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Smith Chart and its Applications

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Abstract

The Smith chart is one of the most useful graphical tools for high frequency circuit applications. The chart provides a clever way to visualize complex functions and it continues to endure popularity decades after its original conception. From a mathematical point of view, the Smith chart is simply a presentation of all possible complex impedances with respect to coordinates defined by the reflection coefficient or it can be defined mathematically as one port scattering parameter S11. The domain of definition of the reflection coefficient is a circle of radius 1 in the complex plane. This is also the domain of the Smith chart. A Smith chart is a circular plot with lot of interlaced circles on it; when used correctly, matching impedances with apparent complicated structures can be made without any computations. The only effort required is the reading and following of values along the circles.

Types of Smith Chart

There are mainly two kinds of Smith chart, the impedance or the Z-Smith chart and the other one admittance or the Y-Smith chart. The superposition of Z or Y-Smith chart gives the Z-Y Smith chart or the complete smith chart. In matching or in designing circuits it is convenient to overlay the impedance Z-Smith Chart and admittance Y- Smith Chart called the Impedance Admittance Z-Y Smith Chart which is basically the superimposition of the individual Z-Smith Chart and Y-Smith Chart. The figure below shows a Z-Y Smith Chart. Impedance and admittance charts are used to calculate the component values needed for device in different parts of the impedance matching circuit.

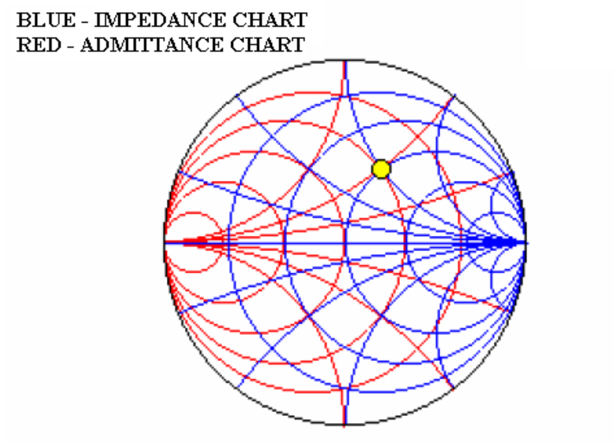


Figure B1: A Complete Smith chart.

Development of a Smith Chart

A Smith chart is developed by examining the load where the impedance must be matched. Instead of considering its impedance directly, you express its reflection coefficient Γ_L, which is used to characterize a load (such as admittance, gain, and trans conductance). The Γ_L is more useful when dealing with RF frequencies. We know the reflection coefficient is defined as the ratio between the reflected voltage wave and the incident voltage wave:

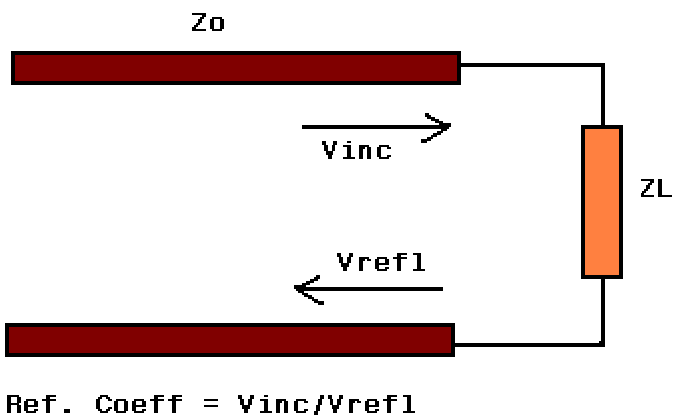


Figure B2: Impedance at the load.

The amount of reflected signal from the load is dependent on the degree of mismatch between the source impedance and the load impedance. Its expression has been defined as follows:

$$\Gamma_L = \frac{Vrefl}{Vinc} = \frac{Z_L - Z_O}{Z_L + Z_O} = \Gamma_r + j.\,\Gamma_I \qquad \text{(eqn B.1)}$$

Because the impedances are complex numbers, the reflection coefficient will be a complex number as well. In order to reduce the number of unknown parameters, it is useful to freeze the ones that appear often and are common in the application. Here Z_o (the characteristic impedance) is often a constant and a real industry normalized value, such as 50 Ω, 75 Ω, 100 Ω, and 600 Ω. We can then define a normalized load impedance by:

$$z = Z_L/Zo = (R + jX) / Zo = r + jx \qquad \text{(eqn B.2)}$$

With this simplification, we can rewrite the reflection coefficient formula as:

$$\Gamma_L = \Gamma_r + j.\Gamma_i = \frac{Z_L - Z_O}{Z_L + Z_O} = \frac{(Z_L - Z_O)/Z_O}{(Z_L + Z_O)/Z_O} = \frac{z-1}{z+1} = \frac{r + j.x - 1}{r + j.x + 1} \qquad \text{(eqn B.3)}$$

Here we can see the direct relationship between the load impedance and its reflection coefficient. Unfortunately, the complex nature of the relation is not useful practically, so we can use the Smith chart as a type of graphical representation of the above equation.

To build the chart, the equation must be rewritten to extract standard geometrical figures (like circles or stray lines).

First, equation B.3 is reversed to give

$$z = r + jx = \frac{1 + \Gamma_L}{1 - \Gamma_L} = \frac{1 + \Gamma_r + j\Gamma_i}{1 - \Gamma_r - j\Gamma_i} \qquad \text{(eqn B.4)}$$

and

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{1 + \Gamma_r^2 - 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.5)}$$

By setting the real parts and the imaginary parts of equation B.5 equal, we obtain two independent, new relationships:

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{1 + \Gamma_r^2 - 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.6)}$$

$$x = \frac{2.\Gamma_i}{1 + \Gamma_r^2 - 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.7)}$$

Equation is then manipulated by developing equations B.8 through B.13 into the final equation, B.14. This equation is a relationship in the form of a parametric equation $(x-a)^2 + (y-b)^2 = R^2$ in the complex plane (Γr, Γi) of a circle centered at the coordinates (r/r+1, 0) and having a radius of 1/1+r.

