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Solving Environmental Economic Dispatch Problem with Lagrangian Relaxation Method

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Abstract

In this paper an Environmental Economic Dispatch (EED) problem is proposed to investigate the emission control. The current energy consumption is weighing heavily on fossil fuels which account for about 70-90% of total energy used. These fossil fuels produce harmful gaseous emissions such as nitrogen oxides (NO_x), sulphur dioxide (SO_2) and carbon dioxide (CO_2) which cause hazardous effects in the atmosphere. Therefore an EED problem which is a non linear bi-objective optimization problem is designed to deal with minimization of two conflicting objectives namely fuel cost and emission from thermal power plants. It is solved with Lagrangian Relaxation (LR) method that effectively handles coupled structures. To validate the effectiveness of proposed method it is tested upon six unit test system.

Index terms: Economic dispatch, Emission control, Lagrangian Relaxation

I. INTRODUCTION

The Economic Dispatch problem seeks the best generation schedule for the generating plants to supply the required demand plus transmission losses with the minimum production cost. Conventionally, the emphasis on performance optimization of fossil–fuel power systems was on economic operation only, using the ED approach, as better solutions would result in significant economical benefits [1, 2]. However, due to the pressing public demand for clean air as well as due to the ‘‘global warming’’ concept, new clean air policies and regulations have been forced on the industries, as environmental effect is a direct consequence of industrial advancement. Thermal power units are responsible in a major way for creating major atmospheric pollution because of high concentrations of pollutants, such as NO_x, SO_2, and CO_2, contained in their emissions.

Therefore, power sectors have realized the importance of maintaining a cleaner environment. Due to this constraint, generation allocation is not only governed by production costs, but also by the maximum allowable emissions level. As a result, a new approach has come up, known as the EED method. EED is an optimization problem that pursues the least emission level of operation of a power system [3]. But operating either at the absolute minimum cost of generation or at the absolute minimum emission level may no longer be a desirable criterion. The obvious approach is to figure out the optimal amounts of the generated powers for the thermal units in the system by minimizing the emission level and cost of generation simultaneously, which is known as EED. Unlike Economic Load Dispatch Problem (ELD), emission in EED is often treated as an additional constraint or included into the optimization function by assigning different weighting factors to emission and cost [4]. To reduce chemical emission, the high-priority tasks of modern power generation systems have found effective ways and means such as to employ improved types of fuels with low emission potential, to enhance the efficiency of system by retrofitting new technologies to upgrade the existing power plant equipment to reduce emission, to integrate of wind/solar systems or to modify the existing power dispatch strategies [5, 6].

In this paper the solution procedure for EED problem with the LR method is provided. The premise of this method is based on duality theory. It has the ability to handle “local” and “coupling” constraints and to decompose larger mathematical pronblems into smaller, easy to solve subproblems. Because of this nature, the EED problem lends itself to the application of LR method. This is done by forming the Lagrangian function and solving “dual” and “relaxed” problems. The convergence of the dual optimization method can be measured by the relative size of the duality gap between the primal and dual solutions. To validate the effectiveness of proposed method it is tested upon six unit test system.

II. MATHEMATICAL MODELLING

The objective of the ED problem is to calculate, for a single period of time, the output power of every generating unit so that all demands are satisfied at minimum cost, while satisfying different technical constraints of the network and the generators. This section deals with mathematical formulation of EED problem considering the power balance contraints, generation limit constraints and the transmission loss constraints [7]. The model aims at minimizing emission and overall cost of operation which includes fuel cost and the emission costs of thermal units.

