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Tuning and Analysis of Fractional Order PID Controller

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Abstract

This paper presents the development of a new tuning method and performance of the fractional order PID controller includes the integer order PID controller parameter. The tuning of the PID controller is mostly done using Zeigler and Nichols tuning method. All the parameters of the controller, namely K_p (Proportional gain), K_i (integral gain), K_d (derivative gain) can be determined by using Zeigler and Nichols method. Fractional order PID (FOPID) is a special kind of PID controller whose derivative and integral order are fractional rather than integer. To design FOPID controller is to determine the two important parameters λ (integrator order) and μ (derivative order).In this paper it is shown that the response and performance of FOPID controller is much better than integer order PID controller for the same system.

Introduction

PID controller is a well known controller which is used in the most application.PID controller becomes a most popular industrial controller due to its simplicity and the ability to tune a few parameters automatically. According to the Japan electric measuring instrument manufacture's association in 1989, PID controller is used in more than 90% of the control loop. As an example for the application of PID controller in industry, slow industrial process can be pointed, low percentage overshoot and small settling time can be obtained by using this controller. This controller provides feedback, it has ability to eliminate steady state offsets through derivative action. The derivative action in the control loop will improve the damping and therefore by accelerating the transient response, a lighter proportional gain can be obtained during the past half century, many theoretical and industrial studies have been done in PID controller setting rules Zeigler and Nichol's in 1942 proposed a method to set the PID controller parameter Hagglund and Astrom in 1955 and cheng-ching in 1999, introduced other technique. By generalizing the derivative and integer orders, from the integer field to non-integer numbers, the fractional order PID control is obtained.

The performance of the PID controller can be improved by making the use of fractional order derivatives and integrals. This flexibility helps the design more robust system. Before using the fractional order controller in design an introduction to the fractional calculus is required. The first time, calculus generation to fractional, was proposed Leibniz and Hopital for the first time afterwords, the systematic studies in this field by many researchers such as Liouville (1832), Holmgren (1864) and Riemann (1953) were performed.

Integer Order PID Controller

The PID refers to the first letter of the names that make up the standard three term controller. These are P for the proportional term, I for the Integral term and D for the derivative term in the controller. Three term or PID controllers are probably used by most widely industrial controller.

A PID controller is essentially a generic closed loop feedback mechanism.

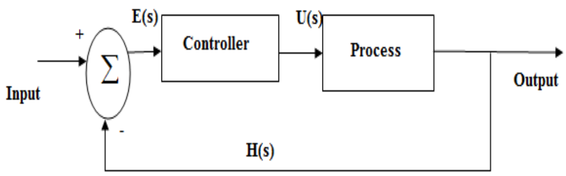


Figure 1: SISO unity feedback controller

Controller monitors the error between a measured process variable and a desired set point; from this error, a corrective signal is computed and is eventually feedback to the input side to adjust the process accordingly. The differential equation for the PID controller is

$$u(t) = K_p e(t) + T_i \int_0^t e_p(t)dt + T_d \frac{d}{dt} e(t)dt \qquad (1)$$

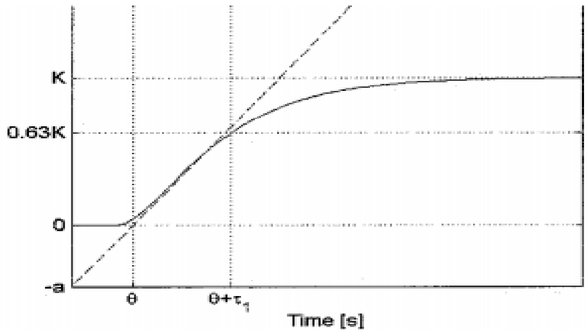
Thus, the PID controller algorithm is described by a weighted sum of the three time functions where the three distinct weights are: K_p (Proportional gain) determines the influence of the present error value on the control mechanism, I (integral gain) decides the reaction based on the area under the error time curve up to the present point and Td (derivative gain) accounts for the extent of the reaction to the rate of change of the error with time.