$$r + r.\Gamma_r^2 - 2.r.\Gamma_r + r.\Gamma_i^2 = 1 - \Gamma_r^2 - \Gamma_i^2 \quad \text{(eqn B.8)}$$

$$\Gamma_r^2 + r.\Gamma_r^2 - 2.r.\Gamma_r + r.\Gamma_i^2 + \Gamma_i^2 = 1 - r \quad \text{(eqn B.9)}$$

$$(1+r).\Gamma_r^2 - 2.r.\Gamma_r + (r+1).\Gamma_i^2 = 1 - r \quad \text{(eqn B.10)}$$

$$\Gamma_r^2 - \frac{2.r}{r+1}.\Gamma_r + \Gamma_i^2 = \frac{1-r}{1+r} \quad \text{(eqn B.11)}$$

$$\Gamma_r^2 - \frac{2.r}{r+1}.\Gamma_r + \frac{r^2}{(r+1)^2} + \Gamma_i^2 - \frac{r^2}{(r+1)^2} = \frac{1-r}{1+r} \quad \text{(eqn B.12)}$$

$$(\Gamma_r - \frac{r}{r+1})^2 + \Gamma_i^2 = \frac{1-r}{1+r} + \frac{r^2}{(1+r)^2} = \frac{1}{(1+r)^2} \quad \text{(eqn B.13)}$$

$$(\Gamma_r - \frac{r}{r+1})^2 + \Gamma_i^2 = (\frac{1}{1+r})^2 \quad \text{(eqn B.14)}$$

Figure B3 given below gives in depth details.

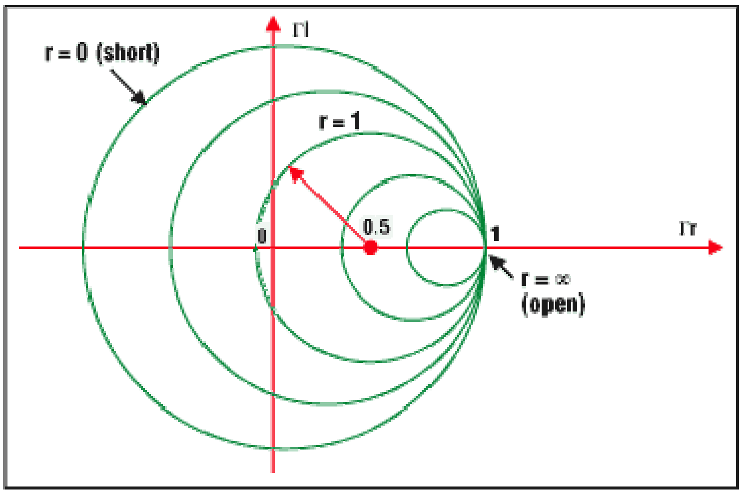


Figure B3: The points situated on a circle are all the impedances characterized by a same real impedance part value. For example, the circle, R = 1, is centered at the coordinates (0.5, 0) and has a radius of 0.5. It includes the point (0, 0), which is the reflection zero point (the load is matched with the characteristic impedance). A short circuit, as a load, presents a circle centered at the coordinate (0, 0) and has a radius of 1. For an open-circuit load, the circle degenerates to a single point (centered at 1, 0 and with a radius of 0). This corresponds to a maximum reflection coefficient of 1, at which the entire incident wave is reflected totally.

Moving on, we use equations B.15 through B.18 to further develop equation B.7 into another parametric equation. This results in equation B.19.

$$x + x.\Gamma_r^2 - 2.x.\Gamma_r + x.\Gamma_i^2 = 2.\Gamma_i \qquad \text{(eqn B.15)}$$

$$1 + \Gamma_r^2 - 2.\Gamma_r + .\Gamma_i^2 = 2.\Gamma_i / x \qquad \text{(eqn B.16)}$$

$$\Gamma_r^2 - 2.\Gamma_r + 1 + \Gamma_i^2 - \frac{2}{x}\Gamma_i = 0 \qquad \text{(eqn B.17)}$$

$$\Gamma_r^2 - 2.\Gamma_r + 1 + \Gamma_i^2 - \frac{2}{x}\Gamma_i + \frac{1}{x^2} - \frac{1}{x^2} = 0 \qquad \text{(eqn B.18)}$$

$$(\Gamma_r - 1)^2 + (\Gamma_i - \frac{1}{x})^2 = \frac{1}{x^2} \qquad \text{(eqn B.19)}$$

Again, B.19 is a parametric equation of the type $(x\text{-}a)^2 + (y\text{-}b)^2 = R^2$ in the complex plane (Γr, Γi) of a circle centered at the coordinates (1, 1/x) and having a radius of 1/x.

Figure B.4 given below gives in depth details

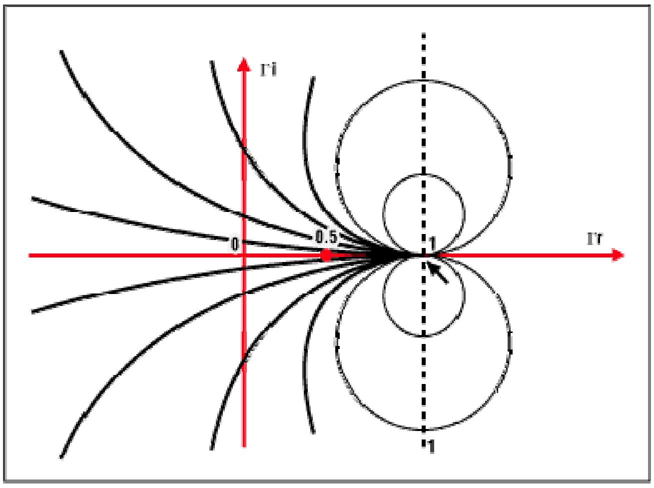


Figure B4: The points situated on a circle are all the impedances characterized by a same imaginary impedance part value x. For example, the circle x = 1 is centered at coordinate (1, 1) and has a radius of 1. All circles (constant x) include the point (1, 0). Differing with the real part circles, x can be positive or negative. This explains the duplicate mirrored circles at the bottom side of the complex plane. All the circle centers are placed on the vertical axis, intersecting the point 1.

To complete our Smith chart, we superimpose the two circles' families. It can then be seen that all of the circles of one family will intersect all of the circles of the other family. Knowing the impedance, in the form of r + jx, the corresponding reflection coefficient can be determined. It is only necessary to find the intersection point of the two circles corresponding to the values r and x.

The reverse operation is also possible. Knowing the reflection coefficient, find the two circles intersecting at that point and read the corresponding values r and x on the circles. The procedure for this is as follows:

- Determine the impedance as a spot on the Smith chart.
- Find the reflection coefficient (Γ) for the impedance.
- Having the characteristic impedance and Γ, find the impedance.
- Convert the impedance to admittance.
- Find the equivalent impedance.
- Find the component values for the wanted reflection coefficient.