1. Objective function

The total cost of the system is the sum of the costs of each of the individual units. The objective function comprises of two terms such as fuel cost equation and emission cost equation.

$$\text{Minimize } \sum_{i=0}^{n} F_i(P_i) + \sum_{i}^{k} E_i(P_i) \tag{1}$$

$F_i(P_i)$ is the fuel cost equation of the ‘i’th plant. It is the variation of fuel cost with generated power (MW) [8]. The input to each unit F_i, represents the generation cost of the unit. The output of each unit P_i is the electrical power generated by that particular unit. $E_i(P_i)$ is the emission cost equation. The fuel cost function of a generator that usually used in power system operation and control problem is represented with a second-order polynomial. Normally fuel cost is expressed as continuous quadratic equation as follows.

$$F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \qquad (2)$$

Where a_i, b_i, c_i are the fuel cost coefficients. The quadratic equation for emission function is given as follows.

$$E_i(P_i) = \alpha_i P_i^2 + \beta_i P_i + \gamma_i \qquad (3)$$

Where α_i, β_i, γ_i are the emission cost coefficients.

Constraints

The above said objective function is subjected to following system constraints.

Power Balance Constraints

The power balance equations constraint suggests that the sum of output powers from all generating units must be equal to the total load demand plus the power losses in the system at each time interval. The equality constraint in (5) gives the power balance between generation and load, P_D including the transmission losses (P_L).

$$\sum_{i=0}^{n} P_i = P_D + P_L \qquad (4)$$

The transmission loss can be determined form either B_{mn} coefficients or power flow.

Generation Limit Constraints

For normal system operation real power output of generators are limited to lower and upper limits. The following constraints limit the power output of thermal units within their lower and upper limits.

$$P_i^{min} \le P_i \le P_i^{max} \qquad (5)$$

Where P_i^{min} and P_i^{max} are the minimum power output and the maximum power outputs of thermal units respectively [9].

Transmission Loss Constraints

The total generation should meet the total demand and transmission loss, where transmission loss is approximated in terms of B_{mn} coefficients which represents the loss coefficients.

$$P_L = 1 - \sum_{i=0}^{n} B_{ij} P_i \qquad (6)$$

Once the B_{mn} coefficients are generated, the system losses become a function of the generator outputs only [10].

III. SOLUTION PROCEDURE

A. Lagrangian Relaxation Method

Lagrangian Relaxation is one of the most effective techniques for handling optimization problems with coupled structures. By introducing Lagrangian multipliers to relax global constraints, a large-scale coupled problem could be transformed into a two-stage dual problem, consisting of a master problem and subproblems.. Based on the sharp bound provided by the Lagrangian dual optimum, it is expected that a sub optimal feasible solution near the dual optimal point can be accepted as a proper solution for primal problem. The subproblems are much smaller than the original problem and can be solved more easily and efficiently in a parallel framework. In the master problem stage, the Lagrangian multipliers are updated to optimize the dual objective function. The initialization and updating strategies for the Lagrangian multipliers have a significant influence on the performance of LR procedures. LR method is used to find the optimal value. It can eliminate the dimentionality problem encountered in the dynamic programming by temporarily relaxing coupling constraints and separately considering each unit [11]. Hence the dual problem of fuel cost and emission minimization can be solved using LR method. The solution procedure using LR method is as follows.

B. LR Method to Solve EED Problem

The demand and the emission constraints are adjoined on to the original cost function using Lagrangian multipliers $\lambda, \mu_1, \ldots\ldots \mu_n$ respectively to yield

$$\mathcal{L}(P,\lambda,\mu) = \sum_{ie\Phi} PROD + \lambda\Psi + \sum_{j=1}^{N} \mu_j \Delta E_j \qquad (7)$$

With $\Psi = P_D + P_L - \sum_{Ie\varphi} P_i$

$and\ \Delta E_j(P) = E_j(P) - E_j^{max}$

The Lagrangian function can be rewritten as follows:

$$\mathcal{L} = \sum_{ie\Phi} Q_i(P_i) + \lambda\Psi - \sum_{j=1}^{N} \mu_j E_j^{max} \qquad (8)$$