Tuning of Integer Order PID Controller

Many tuning method are presented in literature that are based on a few structure of the process dynamics.

Zeigler Nichols Process Reaction Curve Method

In 1942, Zeigler and Nichols presented two classical methods to tune a PID controller. These methods are widely used, due to their simplicity. In the first method, the controller setting are based on two parameter θ and a of the process reaction curve.



The proposed Zeigler Nichols setting is shown Table 1

Controller Type	K_p	T_i	T_d
P	$1/a$	-------	-------
PI	$0.9/a$	$3.33a$	-------
PID	$1.2/a$	2θ	0.5θ

The frequency domain method proposed by Zeigler and Nichols is based on the ultimate gain K_c and the ultimate period T_u. The controller setting is shown Table 2

Controller Type	K_p	T_i	T_d
P	$0.5K_c$	-------	-------
PI	$0.45K_c$	$0.83T_u$	-------
PID	$0.6K_c$	$0.5T_u$	$0.125T_u$

Astrom and Haggland (1985)
Astrom and Haggland recognized that the Zeigler-Nichols continuous cycling method actually identifies the point $(-1/K_u,\ 0)$ on the Nyquist curve, and move it to a predefined point. With PID control, it is possible to move a given point on the Nyquist curve to an arbitrary position. By increasing the gain, the arbitrary point moves in the direction of $G(j\omega)$. By changing I and D action moves the point in the orthogonal direction.

Brief Mathematical Background of Fractional calculus

Orders of fractional calculus are real number. Many different definitions for general integro- differentional operation can be found in the literature. Among them the most commonly used for general fractional Integro- differential expressions are given by chauchy, Riemann-Liouville, Grunwald letnikov and Caputo. These definitions are required for realization of control algorithm.

At first, we generalize the differential and Integral operators in to one fundamental operator ${}_aD_t^{\alpha}$ where

$$
{}_aD_t^{\alpha} = \frac{d^{\alpha}}{dt^{\alpha}} \quad for \quad R(\alpha) > 0
$$
$$
= 1 \quad for \quad R(\alpha) = 0
$$
$$
= \int_a^t (d\tau)^{-\alpha} \quad for \quad R(\alpha) < 0
$$

$R(\alpha)$ Denote real part of the α which is, in general is a complex quantity.

The Grunwald – Letnikov definition is

$$
{}_aD_t^{\alpha} f(t) = Lt_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh)
$$

The fractional order for the first order system is given by in the figure (1)

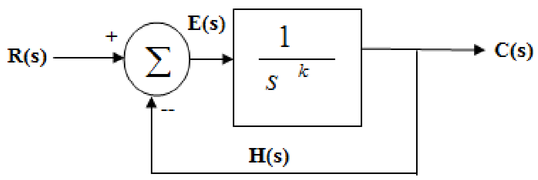


Figure 2

The transfer function is given by

$$
\frac{C(s)}{R(s)} = \frac{1}{s^k + 1} \tag{2}
$$

By putting s= $j\omega$, the closed loop transfer function given by H ($j\omega$)

$$H(j\omega) = \frac{1}{(j\omega)^k + 1}\text{; where } j^k = \cos\frac{\pi K}{2} + j\sin\frac{\pi K}{2}$$

$$H(j\omega) = \frac{1}{\omega^k(\cos\frac{\pi K}{2} + j\sin\frac{\pi K}{2}) + 1}$$

$$H(j\omega) = \frac{1}{(\omega^k\cos\frac{\pi K}{2} + 1) + j\sin\frac{\pi K}{2}\omega^k} \tag{3}$$

For the system design, we need the maximum value of $|H(j\omega)|$ and frequency ω_p where the maximum occur where

$$|H(j\omega)| = \frac{1}{\sqrt{\omega^{2k} + 1 + 2\omega^k\cos\frac{\pi K}{2}}} \text{ and}$$

$$\text{Arg } H(j\omega) = -\tan^{-1}\left(\frac{\omega^k\sin\frac{\pi K}{2}}{1 + \omega^k\cos\frac{\pi K}{2}}\right) \tag{4}$$