Working with Admittance

The Smith chart is built by considering impedance (resistor and reactance). Once the Smith chart is built, it can be used to analyze these parameters in both the series and parallel worlds. Adding elements in a series is straightforward. New elements can be added and their effects determined by simply moving along the circle to their respective values. However, summing elements in parallel is another matter. This requires considering additional parameters. Often it is easier to work with parallel elements in the admittance world.

We know that, by definition, Y = 1/Z and Z = 1/Y. The admittance is expressed in mhos or Ω^{-1} (in earlier times it was expressed as Siemens or S). And, as Z is complex, Y must also be complex.

Therefore, Y = G + jB (B.20), where G is called "conductance" and B the "susceptance" of the element. It's important to exercise caution, though. By following the logical assumption, we can conclude that G = 1/R and B = 1/X. This, however, is not the case. If this assumption is used, the results will be incorrect.

When working with admittance, the first thing that we must do is normalize y = Y/Y_o. This results in y = g + jb. So, what happens to the reflection coefficient? By working through the following:

$$\Gamma = \frac{Z_L - Z_O}{Z_L + Z_O} = \frac{1/Y_L - 1/Y_O}{1/Y_L + 1/Y_O} = \frac{Y_O - Y_L}{Y_O + Y_L} = \frac{1-y}{1+y} \qquad \text{(eqn B.21)}$$

It turns out that the expression for G is the opposite, in sign, of z, and Γ(y) = -Γ (z).

If we know z, we can invert the signs of Γ and find a point situated at the same distance from (0, 0), but in the opposite direction. This same result can be obtained by rotating an angle 180° around the center point (see Figure B.5).

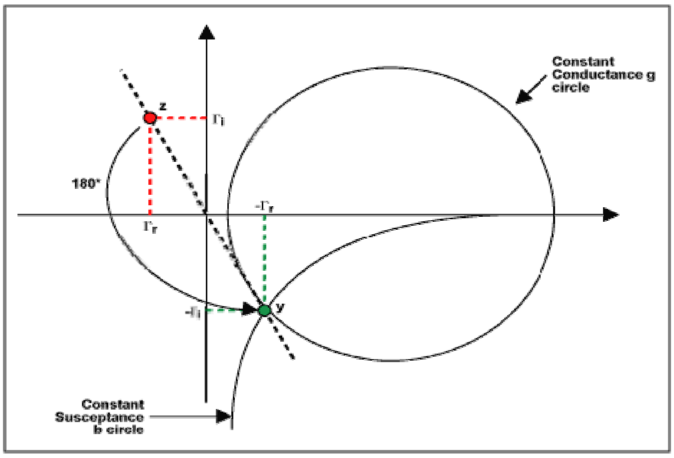


Figure B5: Results of the 180° rotation.

Of course, while Z and 1/Z do represent the same component, the new point appears as a different impedance (the new value has a different point in the Smith chart and a different reflection value, and so forth). This occurs because the plot is an impedance plot. But the new point is, in fact, an admittance. Therefore, the value read on the chart has to be read as mhos. Although this method is sufficient for making conversions, it doesn't work for determining circuit resolution when dealing with elements in parallel.

The Admittance Smith Chart

In the previous discussion, we saw that every point on the impedance Smith chart can be converted into its admittance counterpart by taking a 180° rotation around the origin of the Γ complex plane. Thus, an admittance Smith chart can be obtained by rotating the whole impedance Smith chart by 180°. This is extremely convenient, as it eliminates the necessity of building another chart. The intersecting point of all the circles (constant conductances and constant susceptances) is at the point (-1, 0) automatically. With that plot, adding elements in parallel also becomes easier. Mathematically, the construction of the admittance Smith chart is created by:

$$\Gamma_L = \Gamma_r + j.\Gamma_i = = \frac{1-y}{1+y} = \frac{1-g-jb}{1+g+jb} \qquad \text{(eqn B.22)}$$

then, reversing the equation:

$$y = g + jb = \frac{1-\Gamma_L}{1+\Gamma_L} = \frac{1-\Gamma_r - j\Gamma_i}{1+\Gamma_r + j\Gamma_i} \qquad \text{(eqn B.23)}$$

$$g + jb = \frac{(1-\Gamma_r - j\Gamma_i).(1+\Gamma_r - j\Gamma_i)}{(1+\Gamma_r + j\Gamma_i).(1+\Gamma_r - j\Gamma_i)} = \frac{1-\Gamma_r^2 - \Gamma_i^2 - j.2.\Gamma_i}{1+\Gamma_r^2 + 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.24)}$$

Next, by setting the real and the imaginary parts of equation B.24 equal, we obtain two new, independent relationships:

$$g = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{1 + \Gamma_r^2 + 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.25)}$$

$$b = \frac{-2.\Gamma_i}{1 + \Gamma_r^2 + 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.26)}$$

By developing equation B.25, we get the following,

$$g + g.\Gamma_r^2 + 2.g.\Gamma_r + g.\Gamma_i^2 = 1 - \Gamma_r^2 - \Gamma_i^2 \qquad \text{(eqn B.27)}$$

$$\Gamma_r^2 + g.\Gamma_r^2 + 2.g.\Gamma_r + g.\Gamma_i^2 + \Gamma_i^2 = 1 - g \qquad \text{(eqn B.28)}$$

$$(1+g).\Gamma_r^2 + 2.g.\Gamma_r + (g+1).\Gamma_i^2 = 1 - g \qquad \text{(eqn B.29)}$$

$$\Gamma_r^2 + \frac{2.g}{g+1}.\Gamma_r + \Gamma_i^2 = \frac{1-g}{1+g} \qquad \text{(eqn B.30)}$$

$$\Gamma_r^2 + \frac{2.g}{g+1}.\Gamma_r + \frac{g^2}{(g+1)^2} + \Gamma_i^2 - \frac{g^2}{(g+1)^2} = \frac{1-g}{1+g} \qquad \text{(eqn B.31)}$$

$$(\Gamma_r + \frac{g}{g+1})^2 + \Gamma_i^2 = \frac{1-g}{1+g} + \frac{g^2}{(1+g)^2} = \frac{1}{(1+g)^2} \qquad \text{(eqn B.32)}$$

$$(\Gamma_r + \frac{g}{g+1})^2 + \Gamma_i^2 = (\frac{1}{1+g})^2 \qquad \text{(eqn B.33)}$$

which again is a parametric equation of the type $(x\text{-}a)^2 + (y\text{-}b)^2 = R^2$ (equation B.33) in the complex plane (Γr, Γi) of a circle with its coordinates centered at (-g/g+1, 0) and having a radius of 1/(1+g).

