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Fast Converging Layered Adaptive Beam Forming Algorithm

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Abstract

LMS algorithm is simple and is well suited for continuous transmission systems since it is a continuously adaptive algorithm. However, it is not known for its convergence speed in the presence of Gaussian, spatially white, of null mean and σ_N^2 variance which has prompted people to use other complicated algorithms. In the above scenario LMS has maximum mean square error and minimum error stability. Hence, there is a need for an algorithm, which is simple to implement yet has a fast convergence rate and is not computationally intensive in the presence of noise. Thus, the algorithm featured in this paper is an attempt in achieving this and it will be referred to as the Fast Converging Layered-LMS algorithm. In the FCL-LMS algorithm the process of finding the optimum weights have been divided into two layers where both the layers have different convergence factors, the convergence factor of the upper layer is always greater than the lower layer, hence the larger value of the convergence factor helps in approaching the optimum weights and the smaller value of the convergence factor minimizes the misadjustments thus reducing the excess mean square error, this results in least mean square error, better error stability and faster convergence.

Keywords: Fast convergence, Least Mean Square (LMS), Eigen value, Mean Square Error, Misadjustments, Gaussian noise.

Introduction

Digital Beam forming is a mature technology drawing its roots from the use in Radar systems. Literature survey reveals the extent of its use in radar systems for airborne, space borne, and ground based for both commercial and military purpose [4] [5] [6] [9] [10] [11]. Although the term beam forming has been primarily used for spatial filtering, its combination with temporal filtering is widely used for eliminating ground based clutter for airborne surveillance radars [4] [5] [9]. Studies are being carried out for providing wide area radiation coverage using digital beam forming on proposed High Altitude Platforms (HAPS) [12][13]. Digital Beam forming technology has reached a sufficient level of maturity that it can be applied to communications for improving system performance [3]. One of the main reasons for using digital beam forming algorithm is the capability of forming a main beam in the direction of desired signal while at the same time placing nulls in the direction of the jammer signals. Knowing the signal characteristics and/or direction of the desired signal these smart antennas update a set of calculated weights adaptively to the signal environment, which when applied to the individual system level antenna elements result in the reception of desired signal and elimination of jammer signals. Literature survey reveals that considerable work has been done on improving the convergence speeds and radiation patterns for Least Mean Squares (LMS). Godara [16] [17] provides a comprehensive review of various digital beam forming algorithms and on performance improvement, feasibility and system consideration of a smart antenna system.

This paper presents a comprehensive analysis of FCL-LMS adaptive beam forming algorithm arising from the Least Mean Square (LMS) algorithm. Simulation study for the algorithm will be tested for a uniformly spaced linear array and the adaptability of the algorithm for various interferences angle, Gaussian noise and SNR will be studied. The reminder of the paper is organized as follows: Section II provides a background of smart antennas. Section III reviews the uniform linear space array signal model used and a discussion on the Least Mean Square, and FCL-LMS adaptive beam forming algorithm used in smart antenna array synthesis. The FCL-LMS adaptive beam forming algorithm is also derived and discussed. In Section IV, an overview of the parametric estimation tools used for our analysis will be briefed. Section V deals with the simulation scenario developed using MATLAB® environment for analyzing the algorithms. In Section VI, the result of simulation analysis will be presented and inference on algorithmic performance for different number of elements and misadjustment values for various iterations is tabulated.

Background

Smart antennas have been widely investigated for nearly two decades and are thus a mature technology. The combination of more than one array with the associated system level algorithm has the capability of providing a better directivity than their single antenna counterparts. Also a better control over placement of nulls and thus jammer rejection can be attained than the single antenna counterparts. Digital beam forming algorithms can be classified into non adaptive and adaptive algorithms. Although adaptive algorithms have shown to be more robust in adapting than their non adaptive counterparts, however, LMS algorithm fails to do so with higher efficiency in the presence of Gaussian noise.

Adaptive Antenna System

Signal Model

Consider a uniformly spaced linear array of N elements in a signal environment consisting of M far field discrete sinusoidal sources each of frequency f_o. Let these N isotropic elements be arranged in a linear pattern with the inter-elemental spacing'd'. Let us consider angle è with respect to the normal at which the plane wave impinges upon the array. The signal wave arrives at any antenna element sooner than it's preceding element.