A fractional differential equation for a fractional order control system can be written as

$$a_n\frac{d^{\alpha_n}y(t)}{dt^{\alpha_n}} + a_{n-1}\frac{d^{\alpha_{n-1}}y(t)}{dt^{\alpha_{n-1}}} + - - + a_0\frac{d^{\alpha_0}y(t)}{dt^{\alpha_0}}$$

$$= b_m\frac{d^{\beta_m}x(t)}{dt^{\beta_m}} + b_{m-1}\frac{d^{\beta_{m-1}}x(t)}{dt^{\beta_{m-1}}} + - - + b_0\frac{d^{\beta_0}x(t)}{dt^{\beta_0}}$$

Where $\alpha_n > \alpha_{n-1} > ---- > \alpha_0 \geq 0$ and $\beta_n > \beta_{n-1} > ---- > \beta_0 \geq 0$ are constant.

Basic concept of Fractional order PID controller

Consider the negative feedback control system as shown in fig (2).

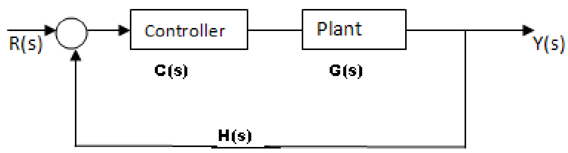


Figure 2

The continuous transfer function of the $PI^{\lambda}D^{\mu}$ controller is obtained through Laplace transform as

$$C(s) = K_p + \frac{T_i}{s^{\lambda}} + T_d s^{\mu} \tag{5}$$

The PID controller expands the integer order PID controller from point to plane, there by adding flexibility to controller design and allowing us to control our real world processes more accurately but only at the cost of increased design complexity.

Tuning method for the fraction order PID controller

To obtain the K_p (proportional gain), a constant of integral term (K_i), the constant of derivative term K_d , the fractional order of differentiator μ and the fractional order of integrator λ. The method uses classical Zeigler – Nichols tuning rule to obtain K_p and K_i. To obtain initial value of K_d , then some fine tuning has been done by using Astrom-Hagglund method described earlier. The fractional order λ and μ are obtained to achieve specified phase margin.

Let ϕ_{pm} be the required phase margin and ω_{cp} be the frequency of the critical point on the Nyquist curve of G(s) at which $\arg(G(j\omega_{cp})) = -180$, then the gain margin defined as

$$g_m = \frac{1}{\left|G(j\omega_{cp})\right|} = K_c \tag{6}$$

In order to make the phase margin of the system equal to ϕ_{pm} and $\left|C(j\omega_{cp})G(j\omega_{cp}) = 1\right|$, the following equation must be satisfied.

$$C(j\omega_{cp}) = \frac{1}{\left|G(j\omega_{cp})\right|} e^{j\phi_{pm}} = K_c \cos\phi_{pm} + jK_c \sin\phi_{pm} \tag{7}$$

Then we write $C(j\omega_{cp})$ using equation

$$C(j\omega_{cp}) = K_p + K_i \omega_{cp}^{-\lambda} \cos(\frac{\pi}{2}\lambda) + K_d \omega_{cp}^{\mu} \cos(\frac{\pi}{2}\mu)$$

$$+\left[-K_i \omega_{cp}^{-\lambda} \sin(\frac{\pi}{2}\lambda) + K_d \omega_{cp}^{\mu} \sin(\frac{\pi}{2}\mu)\right] \quad (8)$$