Furthermore, by developing equation B.26, we show that:

$$b + b.\Gamma_r^2 + 2.b.\Gamma_r + b.\Gamma_i^2 = -2.\Gamma_i \qquad \text{(eqn B.34)}$$

$$1 + \Gamma_r^2 + 2.\Gamma_r + .\Gamma_i^2 = -2.\Gamma_i / b \qquad \text{(eqn B.35)}$$

$$\Gamma_r^2 + 2.\Gamma_r + 1 + \Gamma_i^2 + \frac{2}{b}\Gamma_i = 0 \qquad \text{(eqn B.36)}$$

$$\Gamma_r^2 + 2.\Gamma_r + 1 + \Gamma_i^2 + \frac{2}{b}\Gamma_i + \frac{1}{b^2} - \frac{1}{b^2} = 0 \qquad \text{(eqn B.37)}$$

$$(\Gamma_r + 1)^2 + (\Gamma_i + \frac{1}{b})^2 = \frac{1}{b^2} \qquad \text{(eqn B.38)}$$

which is again a parametric equation of the type $(x\text{-}a)^2 + (y\text{-}b)^2 = R^2$ (equation B.38).

Facts about Smith Chart

One circuit of the SMITH chart is only half a wavelength:

We remember that the SMITH chart is a polar plot of the complex reflection coefficient, which represents the ratio of the complex amplitudes of the backward and forward waves. Imagine the forward wave going past you to a load or reflector, then traveling back again to you as a reflected wave. The total phase shift in going there and coming back is twice the phase shift in just going there. Therefore, there is a full 360 degrees or 2 pi radians of phase shift for reflections from a load HALF a wavelength away. If you now move the reference plane a further HALF wavelength away from the load, there is an additional 360 degrees or 2 pi radians of phase shift, representing a further complete circuit of the complex reflection (SMITH) chart. Thus for a load a whole wavelength away there is a phase shift of 720 degrees or 4 pi radians, as the round trip is 2 whole wavelengths. Thus in moving back ONE whole wavelength from the load, the round trip distance is actually increasing by TWO whole wavelengths, so the SMITH chart is circumnavigated twice.