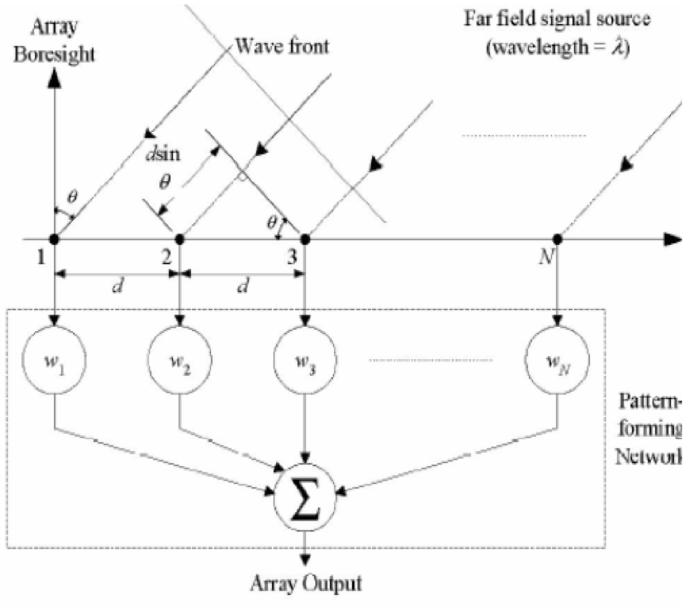

Figure 1: Smart Antenna System for a Uniform Linear Array.

Let us consider the phase received at the first element as the reference phase. Thus the phase lead with which the signal at any element arrives than at its preceding element is given by,

$$2\pi\left(\frac{d}{\lambda}\right)\sin\theta \tag{1}$$

We know that $k = 2\pi/\lambda$, which is also called the wave number. Thus phase delay is now given by,

$$kd\sin\theta \tag{2}$$

Adaptive Beam forming Algorithms

The main objective of beam forming is to form multiple beams towards desired users while nulling to the interferers at the same time by adjusting the beam former's weight vectors. The received signal $x(n)$ from multiple antenna elements are multiplied with the weight coefficients which are a set of amplitude and phase coefficients so as to adjust the phase and amplitude of the incoming signal accordingly.

The weighted signal from individual channels is summed up to obtain the beam former output $y(n)$. Thus this beam former output $y(n)$ is a linear combination of the data at the k sensors and can thus be expressed as,

$$y(n) = w^H x(n) \tag{3}$$

where,

$$w = [w_1, w_2, w_3.....w_K] \tag{4}$$

and,

$$x(n)=[x_1(n),x_2(n),x_3(n)\ldots,x_k(n)] \tag{5}$$

For the given output $y(n)$, the objective for a beam forming algorithms thus becomes minimizing the error $e(n)$ between the desired signal $d(n)$ and the array output $y(n)$.

LMS algorithm

The Least Mean Square (LMS) algorithm is an adaptive algorithm, which uses a gradient-based method of steepest decent. LMS algorithm uses the estimates of the gradient vector from the available data. LMS incorporates an iterative procedure that makes successive corrections to the weight vector in the direction of the negative of the gradient vector which eventually leads to the minimum mean square error. Compared to other algorithms LMS algorithm is relatively simple.

The LMS algorithm can be summarized in following equations,

$$y(n)= w^h x(n) \tag{6}$$

$$e(n) = d^*(n) – y(n) \tag{7}$$

$$w(n+1) = w(n) + \mu x(n)e^*(n) \tag{8}$$

In the above equations, equation (6) represents the output, equation (7) represents error and equation (8) represents weight update. Here, μ is the convergence factor.

Optimum weights are found after much iteration and they are not stable, they keep endlessly wandering around the “bottom of the bowl” (quadratic performance error surface), in the vicinity of the minimum of ξ, producing a noisy weight vector solution and an average misadjustment and also LMS does not produce efficient results in presence of Guassian noise at the input. Hence, there is a need for an algorithm, which is simple to implement yet has a fast convergence rate and is not computationally intensive.