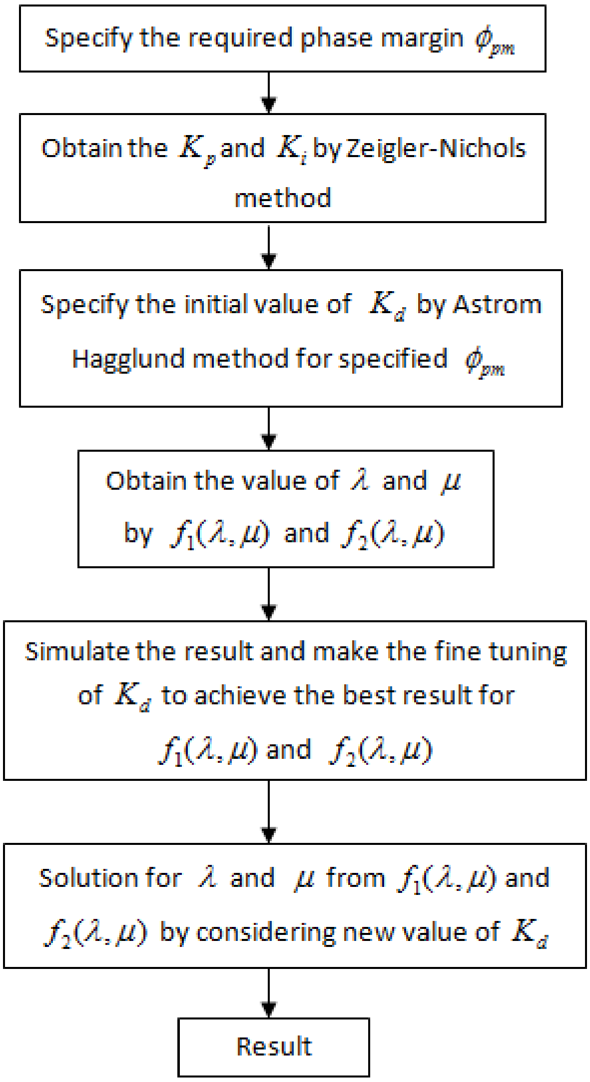
Considering equation (7) and (8) we can write

$$f_1(\lambda,\mu) = K_p + K_i \omega_{cp}^{-\lambda} \cos(\frac{\pi}{2}\lambda) + K_d \omega_{cp}^{\mu} \cos(\frac{\pi}{2}\mu) - K_c(\cos\phi_{pm}) = 0 \quad (9)$$

$$f_2(\lambda,\mu) = -K_i \omega_{cp}^{-\lambda} \sin(\frac{\pi}{2}\lambda) + K_d \omega_{cp}^{\mu} \sin(\frac{\pi}{2}\mu) - K_c(\sin\phi_{pm}) = 0 \quad (10)$$

The numerical solution for λ and μ can be obtained by the equation (9) and (10).

Algorithm for tuning of $PI^{\lambda}D^{\mu}$ controller



Problem formulation

The transfer function consider for the implementation of PID and FOPID controller is given as

$$G(s) = \frac{1}{s(s^2 + 3s + 2)}$$

By using Routh's stability, the phase crossover frequency of the system can be defined as $\omega_{cp} = \sqrt{2}$ and the gain margin of the system can be obtained as K_c=6.

To tune the PID controller, Zeigler Nichols closed loop tuning is used. K_p, K_i *and* K_d of the controller has been obtained are 3.6, 1.63 and 1.98. The PID controller obtain can be given as $C_1(s)$

$$C_1(s) = 3.6 + \frac{1.63}{s} + 1.98s$$

By using classical Astrom Hagglund method, the value of the PID controller parameters have been calculated for the specified phase margin (ϕ_{pm}) at 40^0.

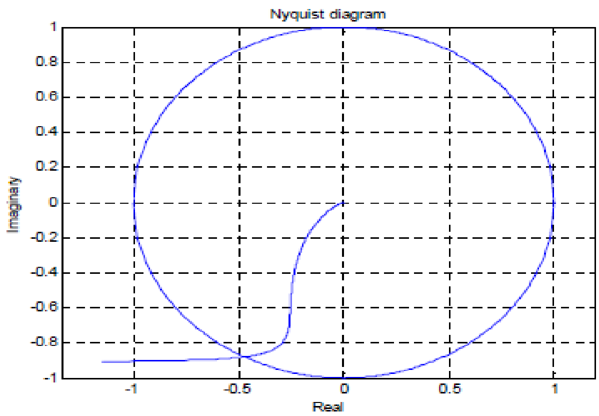


Figure 3

Table 3

Parameter	Z-N Method value	Phase Angle	Astrom method value
K_p	3.6	40^0	4.59
K_i	1.63	40^0	1.51
K_d	1.98	40^0	3.48