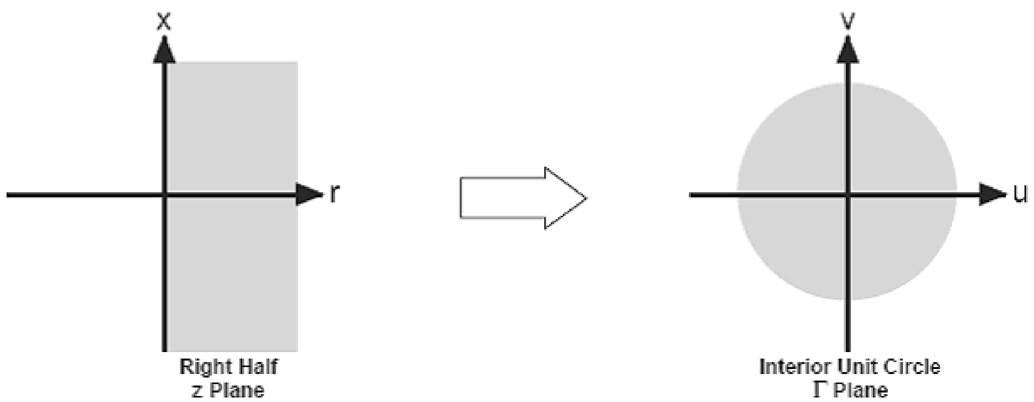
Smith Chart: graphical representation

Mathematical Basis of the Smith Chart

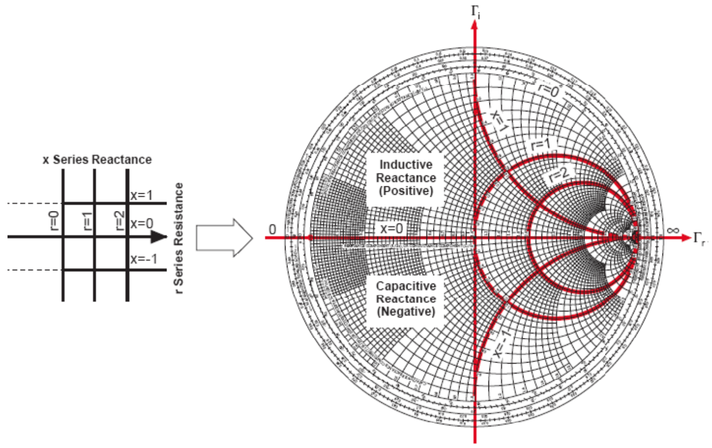
$$\Gamma = \frac{z-1}{z+1}$$

A bilinear conformal complex function

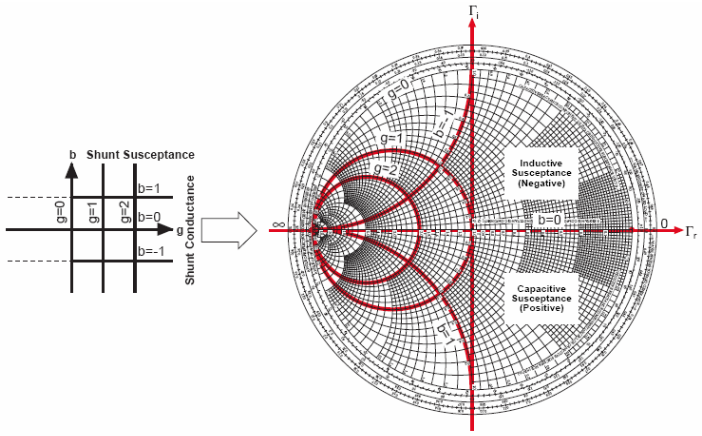
$$u + jv = \frac{(r-1)+jx}{(r+1)+jx}$$



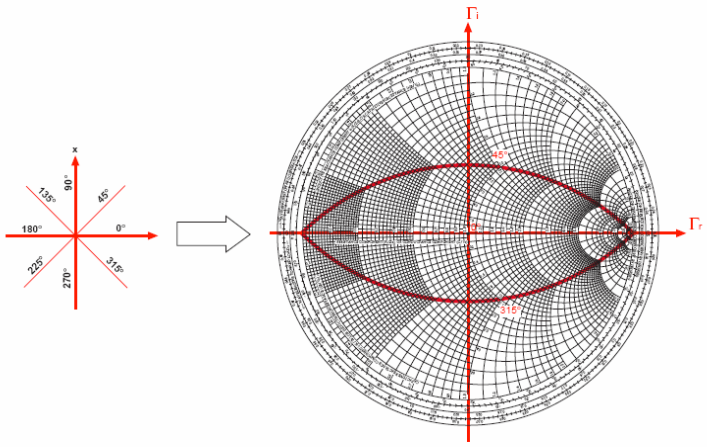
Smith Chart: Impedance Coordinates



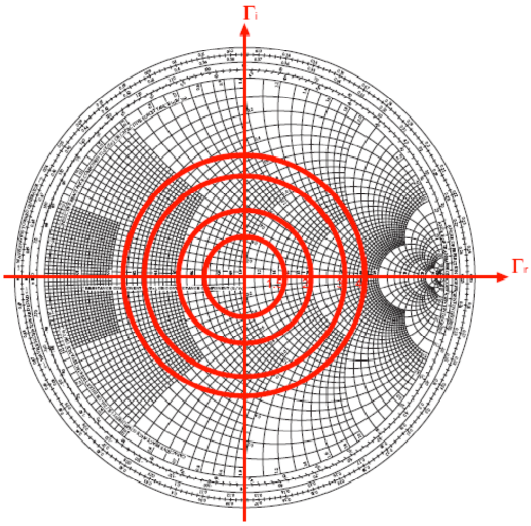
Smith Chart: Admittance Coordinates



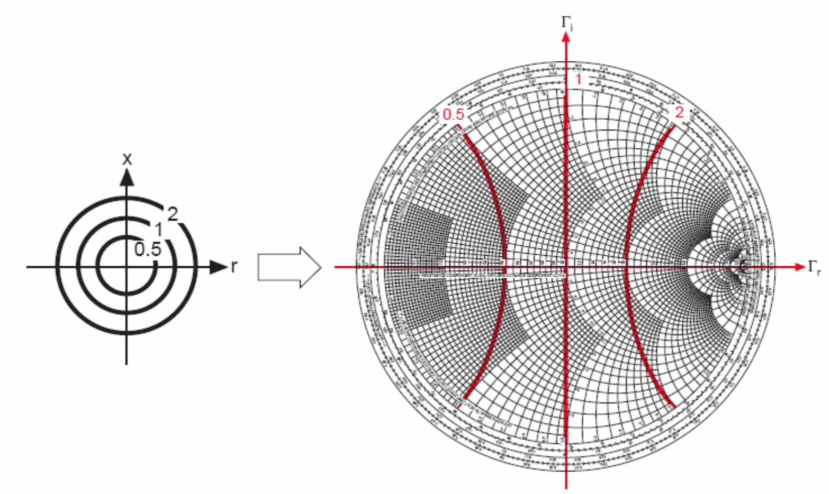
Smith Chart: Constant Impedance Phase Angle Circles



Smith Chart: Constant VSWR circles



Smith Chart: Constant Impedance Magnitude Circles



Smith Chart: for Multiplication, Division, Squares, and Square Roots

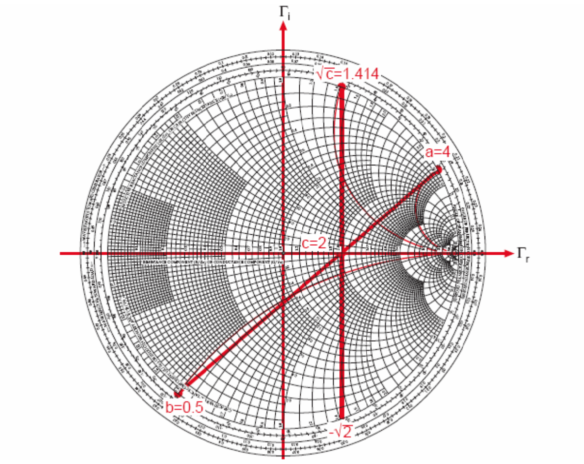
Unary Operators

- squares a^2
- square roots $\sqrt{a}$
- tangents $\tan\ \Theta$
- cotangents $\cot \Theta$
- inverse tangents tan-1 a
- inverse cotangents cot-1 a

Binary Operators

- multiplication a • b
- division c/a
- geometric mean $\sqrt{a\,b}$

Smith Chart: A Nomogram for Math Calculations



The stereographic representations of the complex plane are obtained by real constructions. Instead, the complex analogue of the representation of the trigonometric functions could be used, in which the tangent of an angle is the point of intersection of the radius of the unit circle prolonged to intersect the vertical tangent at *x*=1. The basic representation holds that the *x*-projection of the radius is $\cos\theta$, and that the *y*-projection is $\sin\theta$. The complex equivalent of this mapping needs a complex angle to work with, and ought to correspond to the polar stereographic projection rather than the central stereographic projection. In the former case, the modulus of the projection is rather $\tan\Theta/2$ r than $\tan\Theta$, so the suggested mapping is $W = \tan\Theta/2$. But Constant

$$\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}$$

$$= -i\,\frac{e^{i\theta/2} - e^{\quad i\theta/2}}{e^{i\theta/2} + e^{-i\theta/2}}$$

$$= -i\,\frac{e^{i\theta} - 1}{e^{i\theta} + 1}$$

$$= -i\,\frac{z - 1}{z + 1}$$

after introducing the abbreviation $e^{i\theta} = z$.

Figure: contours of $\tan\theta/2$ generate a useful nomogram, the Smith Chart.

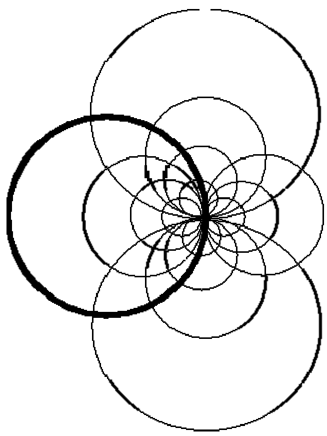
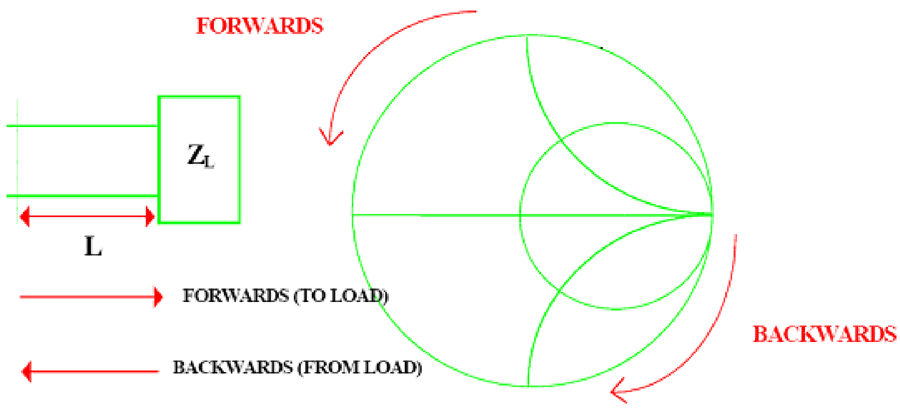


Figure B6: Contour plots for this mapping constitute nomograms which, after having been labeled and drawn artistically, are known as Smith Charts. They are of considerable use in transmission line theory, and are used without the factor i.