FCL-LMS Algorithm

Experimentation with the LMS algorithm indicates that its speed of convergence decays rapidly for small values of MSE or misadjustment. This proposed algorithm *FCL-LMS (Fast Converging Layered Least Mean Square Algorithm),* converges at a very fast rate with minimum misadjustment values. The *figure 2* represents the system under test,

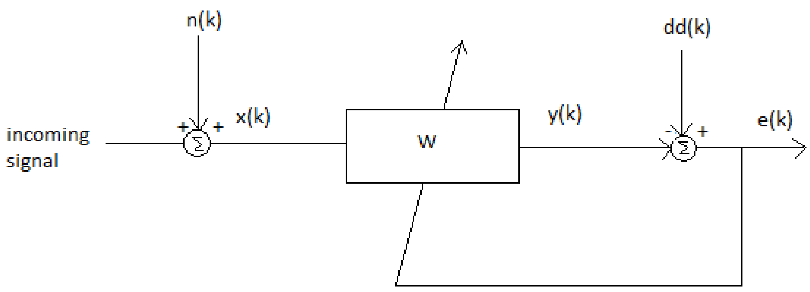

Figure 2: System identification setup (The FCL-LMS algorithm uses the above system).

From above,

$$x(k) = A * S + n(k) \tag{9}$$

Where A is the array factor, S is the signal from the transmitting end and n(k) is the matrix representing the received noise vector which can be considered here as Gaussian, spatially white, of null mean and σ_N^2 variance.

In FCL-LMS, the samples are processed in two different blocks, i.e., the total iterations for which the algorithm runs are divided into two blocks, as shown below:

Iterations	*Block–1 (n_1)*	*Block–2 (n_2)*
≥1000	1 -100	101 - itrn
<1000	1 – 50	51 - itrn

The input correlation matrix is calculated as follows:

$$ER = X*X^T \tag{9}$$

The largest Eigen value of the input correlation matrix (R) is represented as λ_{max}.
In Block-1, the value for the constant adaptation μ_1 is given by:

$$\mu_1 = \left(\frac{100}{2\lambda_{max}}\right) + K \tag{10}$$

In Block-2, the value for the constant of adaptation μ_2 is given by:

$$\mu_2 = \left(\frac{100}{2\lambda_{max}}\right) + \frac{K}{10} \tag{11}$$

Where, K = 0.04.

At any instant of time $\mu_1 > \mu_2$, this has been carefully chosen to achieve rapid convergence during the processing of Block–1 due to μ_1 and the misadjustments have been minimized due to μ_2.

The weight update equation in FCL-LMS is given by:

$$w(k+1)=w(k) + \mu_1 e(k)x(k) \tag{12}$$

Equation (12) is applicable for iterations n_1 and

$$w(k+1)=w(k) + \mu_2 e(k)x(k) \tag{13}$$

Equation (13) is applicable for iterations n_2.

The weights are largely influenced by the convergence factor which in turn is influenced by the number of iterations and the largest eigen value of the R matrix as shown in equations (14) & (15), thus these weight update equations eliminate the guassian noise and compute the optimum weights required to obtain the transmitted signal by automatically locating the appropriate convergence factor required for adaptation. μ_1 decreases as iterations increase and so does μ_2 as seen in *figure 3* this is because,

$$\lambda_{max} \propto \text{iterations} \tag{14}$$

they exhibit a linear behaviour as seen in *figure 4*.But,

$$\mu \propto \left(\frac{1}{\lambda_{max}}\right) \tag{15}$$

Thus, from equation (15), as iterations increase, μ decreases, where μ refers to either μ_1 or μ_2.

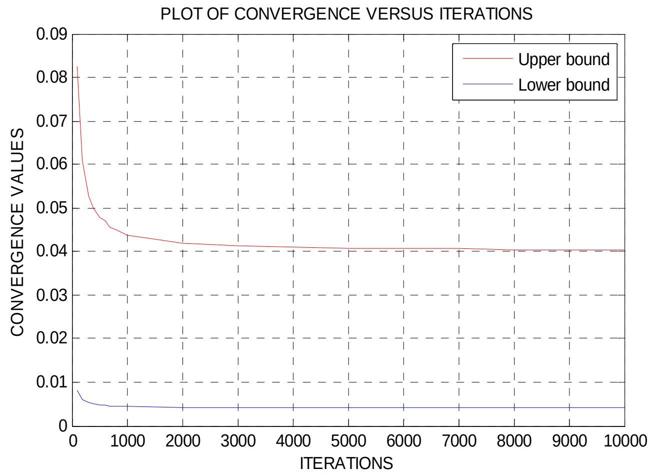

Figure 3: Variation of μ_1(upper bound) and μ_2(lower bound) with increasing iterations.