Where $Q_i(P_i) = Q\alpha_i P_i^2 + Q\beta_i P_i + Q\gamma_i$

The Relaxed Problem

This problem is stated as follows

$$Minimize \mathcal{L}(P,\lambda,\mu_1,\ldots\ldots\mu_n) \qquad (9)$$

$for\ given\ \mu_j \geq 0, j = 1, \ldots, N$

$Subject\ to\ P_i^{min} \leq P_i \leq P_i^{max}$

The optimality conditions can be expressed as follows:

$$\frac{\partial Q(P_i)}{\partial P_i} = 2Q\alpha_i P_i \ + Q\beta_i = \lambda\left(1 - \frac{\partial P_L}{\partial P_i}\right)$$

The equation (9) can be solved for $P^*(\mu)\ and\ \lambda^*(\mu)$ using conventional ED program modified to account for the coefficients in the modified objective function.

The Dual Problem

The dual problem of the original EED problem is expressed as follows:

$$Maximize\ m(\mu) \triangleq \mathcal{L}(P^*(\mu), \lambda^*(\mu)\mu) \qquad (10)$$

Where

$m(\mu) = PROD(P^*) + \lambda^*\Psi(P^*) + \sum_{J=1}^{N} \mu_j \Delta E_j(P^*)$

$$with\ PROD(P^*) = \sum_{ie\Phi} \mathrm{F}_i(P_i^*) \qquad (11)$$

$$\Delta E_j(P^*) = E_j(P^*) - E_j^{max} \qquad (12)$$

The condition for optimality is

$$\frac{\partial m(\mu)}{\partial \mu_j} = 0 \; if \; \mu_j > 0 \tag{13}$$

$$\frac{\partial m(\mu)}{\partial \mu_j} < 0 \; if \; \mu_j = 0 \tag{14}$$

Where equations (13) and (14) correspond to active and inactive constraints respectively. Since $\{P^*, \lambda^*\}$ is the solution of the relaxed problem, optimality condition yields

$$\frac{\partial Q(P_i)}{\partial P_i} = \lambda^* \left(1 - \frac{\partial P_L}{\partial P_i}\right) \; for \; P_i^* = P_i^{min} \; or P_i^{max} \tag{15}$$

Where P_i^* is given as

$$\sum_{ie\Phi} P_i^* = P_D + P_L$$

The optimality condition in (13) and (14) can be restated as

$$E_j(P^*) = E_j^{max} \; if \; \mu_j > 0$$

$$E_j(P^*) < E_j^{max} \; if \; \mu_j = 0 \tag{16}$$

The solution of these equations corresponding to the elements of the set of active constraints reduces to updating μ_j's by solving the following system of linear equations:

$$\begin{pmatrix} \Delta E_{s1} \\ \Delta E_{s2} \\ \vdots \\ \Delta E_{sr} \end{pmatrix} = \begin{pmatrix} \frac{\partial E_{s1}}{\partial \mu_{s1}} & \frac{\partial E_{s1}}{\partial \mu_{s2}} & \dots\dots & \dots & \frac{\partial E_{s1}}{\partial \mu_{sr}} \\ \frac{\partial E_{s2}}{\partial \mu_{s1}} & \frac{\partial E_{s2}}{\partial \mu_{s2}} & \dots\dots & \dots & \frac{\partial E_{s2}}{\partial \mu_{sr}} \\ \frac{\partial E_{sr}}{\partial \mu_{s1}} & \frac{\partial E_{sr}}{\partial \mu_{s2}} & \dots\dots & \dots & \frac{\partial E_{sr}}{\partial \mu_{sr}} \end{pmatrix} \begin{pmatrix} \Delta \mu_{s1} \\ \Delta \mu_{s2} \\ \vdots \\ \Delta \mu_{sr} \end{pmatrix} \tag{17}$$

