International Journal of Electronic and Electrical Engineering.
ISSN 0974-2174 Volume 8, Number 1 (2015), pp. 57-65

http://www.irphouse.com

Designing Passive Filter Using Butterworth Filter Technique

Zubair S. Nadaph* and Vijay Mohale#

**UG Student, Electrical Engineering, Walchand College of Engineering, Vishrambag, Sangli, Maharashtra- 416415 , India)*
E-mail: zubairsnadaph@hotmail.com
 # Assitant Professor, Electrical Engineering, Walchand College of Engineering, Vishrambag, Sangli, Maharashtra-416415, India)
E-mail: vijay.mohale@walchandsangli.ac.in

Abstract—

Harmonics and its mitigation technique are headache for most of the power engineers for the obvious reasons; its adverse effects on power system. In power system, various techniques exist for mitigation, but most of them are based on the study of harmonic pattern at the site. Beside this, there is no passive filter design for ASDs (Automatic Speed Drives). This paper focuses on filter design technique based on Butterworth filter. It does not need any prior study of harmonics at the site. It is applicable for both VFDs (Variable Frequency Drives) and PCCs (Point of Common Coupling). Butterworth filter provides perfect mitigation technique bringing THD (Total Harmonic Distortion) well fewer than 5%. This paper mostly focuses on mathematical tools of Butterworth filter. But it also provides software analysis of design.

Keywords—Passive Filters, Power System Harmonics, Harmonic Distortion, Butterworth Filter

I. INTRODUCTION

During the last thirty years, much attention has been focused on power system harmonics. This is one of the most severe issue affecting power quality, because it affects both the utility company and consumers. Power system harmonics are considered to be one of the major concern with immediate effects, such as reduction of life of the electrical equipment and early aging of insulators [1]. Various ideas and techniques have been formulated to reduce harmonics and its effects.

A simple mitigation action such as adding, resizing, or relocating can effectively modify an unfavourable system frequency response, and thus drop harmonic distortion by some level [3]. But for healthy and safe system distortions should be within the standards mentioned in IEEE-519. Much ignorance still surrounds the subject of the application of harmonic filters to correct the degradation of power quality produced by various power electronic devices [2]. Harmonic filters are classified into active and passive filter depending on the use of supply for filtration technique.

Active filter: this concept comprises of power electronic devices that generate harmonics with phase shift of 180^0. These filters are relatively new and prove costly as compared to passive. But they have distinct advantages over passive; firstly they do not have any resonating component and also the accuracy of active is more compared to passive. Use of switching devices and other electronic devices restrict the use of active filter to ASDs and PCCs where harmonic distortions are huge [4].

Passive filter: this area has been trending for last two decades or so, for the obvious reason cost. Passive filters are inductance, capacitance, and resistance elements configured and tuned to control harmonics. They are used either to shunt the harmonic currents off the line or to block their flow in the system by tuning the components to a fixed resonating frequency. As they do not have any electronic components, passive filters are relative cheaper compared to active. Frequently used passive filters are

- Single-tuned “notch” filter
- Series passive filters
- Low pass broadband filter
- C filters

All these concepts are frequently and traditionally used at the utility system. Along with the merits passive filters have few demerits as well. One of the unavoidable deficit is the aging of the components, this not only deteriorates the performance but also off tunes the passive design. This aging of components is seen after a long period of time; say after a period of 20-30 years. Hence for this period passive prove to be reasonable as compared to active.

The only area where the active is more reliable than passive is the type of application. Passive filters are usually employed at the PCC of industry. It prevents the harmonic components from entering utility. The passive filters mentioned above cannot be used directly to filter harmonics form VFD. Even if used, the design becomes bulky; also the cost of installation becomes costly as compared to active filter.

This paper encompasses the designing and implementation of “Butterworth filter” technique. Even though this idea is conventionally used in low voltage electronic circuits, it can be equivalently implemented in medium and high voltage circuits using the basic concept of Butterworth filter.

The objectives of this paper are 1) to provide the concept of Butterworth filter 2) mathematical tools for designing of Butterworth filter and 3) to study practical consideration of each order of the Butterworth filter. The verification of the orders of the Butterworth filter has been done under the guidelines of IEEE-519. Keeping the practical consideration of the order of harmonics, for the study of mitigation technique real time simulation has been carried out.