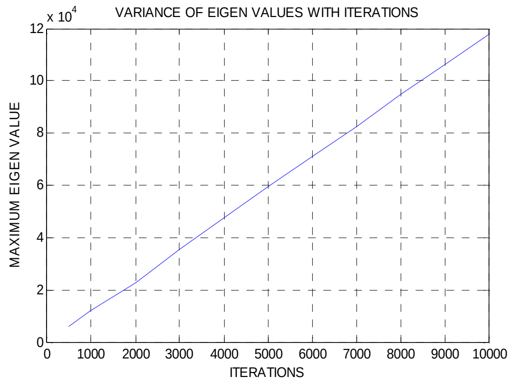

Figure 4: Variation of λ_{max} with increasing iterations.

Performance Metrics

Mean square error

The objective of mean square error consists of adjusting the parameters of a model function to best fit a data set. A simple data set consists of n points (data pairs)(x_i, y_i),i=1, 2….n, where x_i is an independent variable and y_i is a dependent variable whose value is found by observation. The model function has the form $f(x, \beta)$, where the m adjustable parameters are held in the vector β. The goal is to find the parameter values for the model which "best" fits the data. The least squares method finds its optimum when the sum, S, of squared residuals,

$$s = \sum_{i=1}^{n} r_i^2 \tag{16}$$

is a minimum. A residual is defined as the difference between the value predicted by the model and the actual value of the dependent variable,

$$r_i = y_i - f(x_i, \beta) \tag{17}$$

The mean square error plays a vital role in DBF, and this is a performance metric in FCL-LMS study.

Misadjustments

The performance of a system is measured in terms of its misadjustment M, which is a normalized mean square error, is defined as the ratio of the steady state excess mean square error (EMSE) to the minimum mean-square error,

$$M = \frac{EMSE(\infty)}{MSE_{min}} \tag{18}$$

The EMSE at the k^{th} iteration is given by,

$$EMSE(k) = MSE(k) - MSE_{min} \tag{19}$$

Thus,

$$EMSE(\infty) = MSE(\infty) - MSE_{min} \tag{20}$$

The value of MSE_{min} , obtained when the coefficients of the unknown system and the filter match, is equal to an irreducible noise variance σ_N^2 , for a zero mean noise N.

Simulation Scenario

Simulation environment

The FCL-LMS algorithm was tested in a simulation scenario developed using MATLAB platform. Results were obtained for the convergence plot, weight convergence and radiation pattern for both the LMS and the FCL-LMS algorithm as shown in figure (5) (6) (7) (8) (9) (10) (11) (12) & (13).In LMS algorithm two cases have been considered one with a higher value of μ and the other with a lower value of μ. An environment is simulated consisting of a single desired signal arriving at an angle of 90^0 and the interferences are at 40^0, 60^0, 120^0 and 140^0. The Signal to Noise ratio is kept constant at 20dB. Each realization is run for 1000 Monte Carlo trials. The variation of the performance metrics was obtained and tabulated in Table 1 and Table 2 below.

LMS Results

Case (1): μ=0.05

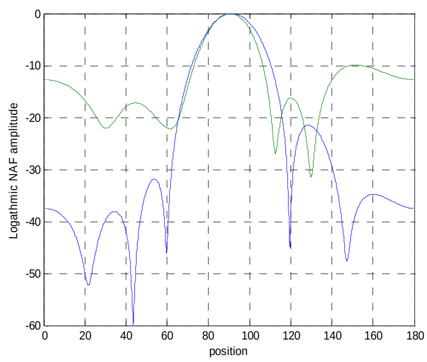

Figure 6: Radiation pattern for LMS algorithm.

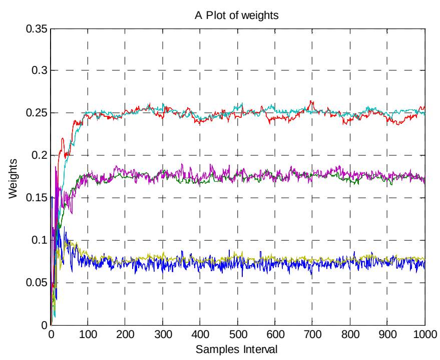

Figure 7: Convergence of weights for LMS algorithm.