And the vector of Lagrangian multipliers corresponding to the active constraints in the dual problem is updated as follows:

$$\mu_{s_j}^{(new)} = \mu_{s_j}^{(old)} + \Delta \mu_{s_j} \; j = 1, 2, \dots . T \tag{18}$$

$$\frac{\partial (P_i)}{\partial \mu_{sk}} = \frac{1}{2Q\alpha_i} \left[\frac{\partial (\lambda)}{\partial \mu_{sk}} - (2\alpha_i P_i + \beta_i)\right] \tag{19}$$

$$\frac{\partial (\lambda)}{\partial \mu_{sk}} = \frac{\sum_{i \in \Phi_d} \frac{2\alpha_i P_i + \beta_i}{2Q\alpha_i}}{\sum_{i \in \Phi_d} \frac{1}{2Q\alpha_i}} \tag{20}$$

In the procedural steps which follow, active and inactive constraints are identified using the variable Z_j such that

$$Z_j = \begin{Bmatrix} 1 \; if \; the \; constraint \; is \; included \; in \; active \; set \\ 0 \; otherwise \end{Bmatrix}$$

C. Algorithmic Steps

Step 1. Set v = 0.

Step2. Set all $Z_j = 0$ for j = 1……, N. All environmental constraints are assumed to be initially inactive.

Step 3. Set $\mu_j = 0$ for all j=1, 2, …., N, and solve the relaxed problem for $P^{(0)}$ and $\lambda^{(0)}$ and calculate the corresponding $E_j^{(0)}$, for j = 1, …., N.

Outer Loop

Step 4. If $\mu_j > 0$, $Z_j = 1$ continue,
Else for $\mu_k < 0$ set $\mu_k = 0$ and $Z_k = 0$(switch constraint types). Go to step 6.
Step 5. If $E_j < E_j^{max}$ for all j's with x, then stop. Solution is obtained. Else identify the index, m, of the constraint which has the maximum relative violation and set $Z_m = 1$. Go to step 8.
Step 6. If $Z_j = 0$ for j=1, ….N, then stop. No solution exists.
Step7. Solve equation (16) which corresponds to binding constraints.

Inner Loop

For n = 1, …..n^{max}, Do

i. If n = 1, set $P = P^{(0)}$, and $\lambda = \lambda^{(0)}$, and $E_j = E_j^{(0)}$. Go to iv.
ii. Solve the relaxed problem to find P and λ.
iii. Calculate E_j for $Z_j = 1$
iv. If for all binding constraints ($Z_j = 1$) the relative error $\leq \delta$, then go to step 8.
v. Update all μ_j corresponding to $Z_j = 1$.

End
Step 8. v = v + 1. Go to step 4.
Followed by the algorithm the process flow is represented by a flowchart in Fig. 1.