II. BUTTERWORTH FILTER

A. Concept

Basically it is a signal processing filter designed to have as flat a frequency response as possible in pass band. It is a type of low pass filter which passes all the frequency components which are less than cutoff frequency with minimum attenuation. It is also known as maximally flat magnitude filter. The frequency response (gain) of a general n^{th} order Butterworth filter with a cutoff frequency of 1 radian per second is given by

$$G(\omega) = \sqrt{\frac{1}{1+\omega^{2n}}} \qquad (1)$$

Here ω is the angular frequency in radians per second n is the order of the filter

Gain plot of the Butterworth filter with the frequency response represented earlier is shown in fig. 1

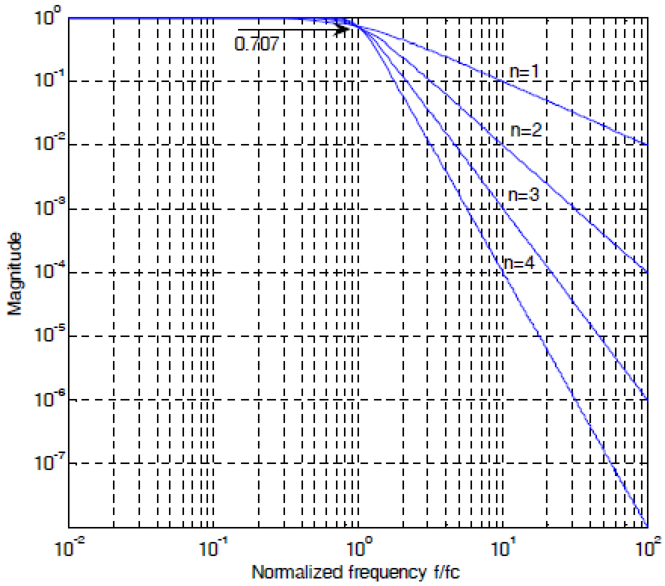


Fig. 1. Gain curve of n^{th} order Butterworth filter

Conceptually a low pass filter allows all signals to pass which are less than cutoff frequency, and those which are higher than cutoff are completely attenuated. But in real world this does not happen, frequency components beyond the cutoff do not get completely attenuated and are rather seen at the input with some magnitude. This can be proved from fig. 1, for a first order system, the curve drops down at a constant slope of -20db/decade. Hence for a harmonic order 5 20% of the magnitude is still seen at the input. The does not bring the THD under limit.

The remedy for this is to increase the order of the system. On increasing the order the slope of the curve after cutoff frequency increases by -20db/decade for every increase in order of the system. For higher orders the curve tends to match the ideal concept. The order "n" stated above is the number of reactive elements in the filter.

B. ***Mathematical Analysis of Butterworth Filter***

The gain G(ω) or the frequency response of an n-order Butterworth filter is given in s domain in terms of the transfer function H(s) using equation (2), which is given as

$$G^2(\omega) = |H(j\omega)|^2 = \frac{G_0^2}{1+\left(\frac{\omega}{\omega_c}\right)^{2n}} \quad (2)$$

Where

n = order of the filter

ω_c = cutoff frequency

G_0 is the gain when frequency tends to zero

Here all signals or rather all the frequency components below cutoff frequency will be seen with magnitude G_0, while frequencies above it will be suppressed. The extent of suppression depends on value of n. The curve of the above relation is a circle with radius ω_c, and the n poles of the above relation lie on that circle.

