22,23,24,25,27,28,29,30	Complete
20	1,5,7,8,9,10,12,15,16,17,19,22,25,27,28,29,30	Complete
21	1,5,7,8,9,10,12,15,16,17,20,22,23,24,25,27,28,29,30	Complete
22	1,5,7,8,9,10,12,15,16,17,20,21,23,24,25,27,28,29,30	Complete
23	2,5,8,9,10,13,14,15,17,19,21,25,29,30	Complete
24	2,4,5,7,8,9,14,15,16,17,20,21,22,23,25,27,28,29,30	Complete
25	3,5,6,7,8,9,13,15,16,17,20,21,23,24,25,27,28,29,30	Complete
26	1,5,7,8,9,10,12,15,16,17,20,21,23,25,27,28,29,30	Complete
27	1,5,7,8,9,10,12,15,16,17,20,21,23,25,28,29,30	Complete
28	2,4,5,7,8,9,14,15,16,17,20,21,22,23,25,27,28,30	Complete
29	2,4,5,7,8,9,14,15,16,17,20,21,22,23,25,27,28,30	Complete
30	2,4,5,7,8,9,14,15,16,17,20,21,22,23,25,27,28,29	Complete

Results for Minimum Channel PMU for IEEE 30 bus System (2 channel)

System	Location of PMU(with out considering zero injection buses)	Location of PMU ( considering zero injection buses)
IEEE14 bus system	2,6,7,9	2,6,9
IEEE 30 bus system	1,7,9,10,12,15,19,25,28,30	1,7,10,12,15,19,30

#### Results for Optimum PMU placement using Spanning Tree Algorithm

System	Spanning tree algorithm	B and B algorithm	ILP	Genetic algorithm
14 bus system	2,6,9	2,6,9	2,6,9	2,6,9
30 bus system	1,7,10,12,15,19,30	3,5,10,12,19,24,27	1,2,10,12,15,18,25	1,2,10,12,15,20,27
Average for 30 bus system	3.2857	3.428	3.714	3.857

#### Comparison of Results obtained with other Algorithm

## VI. CONCLUSION

From the results it can be concluded that spanning tree can be used for big and small systems. This makes spanning tree algorithm a feasible option for use in locating the PMUs. The average of spanning tree algorithm is less compared to other algorithms which means spanning tree algorithm efficiently reduce overall cost of system. Not only 14, 30 bus system spanning tree can also be used for any of large system within short elapsed time. The results obtained in this project can be used practically for placing PMU in large power systems so that the power system is secured and it can be reliably operated.

## REFERENCES

- [1] Abdul-Aziz Fish, S.Chowdhury, and S.P.Chowdhury, "optimal PMU placement in a power network for full system observability" proc IEEE, vol 978.no.1.pp. 4577-1022.nov. 2011.
- [2] Reynaldo F. Nuqui, Member, IEEE, and Arun G. Phadke, Life Fellow, IEEE "Phasor Measurement Unit Placement Techniques for Complete and Incomplete Observability" IEEE TRANSACTIONS ON POWER DELIVERY, VOL. 20, NO. 4, OCTOBER 2005.

- [3] Sanjay Dambhare, Devesh Dua, Rajeev Kumar, Gajbhiye S. A., Soman “Optimal Zero Injection Considerations in PMU Placement: An ILP Approach”.
- [4] D.Dua, S.Dambhare, R.K.Gajbhiye and S.A.Soman, “Optimal Multistage Scheduling of PMU Placement: An ILP Approach”, IEEE Transactions on Power Delivery, Vol.23, Issue 4, pp.1812-1820, 2008”.
- [5] D. Novosel, K.Vu, V. Centeno, S. Skok, M. Begovic, “Benefits of synchronized measurement technology for power grid application”, Proceeding of the 40<sup>th</sup> international conference on system sciences, 2007.
- [6] M. Hajian, A. M. Ranjbar, T. Amraee and A. R. Shirani, “Optimal Placement of Phasor Measurement Units: Particle Swarm Optimization Approach”, Proc. of Int. Conf. on Intelligent Systems Applications to Power Systems 2007, Toki Messe, Niigata, pp.1-6, November 2007.
- [7] R. F. Nuqui, “State estimation and voltage security monitoring using synchronized phasor measurementSSs,” Ph.D. dissertation, Dept. Elect. Computer. Eng., Virginia Polytechnic Inst. State Univ., Blacksburg, VA, 2001.
- [8] T. L. Baldwin, L. Mili, M. B. Boisen, and R. Adapa, “Power system observability with minimal phasor measurement placement,” IEEE Trans. Power Syst., vol. 8, no. 2, pp. 701–715, May 1993.
- [9] F.Aminifar, C.Lucas, A.Khodaei and M.Fotuhi-Firuzabad, “Optimal Placement of Phasor Measurement Units Using Immunity Genetic Algorithm”, IEEE Transactions on Power Delivery, Vol.24, Issue 3, pp.1014-1020, July 2009.
- [10] M.Hurtgen and Jean-Claude Maun, “Advantages of power system state estimation using Phasor Measurement Units”, Proc. of 16th PSCC, Glasgow, Scotland, pp.1-7, July 2008.

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