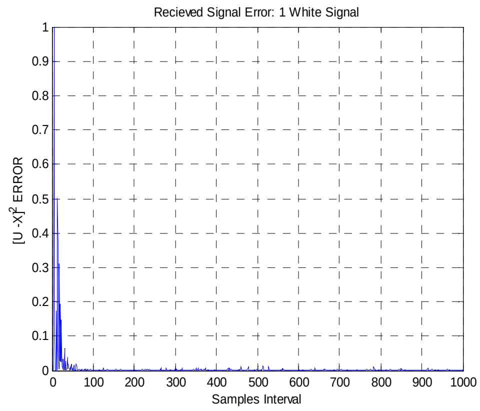

Figure 8: Mean square error for LMS algorithm.

Case (2): μ=0.005

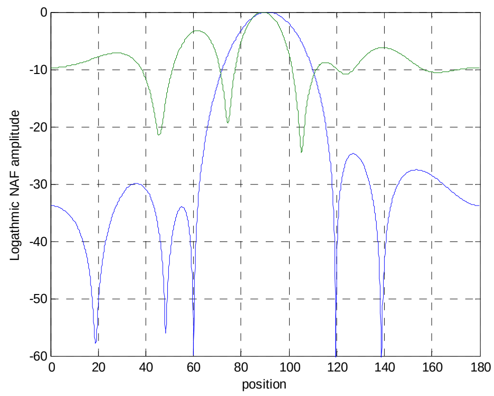

Figure 9: Radiation pattern for LMS algorithm.

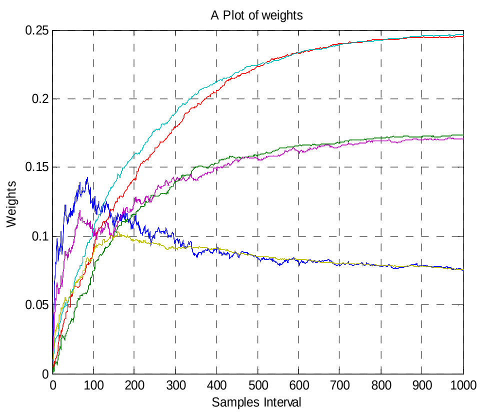

Figure 10: Convergence of weights for LMS algorithm.

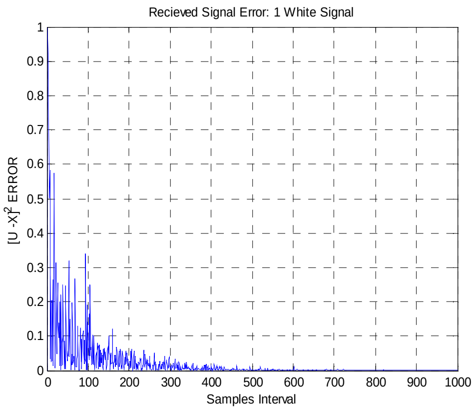

Figure 12: Mean square error for LMS algorithm.

FCL-LMS Results

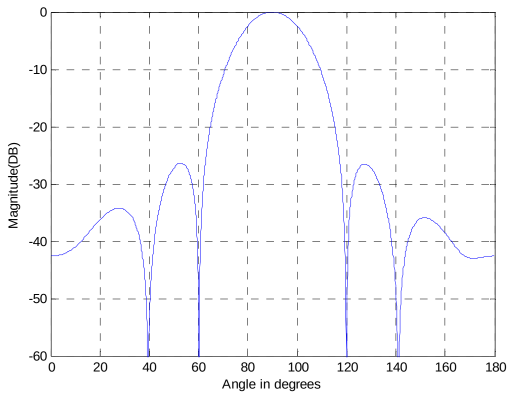

Figure 13: Radiation pattern for FCL-LMS algorithm.

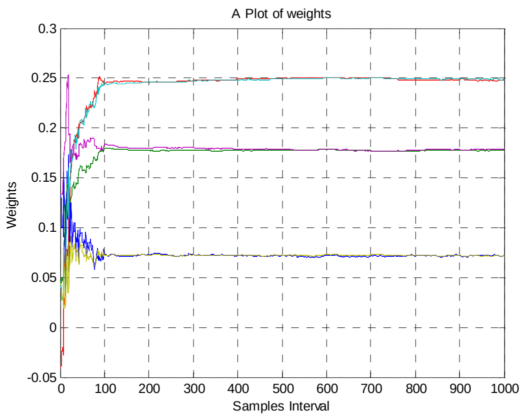

Figure 14: Convergence of weights for FCL-LMS algorithm.