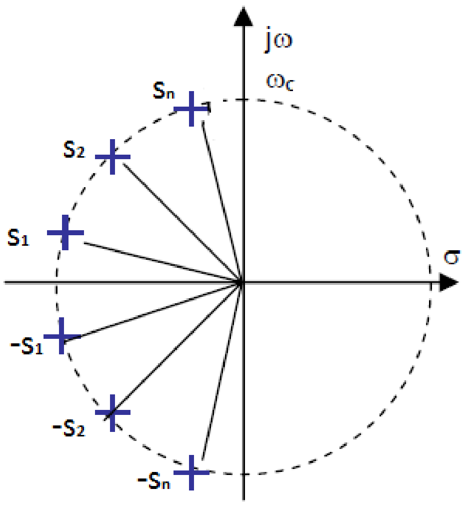


Fig. 2. Poles of Butterworth polynomial

From the above relation on taking s = *j* ω frequency response in s domain can be computed [5]. The transfer function H(s) is given as

$$H(s) = \frac{G_0}{\prod_{k=1}^{n}(s-s_k)/\omega_c} \quad (3)$$

The denominator is a Butterworth polynomial in s, where as the term s_k represents the k^{th} pole of the polynomial, and ω_c the cutoff frequency. Here again the poles of the polynomial lie on the circle of radius ω_c, as shown in fig. 2.

III. MATHEMATICAL ANALYSIS OF THIRD ORDER FILTER

A first order Butterworth filter is most simple and cost friendly, as it consist of inductor, capacitor and resistor with inductor in series and both capacitor and resistor in shunt. But as studied earlier for proper filtration technique there is need to increase the order of system. Depending on the type of application and budget, the order of the filter can be chosen. Order chosen in this paper is third as an intermediate choice between cost and design effectiveness. The circuit of the third order Butterworth filter is shown in fig. 3.

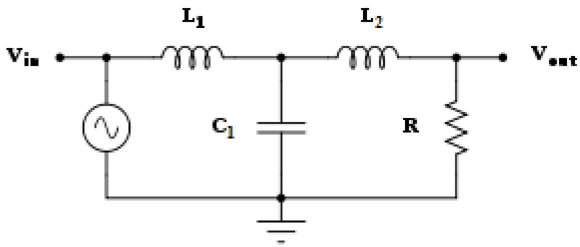


Fig. 3. Circuit of third order Butterworth filter

Here Vin is input sinusoidal voltage where as Vout is the output or the load side non-linear voltage, and the other terms in the fig. have their literal meanings. The transfer function of the circuit is

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} \tag{4}$$

$$H(s) = \frac{R}{L_1L_2C_1s^3 + L_1C_1R \times s^2 + (L_1+L_2) \times s + R} \tag{5}$$

Here the transfer function is in s domain; similarly the general Butterworth filter of third order is given as

$$H(s) = \frac{G_0 \times \omega_C^3}{s^3 - (s_1+s_2+s_3)s^2 + (s_1s_2+s_2s_3+s_3s_1)s - s_1s_2s_3} \tag{6}$$

On equating equations (5) and (6) the values of L_1, L_2, C_1 and R in terms of s_1, s_2 and s_3 can be obtained. Here the values of s_1, s_2 and s_3 can be taken from circular locus of its polynomial, where its radius equals to cutoff frequency in radians per second. The equations of L_1, L_2, C_1 and R are

$$\frac{R}{L_2} = -(s_1 + s_2 + s_3) \tag{7}$$

$$L_1C_1 = \frac{-(s_1+s_2+s_3)}{G_0 \times \omega_c^3} \tag{8}$$

$$\frac{L_1}{L_2} = -1 + \frac{(s_1s_2+s_2s_3+s_1s_3)(s_1+s_2+s_3)}{G_0 \times \omega_c^3} \tag{9}$$

Here for stability reasons as stated above the values of s_1, s_2 and s_3 lie on left half of s-plane. The other important term in all of the above equations is the value of G_0; it is log of ratio of Vout to Vin at zero frequency or before cutoff frequency, as it is maximally flat response system. Slight variations in the values of filtering elements can be altered by varying the values of G_0. This might help in bringing the values of active and reactive elements within practical limits, but the variation in G_0 should be within prescribed constraint as it affects the voltage variations in the system. Normally the assumed value of G_0 is 1.