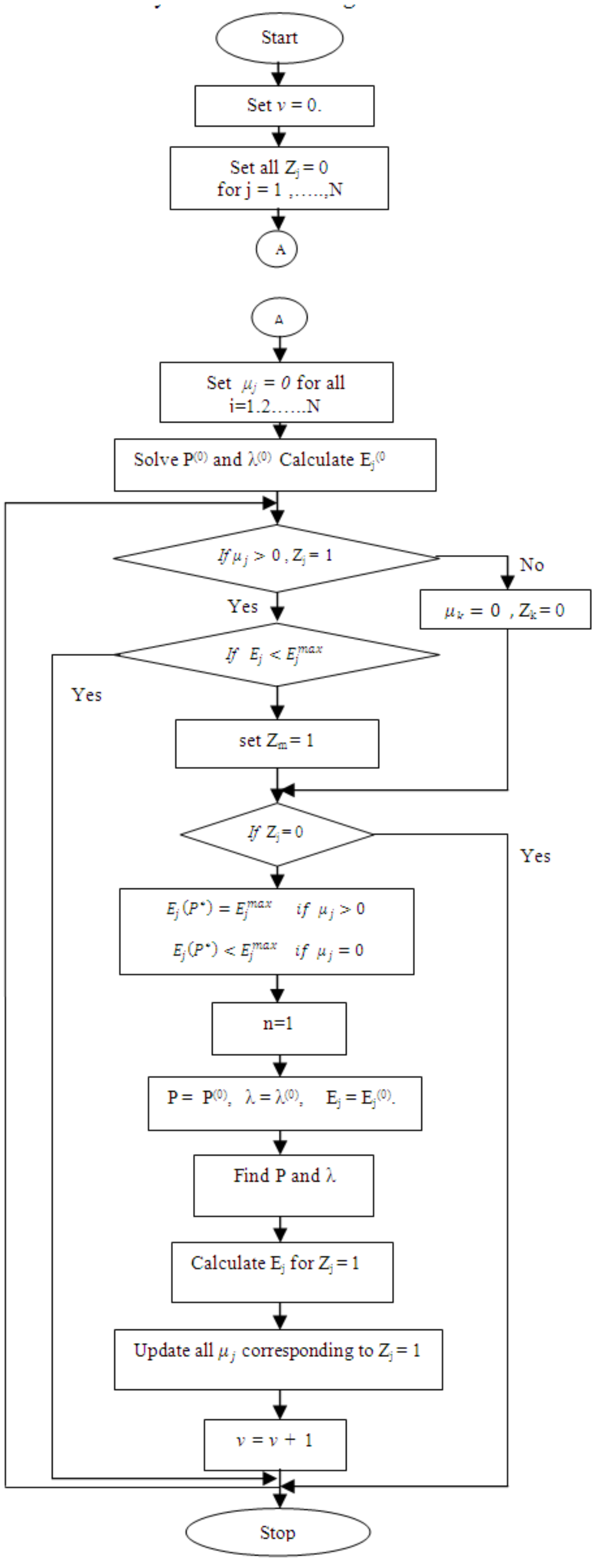

Fig. 1 Flowchart of Lagrangian Relaxation Method for Environmental Economic Dispatch Problem

IV SIMULATION RESULTS

The input data of economic load dispatch and emission dispatch of a 6 unit thermal test system, hourly power demand data, the weighting factors for economy and emission and the B_{mn} loss coefficients data are given. The thermal unit should be committed to minimum demand in hourly load demand and 12 hours committed to generation in thermal plant. The economic load dispatch data for a 6 unit test system are given in Table I.

TABLE I GENERATOR OPERATING LIMITS AND QUADRATIC COST FUNCTION COEFFICIENTS

	Parameter				
Unit i	a_i Rs/√MW	b_i Rs/MW	c_i Rs	P_i^{min} MW	P_i^{max} MW
1	0.15247	38.53973	756.798	10	125
2	0.10587	46.15916	451.325	10	150
3	0.02803	40.3965	1049.997	35	225
4	0.03546	38.30553	1243.531	35	210
5	0.02111	36.32782	1658.559	130	325
6	0.01799	38.27041	1356.659	125	315

The emission data for a 6 unit test system are provided in Table II.

TABLE II POLLUTANT EMISSION COEFFICIENTS

Unit i	α_i Kg/√MW	β_i Kg/MW	γ_i Kg
1	0.00419	0.32767	13.85932
2	0.00419	0.32767	13.85932
3	0.00683	-0.54551	40.2669
4	0.00683	-0.54551	40.2669
5	0.00461	-0.51116	42.89553
6	0.00461	-0.51116	42.89553

The B_{mn} coefficient matrix is the following

$$B = 10^{-4} \times \begin{bmatrix} 1.4 & 0.17 & 0.15 & 0.19 & 0.26 & 0.22 \\ 0.17 & 0.6 & 0.13 & 0.16 & 0.15 & 0.2 \\ 0.15 & 0.13 & 0.65 & 0.17 & 0.24 & 0.17 \\ 0.19 & 0.16 & 0.17 & 0.71 & 0.3 & 0.25 \\ 0.26 & 0.15 & 0.24 & 0.3 & 0.69 & 0.32 \\ 0.22 & 0.2 & 0.19 & 0.25 & 0.32 & 0.85 \end{bmatrix}$$

Since emission is considered to be an additional constraint in EED problem, they are included into optimization function by assigning different weighting factors for fuel cost emission.