The controller obtained by classical Astrom Hagglund method is given by

$$C_2(s) = 4.59 + \frac{1.51}{s} + 3.48s$$

The proposed method takes the value of K_p and K_i from Zeigler and Nichols method. The value of K_d have been obtained by using Astrom - Hagglund method for specified Phase Margin ($\phi_{pm} = 40^0$). By using fine tuning for K_d to achieve the best solution by equation (9) and (10) for the specified phase margin.

These two equations can be solved by using "fsolve" toolbox of the MATLAB to obtain the value of λ and μ for the new value of K_d for the phase margin ($\phi_{pm} = 40^0$).

Phase Margin	Proposed Method				
ϕ_{pm}	K_p	K_i	K_d	λ	μ
40^0	3.6	1.63	3.75	1.39	0.79

The $PI^{\lambda}D^{\mu}$ controller obtained can be written as

$$C_3(s) = 3.6 + \frac{1.63}{s^{1.39}} + 3.75 s^{0.79}$$

Step response of the system for $C_1(s)$, $C_2(s)$ and $C_3(s)$ for 40^0 phase margin are given in figure (3).

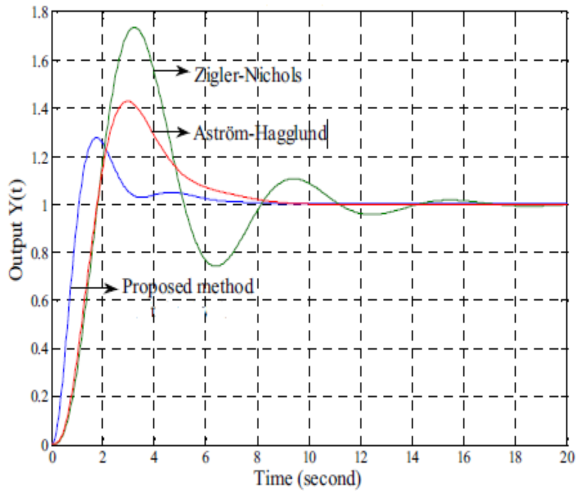


Figure 3

Step response of the system gives valuable information such as Maximum overshoot ($M_p\%$), rise time (T_r), peak time (T_p) and settling time (T_s).

Step Response specification	Maximum Overshoot $M_p\%$	Peak time T_r	Rise Time T_p	Settling time T_s
Z-N PID	73.5	3.25	1.67	12.5
A-H PID	43.0	2.95	1.66	6.67
FO-PID	27.9	1.74	0.96	4.65

From the table the proposed method gives much better performance with respect to Z-N method and Astrom-Hagglund method especially for Maximum overshoot ($M_p\%$), rise time(T_r), peak time(T_p) and settling time(T_s).

Conclusion

A method for tuning of PID and fractional order PID controller has been proposed. The presented method is based on idea of using Zeigler-Nichols for K_p and K_i while Astrom-Hagglund method is used for determining K_d for the conventional PID. Similarly K_p and K_i parameter for fractional order PID controller have been computed from Zeigler and Nichols method and the remaining parameter K_d, λ and μ have been found from Astrom - Hagglund method. The simulation result demonstrated that the fractional order PID controller has better response than the conventional PID controller. The comparison study of the proposed method for tuning of fractional order PID controller certainly will be very important.FOPID controller provides stability region even when an Integer order PID controller cannot provide any stability region.

References

[1] J.G. Zeigler and N.B. Nichols," Optimum setting for automatic controller', ASME Trans., pp. 759-768, 1942

[2] S.Manabe, "The non – integer integral and its application to control systems," ETJ of Japan, vol.6, no.3-4, pp.83-87, 1961.4.