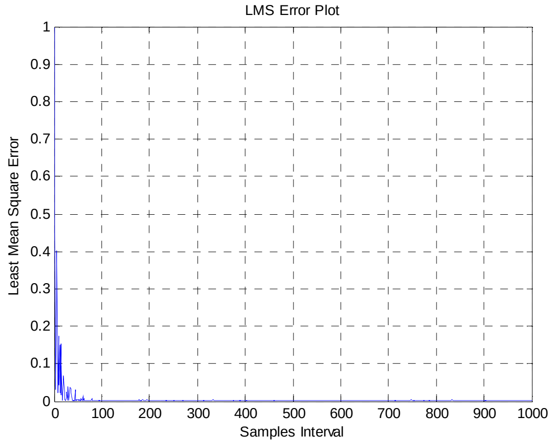

Figure 15: Mean square error for LMS algorithm.

Table 1: Performance of FCL-LMS algorithm for variation in number of elements at $\theta_d=90^0$, θ_i= [40 60 120 140].

Sl. No.	No. of elements	Half power beam width(degrees)	Null depth at interferences(dB)				MSE (dB)
			40^0	60^0	120^0	140^0	
1	6	22	-60	-57.7	-60	-60	0.004725
2	8	12.8	-60	-46.62	-51.19	-58.37	0.006592
3	10	10.6	-60	-60	-60	-60	0.006417
4	12	8.6	-60	-60	-60	-60	0.001814

Table 2: Misadjustment values (M) for various iterations using FCL-LMS.

ITERATIONS	MISADJUSTMENT
5000	3.13583
4000	5.52
3000	5.51269
2000	4.523
1000	2.146

Results and Discussions

Processing algorithms for narrowband beam forming is tested. Highly efficient adaptive beam forming technique to adapt the weights of the antenna array using FCL-LMS algorithm is discussed and implemented. LMS has been conventionally used for larger iterations of the order 1000 but FCL-LMS algorithm can be used even for 100 iterations and here weights converge at a faster rate.

The radiation pattern for the LMS algorithm showed a main beam in the desired angular direction of 90^0 and nulls in the interfering direction of 40^0, 60^0, 120^0 and 140^0. As is evident from *figure (6) & (9),* the nulls are obtained at the desired direction of interferences. The performance of LMS algorithm indicated that more time was required for convergence as shown in *figure (8) & (9).* Less than 1% error was reported only after the number of iterations crossed 350 and the weights converged after much iteration in *case 2* and optimum weights were not found in *case 1* which can be observed in *figure (7) & (10)*. The radiation pattern for the FCL-LMS algorithm also showed a main beam directed at 90^0 and interferences at 40^0, 60^0, 120^0 and 140^0, and how at every null depth in the interfering direction was deeper at about -60dB as shown in *figure (13)*. Also the solution converged from 50 iterations itself for all values of $n \geq 6$ and the weights also converged as shown in *figure (14).* Thus the superior performance of the FCL-LMS algorithm over the LMS algorithm is evident. This characteristic of the FCL-LMS algorithm can be employed easily in radar applications where the target environment changes rapidly and has to be tracked with greater speeds and accuracy. Thus, the FCL-LMS algorithm will perform better than the LMS when employed in radar systems for rejecting intentional wide jammers.

The performance of the algorithm was critically observed for uniformly spaced linear array by varying the number of elements in an array and varying the number of iterations. *Table 1* indicates the performance of the FCL-LMS algorithm for variation in number of elements. Observe that with the increase in number of elements, the half power beam width decreases thereby making more and more directive. Also, the null depth increases thereby improving the performance of the beam forming system in rejecting the interference. Thus, the Mean Square error decreases as we increase the number of elements. Thus in order to provide a better gain and the least error for the FCL-LMS algorithm, maximum possible number of elements must be used. As seen in *table 2*, the misadjustment was reduced by incorporating FCL-LMS.

FCL-LMS can be used in adaptive linear array systems for digital beam forming, enhancing the performance of smart antennas, can be hence used in cellular communication systems and in military applications.

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