The weighting factor for economy H1= 1

The weighting factor for emission H2 = 44.788

The fuel cost, emission and power loss for hourly demand is obtained in Table III. It is seen that the fuel cost, emission cost and power loss is increased as the hourly demand increases.

TABLE III FUEL COST, EMISSION AND POWER LOSS BY LR METHOD

Hour	Load Demand (MW)	Fuel Cost (Rs/MW)	Emission (Rs/kg)	Power Loss (MW)
1	410	23418	212.244	6.409
2	435	24546	223.3	7.08
3	475	26442	246.25	8.10
4	530	29041	284.577	9.90
5	558	30390	306.287	10.99
6	580	31462	324.45	11.85
7	608	32842	348.98	12.96
8	626	33739	365.25	13.72
9	654	35148	392.13	14.93
10	680	36472	418.45	16.22
11	690	36985	428.9343	16.58
12	700	37501	439.60	17.05

The variation in weighting factors of economy and emission influences the fuel cost, emission and transmission loss and thus provides the best obtained result for the respective weighting factors for economy and emission. As the weighting factor for emission constantly increases, the emission cost reduces and the fuel cost is increases. Thus this satisfies the objective of minimizing emission. But the objective of satisfying the fuel cost minimization is not achieved. Hence the optimum value of fuel cost and emission has to be chosen. This can be illustrated in the following graphical representations as in Fig.2 and Fig.3.

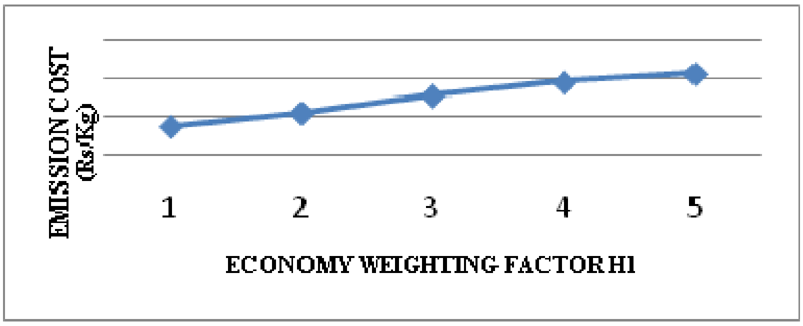

Fig. 2 Change in Economy Weighting Factor

Fig. 2 illustrates that the emission cost increases as the weighting factor for economy is increased. Thus the objective of minimizing the emission is not satisfied.

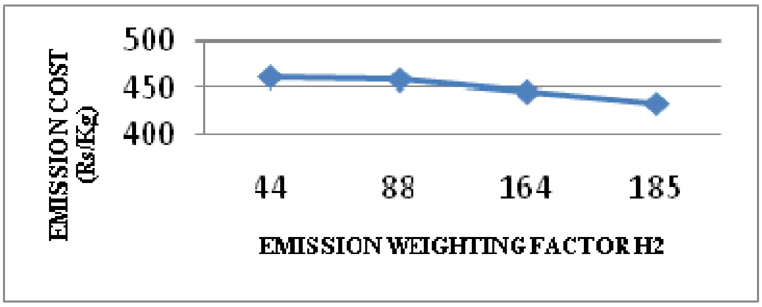

Fig. 3 Change in Emission Weighting Factor

Fig. 3 illustrates that the emission cost is constantly reducing as the emission weighting factor is increased. Thus the objective of minimizing emission cost is achieved. With the help of change in weighting factors for emission and economy the minimization of fuel cost and emission cost have been achieved. Thus the emission level of NO_2 is reduced.

V CONCLUSION

This paper presents the Environmental Economic Dispatch problem with the main objective of minimizing the operating fuel cost and emission in thermal power plants. The Lagrangian Relaxation method was used to solve the EED problem of fuel cost and emission minimization. The proposed method had been tested upon a six unit test system. The fuel cost and emission were minimized in accordance to the variations in their weighting factors. The simulation results have shown that the LR method provided better results than the other existing methods.

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