IV. DESIGN ANALYSIS OF THIRD ORDER BUTTERWORTH FILTER

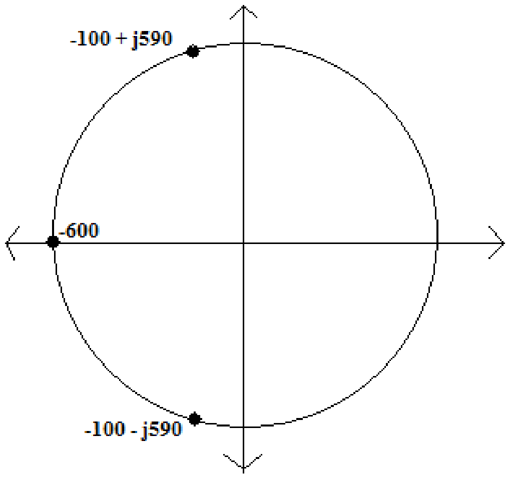
Using the design specifications, practical values of have been computed and tested.

The testing has been done using p-spice.

A. *Selection of Cutoff Frequency*

In the system for the power frequency, the most common harmonics are of order five. Hence selection of cutoff frequency is a tough job. Selection of cutoff frequency very near to system frequency increases the values of reactive elements and makes the design bulky, whereas on selecting value very far from power frequency say for near about 150 radians per second or more, the mitigation is not so as desired. The cutoff frequency so taken here is midway between the two constraints. The cutoff frequency in this paper is 95.5 Hz or 600radians per second.

Using this frequency, the poles of Butterworth polynomial can be plotted from the circle of radius equivalent to cutoff frequency. The poles so chosen in this paper are shown in fig. 4.



$s_1 = -600$
$s_2 = -100 + j590$
$s_3 = -100 - j590$

Fig. 4. Plotting of roots of the Butterworth polynomial

B. *Computation of Values with the Effect of G_0*

From fig. 4, using the values of s_1, s_2 and s_3 the values of the reactive elements are

For $G_0 = 1$

$$\frac{L_1}{L_2} = 0.77$$
$$L_1 C_1 = 3.7 \times 10^{-6}$$
$$\frac{R}{L_2} = 800$$

Taking L_2=117mH, the values of other components are L_1=90mH, C_1=41uF and R=94Ω.

For $G_0 = 1.06$

$$\frac{L_1}{L_2} = \frac{2}{3}$$
$$L_1C_1 = 3.49 \times 10^{-6}$$
$$\frac{R}{L_2} = 800$$

Taking L_2=117mH, the values of other components are L_1=78mH, C_1=45uF and R=94Ω.

Here it can be observed that slight variation in value of G_0 affects the values of the components considerably; on increasing it there is decrease in the value of inductor but increase in value of condenser. So there is need to choose appropriate value of G_0. Also G_0 is the ratio of voltages which should approximately be equal; hence this factor should also be considered while designing. For rest of the paper all analysis has been done for G_0=1.06.

C. Circuit Analysis

P-spice simulation of the above design has been shown in fig. 5. Here the fundamental frequency taken is 60Hz.

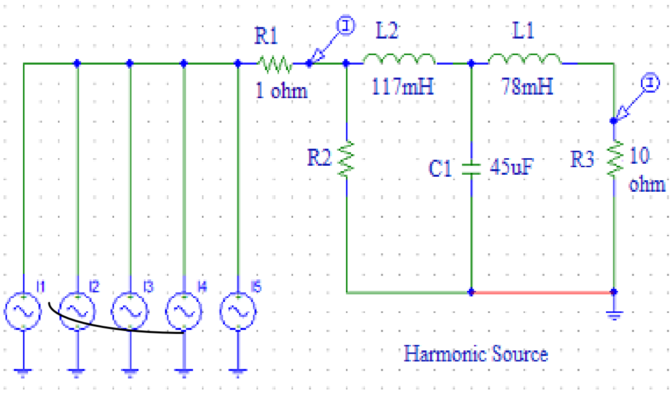


Fig. 5. Circuit analysis of third order Butterworth filter

In fig. 5 R1 is the load side resistance, where as R3 is source resistance; rest are the filter components stated earlier. Here for the testing of filter four orders have been taken; since they are the major ones, hence they are enough to verify the working of the system. The orders taken are namely 5^{th}, 7^{th}, 11^{th} and 13^{th}. Filtered waveform of the circuit is shown in fig. 6.