One great advantage of this representation is that the whole right half-plane, the one whose numbers have positive real parts, is mapped into the unit circle, the imaginary axis taking up residence on its circumference. The real axis maps into the real axis, but given that infinity maps into 1, the whole coordinate grid of lines parallel to the real and to the imaginary axis ends up as two families of mutually orthogonal circles, all passing through 1.

Moving along the Smith chart

(1) Moving along the T.L = rotating around the Smith Chart



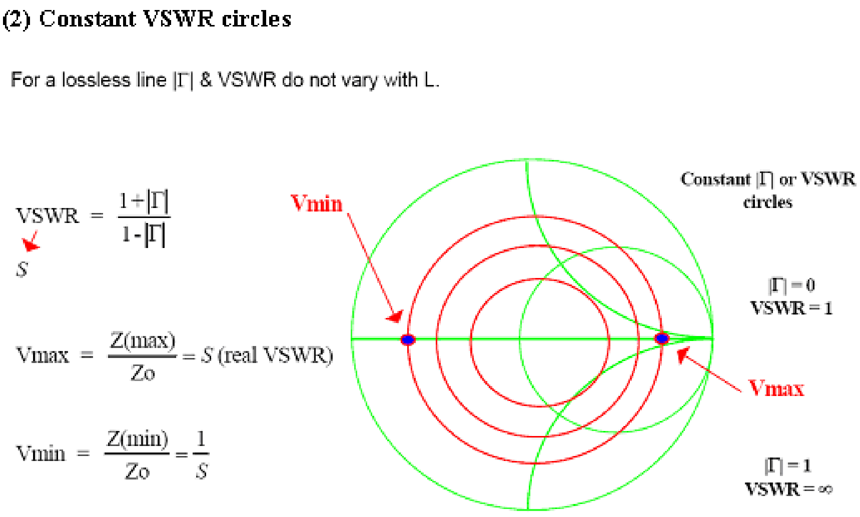


Figure B 7: Movement along the Smith chart.

Problem solving using Smith chart

Given below are the basic Smith Chart techniques for loss-less transmission lines:

- Given Z(d), Find Γ(d)
- Given Γ(d), Find Z(d)
- Given ΓR and ZR, Find Γ(d) and Z(d)
- Given ΓR and ZR, Find the Voltage Standing Wave Ratio (VSWR)
- Given Z(d), Find Y(d)
- Its use for solving line admittances
- Its use in finding Q-factor

Given Z(d), Find Γ(d)

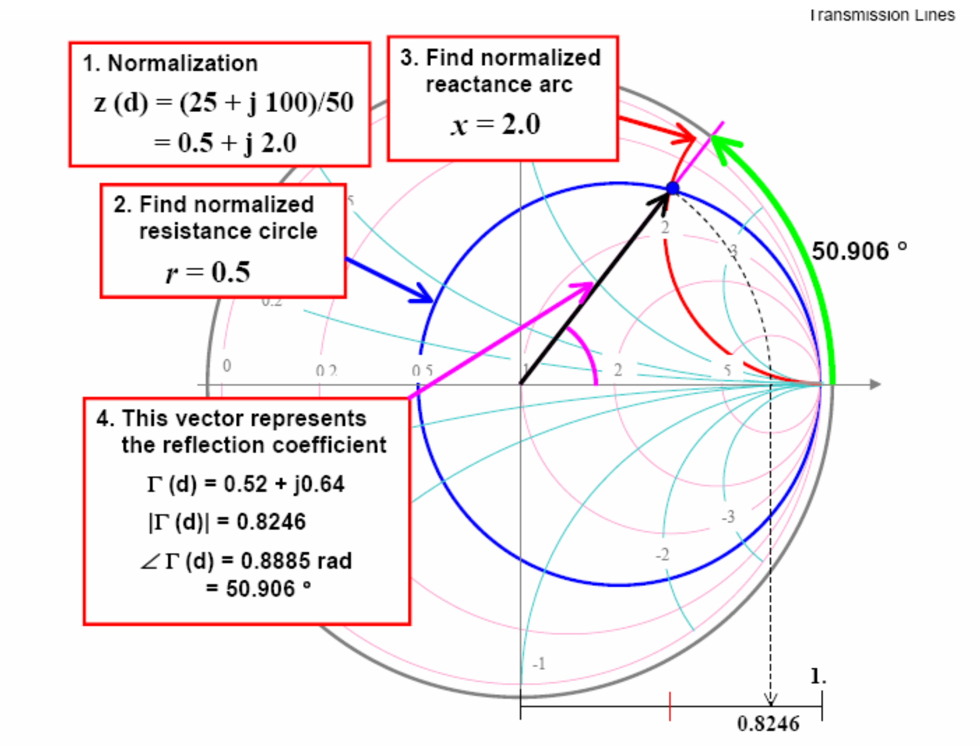
1. Normalize the impedance

$$z(d) = \frac{Z(d)}{Zo} = \frac{R}{Zo} + j\frac{X}{Zo} = r + jx$$

2. Find the circle of constant normalized resistance r
3. Find the arc of constant normalized reactance x
4. The intersection of the two curves indicates the reflection coefficient in the complex plane.