[3] K.J.Astrom, CC.Hang, P.Pearson and W.K Ho:" A self tuning controller with PID structure." Inst.J.of Control, 38, pp. 401-417(1983)

[4] K.J.Astrom and T. Haggland;" Automatic tuning of PID controllers", Instrument Society of America (1988).

[5] G. Maione and P. Lino, "New Tuning Rules for Fractional $PI^{\lambda}D^{\mu}$ controllers", Nonlinear Dynamics, 2006.

[6] M. Zhuang and D P Atherton, "Automatic tunning of Optimum PID controllers," in IEEE Proceeding- Control theory and Application , vol. 140, 1993,pp.216-224

[7] K.J.Astrom and T. Haggland, "PID Controllers: Theory, Design and Tuning." Research Triangle Park, NC: International Society for Measurement and Controls, 1995

[8] I.Petras, "The fractional-order controllers: Methods for their synthesis and application", J.Elect.Eng, vol.50, no.9-10, pp.284-288, 1999.

[9] I.Podlubny, "Fractional-order systems and $PI^{\lambda}D^{\mu}$- CONTROLLERS," IEEE Trans. Automat. Contr., vol.44, pp.208-214, Jan.1999.

[10] Podlubny,I.: "Fractional – order Systems and fractional order controllers."Inst.Exp.Phys. Slovak Academy of Science N-1UEF-03-94,Kosice

[11] O. Lequin , M. Gevers, M.Mossberg, E.Bosmans and L. Triest , " Iterative Feedback tuning of PID parameters : Comprision with classical tuning rules," Control engineering Practice, vol. 11,2003,pp. 1023-1033.

[12] YangQuan Chen , Ivo Petras and Dingyu Xue ," Fractional order Control – A Tutorial" American Control Conference , June 10-12, 2009 , pp. 1397-1411

[13] W. Jifeng, L. Yuankai, "Frequncy domain Analysis and Applications for Fractional Order Control System", IOP,Journal of Physics:Conf. 13, pp.268-273, 2005.

[14] K. Bettou, A. Charef, F. Mesquine, "A new design method for fractional $PI^{\lambda}D^{\mu}$ controller", IJ-STA, vol. 2, pp.414-429, 2008.

[15] B. Vinagre, I. Podlubny, L Dorcak and V. Feliu, "On fractional PID controllers: A frequency domain approach",IFAC workshop on digital control. Past, present and future of PID control, pp. 53–58, Terrasa, Spain, 2000.

[16] Subhransu Padhee, Abhinav Gautam, Yaduvir Singh, and Gagandeep Kaur "A Novel Evolutionary Tuning Method For Fractional Order PID Controller", International Journal of soft computing and Engineering(IJSCE),Issue-3,July 2011

[17] S.E. Hamamci "An algorithm for stabilization of fractional order time delay systems using fractional-order PID controllers", IEEE Trans. On Automatic Control, vol. 52,pp. 1964-1969, 2007

[18] R. S. Barbosa, J. A. Tenerio, Machado and Isabel. M. Ferreira, "Tuning of PID controllers based Bode's Ideal transfer function, Nonlinear Daynamics, vol. 38, pp.305 - 321, 2004.

[19] W. Jifeng, L. Yuankai, "Frequncy domain Analysis and Applications for Fractional Order Control System", IOP,Journal of Physics:Conf. 13, pp.268-273, 2005.

[20] D. Xue, Y.Q. Chen, D. P. Atherton "Linear Feedback Control Analysis and Design with MATLAB", Advance sin Design and Control, Siam, 2007.

[21] Valerio, "Ninteger v. 2.3 Fractional control toolbox for MatLab", http://web.ist.utl.pt/~duarte.valerio, 2005.

[22] K. Ogata, "Modern Control Engineering", Prentice Hall,New Jersey, 2002.

[23] Mohd Fua'ad Rahmat , Amir Mehdi Yazdani , Mohammad Ahmadi Movahed and Somaiyeh Mahmoudzadeh ," Temperature control of continuous stirred tank reactor by means of two different Intelligent Strategies" IJSSIS, Vol. 4 No.2 , June 2011