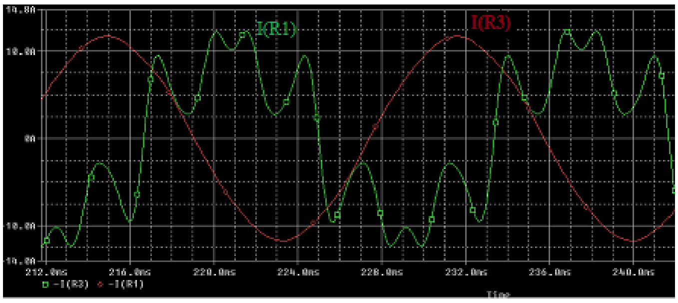


Fig. 6. P-spice waveform results.

The wave form in green is the harmonic current waveform with I_{max} equal to 11.225A. It is non-sinusoidal and the waveform in red is the filtered waveform with peak current equal to fundamental component (10A). The Fast Fourier Analysis of the system is shown in fig. 7.

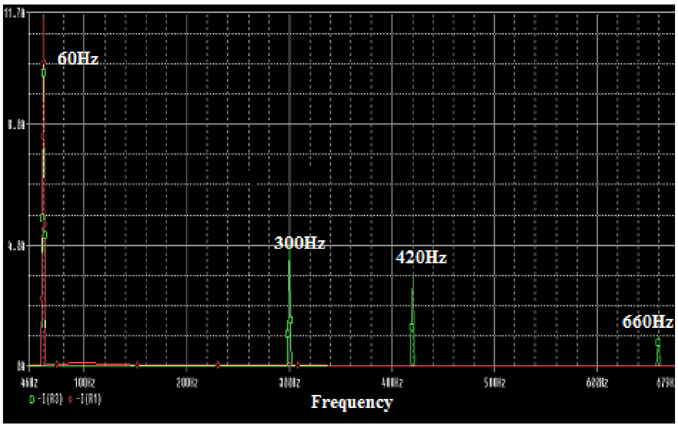


Fig. 7. FFT analysis of the waveforms

From fig. 7 it can be observed that the green and red components match only for frequency 60Hz, for rest of the frequencies only green component exist and red are very near to zero. This proves that the designed filter blocks higher order frequencies without affecting the fundamental component. From above fig. the THD for green is 112.25% where as the THD for red is 0.1%. This also verifies that this design brings THD of current lower than 5%.

V. CONCLUSION

Butterworth filter design has been included in this paper for the introduction of an analogous concept in the field of harmonic mitigation techniques. This paper allows implementation of concept which is either ment for digital circuits or for analog system with higher frequency. This paper focuses on improving the field of harmonic filters in power system. It just does not provide mathematical tool for design but also shows testing and verification of prepared design.

REFERENCES

[1] Nassif, A.B.; Wilsun Xu, "Passive Harmonic Filters for Medium-Voltage Industrial Systems: Practical Considerations and Topology Analysis," *Power Symposium, 2007. NAPS '07. 39th North American* , vol., no., pp.301,307, Sept. 30 2007-Oct. 2 2007.

[2] Aleem, S.H.E.A.; Zobaa, A.F.; Sung, A.C.M., "On the economical design of multiple-arm passive harmonic filters," *Universities Power Engineering Conference (UPEC), 2012 47th International* , vol., no., pp.1,6, 4-7 Sept. 2012.

[3] Roger C. Dugan, Mark F. McGranaghan, H. Wayne Beaty, “Electrical Power Systems Quality”,

[4] Fujita, H.; Yamasaki, T.; Akagi, H., "A hybrid active filter for damping of harmonic resonance in industrial power systems," *Power Electronics Specialists Conference, 1998. PESC 98 Record. 29th Annual IEEE* , vol.1, no., pp.209,216 vol.1, 17-22 May 1998.

Zubair S. Nadaph is currently pursuing his B. Tech. in electrical engineering in WCE, Sangli. His areas of interest are power electronics and drives, power system, mechatronics.

V.P.Mohale received the B.E. in electrical engineering and the M.E. in electrical power systems from Shivaji University, Kolhapur, India,in 2011 and 2013 respectively. Currently, he is Assistant Professor in the Department of Electrical Engineering, Walchand College of Engineering, Sangli. His research interests include power system protection, Power Quality.