The chart provides directly the magnitude and the phase angle of Γ(d). Example: Find Γ(d),

Given Zd=25 +j100 with Z0 =50 Ω



Given Γ(d), Find *Z*(d)

1. Determine the complex point representing the given reflection coefficient G(d) on the chart.
2. Read the values of the normalized resistance *r* and of the normalized reactance *x* that correspond to the reflection coefficient point.
3. The normalized impedance is z(d) = r + j x and the actual impedance is
   Z(d) = Zo. z(d) = Zo.(r + jx) = Zo.r + j Zo.x

Given ΓR and ZR, Find Γ(d) and *Z*(d)

The magnitude of the reflection coefficient is constant along a loss-less transmission line terminated by a specified load, since

$$|\Gamma(d)| = |\Gamma_R \exp(-j2\beta d)| = |\Gamma_R|$$

Therefore, on the complex plane, a circle with center at the origin and radius | R| represents all possible reflection coefficients found along the transmission line. When the circle of constant magnitude of the reflection coefficient is drawn on the Smith chart, one can determine the values of the line impedance at any location. The graphical step-by-step procedure is:

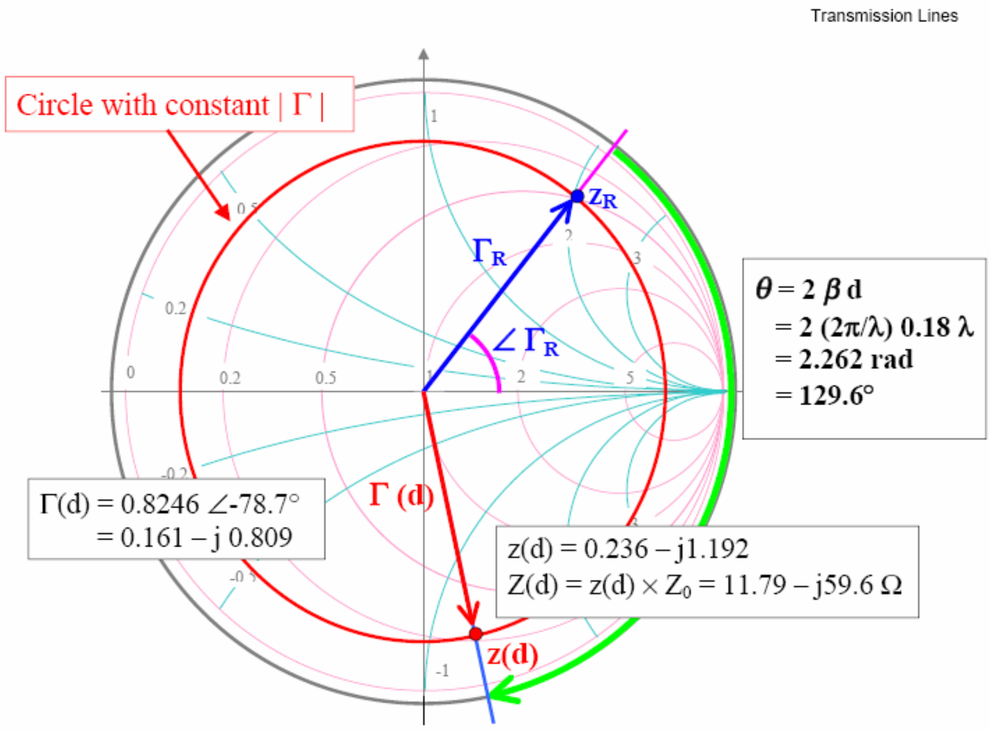
1. Identify the load reflection coefficient R and the normalized load impedance *Z*R on the Smith chart.
2. Draw The magnitude of the reflection coefficient is constant along a the circle of constant reflection coefficient amplitude | Γ(d)| =| ΓR|.

3. Starting from the point representing the load, travel on the circle

$$\Theta = 2\beta d = 2\frac{2\Pi}{\lambda}d$$

The new location on the chart corresponds to location d on the transmission line. Here, the values of Γ(d) and Z(d) can be read from the chart as before.

Example: Given $Z_R = 25 + j100\Omega$ with Zo = 50Ω find Z(d) and Γ(d) for d = 0.18λ



Given ΓR and ZR, Find the Voltage Standing Wave Ratio (VSWR)

The Voltage standing Wave Ratio or VSWR is defined as

$$VSWR = \frac{V\max}{V\min} = \frac{1+|\Gamma_R|}{1-|\Gamma_R|}$$

The normalized impedance at a maximum location of the standing wave pattern is given by

$$z(d_{\max}) = \frac{1+\Gamma(d_{\max})}{1-\Gamma(d_{\max})} = \frac{1+|\Gamma_R|}{1-|\Gamma_R|} = VSWR!$$

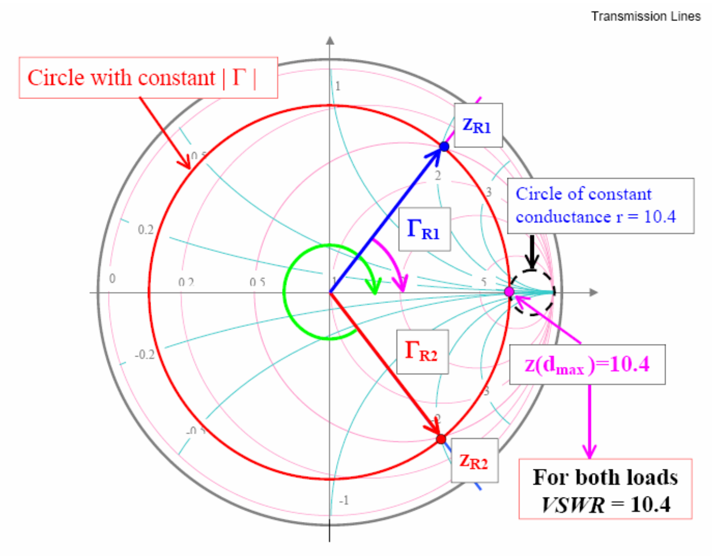
This quantity is always real and . 1. The VSWR is simply obtained on the Smith chart, by reading the value of the (real) normalized impedance, at the location dmax where . is real and positive.

The graphical step-by-step procedure is:

1. Identify the load reflection coefficient R and the normalized load impedance *Z*R on the Smith chart.
2. Draw the circle of constant reflection coefficient amplitude $|\Gamma(d)| = |\Gamma R|$.
3. Find the intersection of this circle with the real positive axis for the reflection coefficient (corresponding to the transmission line location dmax).
4. A circle of constant normalized resistance will also intersect this point. Read or interpolate the value of the normalized resistance to determine the *VSWR*.

Example

Find the *VSWR* for ZR1 = 25 + j100 Ω ; ZR2 = 25 – j100 Ω (Zo = 50 Ω)



Given *Z*(d), Find *Y*(d)

The normalized impedance and admittance are defined as

$$z(d) = \frac{1+\Gamma(d)}{1-\Gamma(d)}$$

$$y(d) = \frac{1-\Gamma(d)}{1+\Gamma(d)}$$

$$\text{since},$$

$$\Gamma(d+\frac{\lambda}{4}) = -\Gamma(d)$$

$$\Rightarrow z(d+\frac{\lambda}{4}) = \frac{1+\Gamma(d+\frac{\lambda}{4})}{1-\Gamma(d+\frac{\lambda}{4})} = \frac{1-\Gamma(d)}{1+\Gamma(d)} = y(d)$$

It is important to note equality $z(d+\frac{\lambda}{4}) = y(d)$ which is valid for normalized impedance and admittance. The actual values are given by

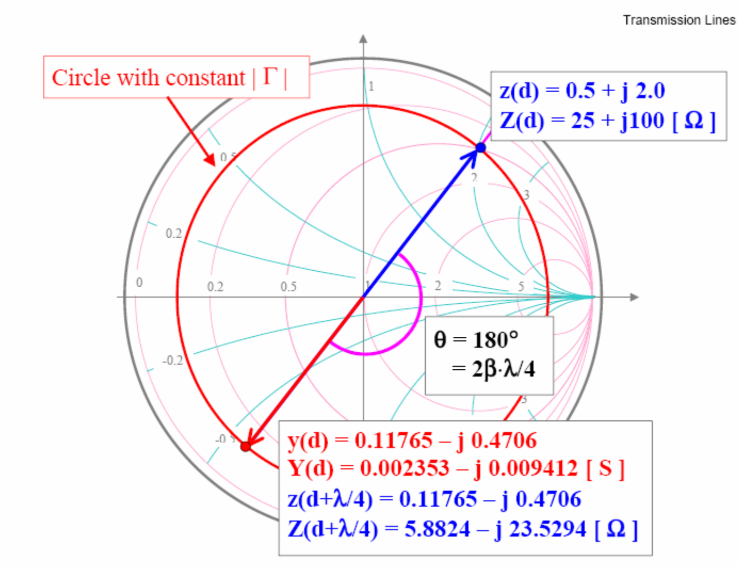
$$Z(d+\frac{\lambda}{4}) = Zo.z(d+\frac{\lambda}{4})$$

$$Y(d) = Yo.y(d) = \frac{y(d)}{Zo}$$

Where, *Y0=1 /Z0* is the characteristic admittance of the transmission line.
The graphical step-by-step procedure is:

1. Identify the load reflection coefficient R and the normalized load impedance ZR on the Smith chart.
2. Draw the circle of constant reflection coefficient amplitude | Γ(d) | = | ΓR |.
3. The normalized admittance is located at a point on the circle of constant | Γ | which is diametrically opposite to the normalized impedance

Example
Given ZR = 25 + j100 Ω with Zo = 50 Ω, find YR

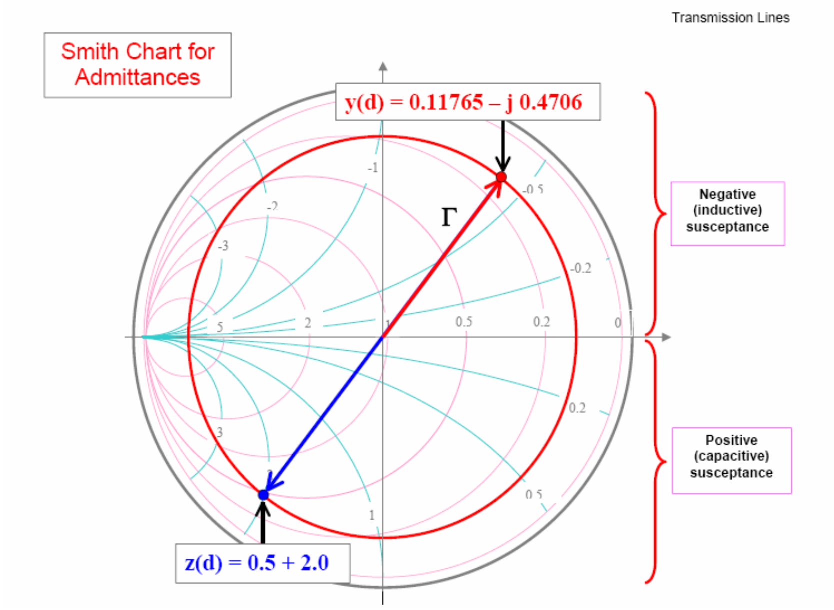


Calculation of line admittances
By shifting the space reference to the admittance location, one can move on the chart just reading the numerical values as representing admittances. Let's review the impedance-admittance terminology:

Impedance = Resistance + j Reactance, Z = R + jX

Admittance = Conductance + j Susceptance, Y = G + jB

On the impedance chart, the correct reflection coefficient is always represented by the vector corresponding to the normalized impedance. Charts specifically prepared for admittances are modified to give the correct reflection coefficient in correspondence of admittance.



Since related impedance and admittance are on opposite sides of the same Smith chart, the imaginary parts always have different sign. Therefore, a positive (inductive) reactance corresponds to a negative (inductive) susceptance, while a negative (capacitive)reactance corresponds to a positive (capacitive) susceptance. Numerically, we have

$$z = r + jx$$

$$y = g + jb = \frac{1}{r + jx}$$

$$y = \frac{r - jx}{(r + jx)(r - jx)} = \frac{r - jx}{r^2 + x^2}$$

$$g = \frac{r}{r^2 + x^2}, b = \frac{-x}{r^2 + x^2}$$

Calculation of Q-factor

After having located the impedance the Q-factor can be directly read off from the Smith Chart as mentioned below:

If using an impedance Z-Smith Chart

Q_n=|x|/r, where, Q_n is the nodal quality factor and Z=r+jx is the normalized impedance.

In order to derive this consider,

$$r = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{1 + \Gamma_r^2 - 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.6)}$$

$$x = \frac{2.\Gamma_i}{1 + \Gamma_r^2 - 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.7)}$$

So by dividing these two equations we get Q_n.

If using the admittance Y-Smith Chart

Q_n=|b|/g where, Q_n is the nodal quality factor and Y=g+jb is the normalized admittance.

In order to derive this consider,

$$g = \frac{1 - \Gamma_r^2 - \Gamma_i^2}{1 + \Gamma_r^2 + 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.25)}$$

$$b = \frac{-2.\Gamma_i}{1 + \Gamma_r^2 + 2.\Gamma_r + \Gamma_i^2} \qquad \text{(eqn B.26)}$$

So by dividing these two equations we get Q_n.

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