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Performance of Error Normalized Step Size LMS and NLMS Algorithms: A Comparative Study

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Abstract

This paper presents a Comparative Study of NLMS (Normalized Least Mean Square) and ENSS (Error Normalized Step Size) LMS (Least Mean Square) algorithms. For this System Identification (An Adaptive Filter Application) is considered. Three performances Criterion are utilized in this study: Minimum Mean Square error (MSE), Convergence Speed, the Algorithm Execution Time. The Step Size Parameter (μ) in both algorithms is chosen to obtain the same exact value of Misadjustment (M) equal to 2.5%. Simulation Plots are obtained by ensemble averaging of 200 independent simulation runs.

Keywords: MSE, Convergence Speed, Execution Time, System Identification, Misadjustment, Step Size, EMSE.

Introduction

An Adaptive filter is very generally defined as a filter whose characteristics can be modified to achieve some end or objective, and is usually assume to accomplish this modification (or "Adaptation") automatically, without the need for substantial intervention by the user. Adaptive filter algorithms have been very popular since last few decades and still it is very useful in many fields of image, speech and signal processing and communication [1].

The choice of one algorithm over other is determined by one or more factors like Convergence speed, Misadjustment, Robustness and Execution Time [1]. Convergence speed is Number of iterations required in response to stationary inputs, to converge "close enough" to the optimum Wiener solution in the Mean-Square error (Mean Square value of the difference between the desired response and actual output) sense. Misadjustment provides a quantitative measure of the amount by which the final values of mean square error, averaged over an ensemble of adaptive filters, deviates from the minimum mean square error produced by the Wiener filter. For an

Adaptive filter to be robust, small disturbances can only result in small estimation errors. Execution Time is total time required for the execution of the algorithm.

System Identification – The notion of a mathematical model is fundamental to science and engineering. In the class of application dealing with identification, an adaptive filter is used to provide a linear model that represents the best fit (in some sense) to an Unknown System. The Unknown System and adaptive filter are driven by the same input. The plant output supplies the desired response for the adaptive filter [2]. A block of system identification setup is shown in Fig.1. The aim is to estimate the impulse response, h, of the unknown system. The adaptive filter adjusts its weights, w, using one of the LMS-like algorithms, to produce an output y(n) that is as close as possible to the plant output d(n). When MSE is minimized, the adaptive filter coefficients, w, are approximately equal to the unknown system coefficients, h. x(n) is the input signal for both unknown system and adaptive filter. The internal plant noise is represented as a additive noise n(n) [3].

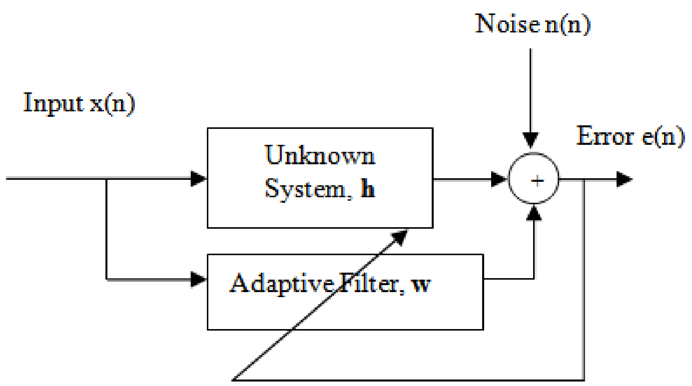

Figure 1: System Identification

The performance of an algorithm for system identification can be measured in the terms of its misadjustment M, which is a normalized mean square error defined as the ratio of the steady state excess mean-square error (EMSE) to the minimum MSE [5].

$$M = \frac{EMSE_{ss}}{MSE_{min}} \tag{1}$$

The MSE at the nth iteration is given by:

$$EMSE(n) = MSE(n) - MSE_{min} , \tag{2}$$

Where

$$MSE(n) = E[\ |e(n)|^2\ ]$$

However, the MSE in (3) is approximately estimated by averaging $|e(n)|^2$ over J independent trials of the experiment. Thus, (3) can be estimated as:

$$MSE(n) = \frac{1}{J} \sum_{n=1}^{J} |e(n)|^2 \qquad (3)$$

From (2), we can write:

$$EMSE_{ss} = MSE_{ss} - MSE_{min} \qquad (4)$$

The value of MSE_{min} obtained when the coefficients of the unknown system and the filter match, is equal to irreducible noise variance σ_n^2 for zero mean noise n.

Adaptive Algorithms

The Adaptive NLMS Algorithm:

The adaptive NLMS algorithm takes the following form:

$$w(n+1)=w(n)+(\mu\, e(n)x(n)/ (\varepsilon + x^T(n)x(n))) \qquad (5)$$

$$y(n)=w^T(n)x(n) \qquad (6)$$

$$e(n)=d(n)-y(n) \qquad (7)$$

where $w(n)=[w_0(n)\ w_1(n)\ldots\ldots\ldots w_N(n)]^T$ (N+1 being the filter length) is the weight vector, μ is the convergence parameter(sometimes referred to as step size),e(n) is the error, d(n) is the desired output, y(n) is the filter output, ε is a constant prevents division by a very small number of data norm, $x(n)=[x(n)\ x(n-1)\ldots\ldots\ldots..x(n-N+1)]^T$ is input vector [6].

The Error Normalized Step Size LMS (ENSS-LMS) Algorithm:

The ENSS-LMS algorithm takes the following form:

$$w(n+1)=w(n)+ \mu x(n)e(n)/(1+\|e_L(n)\|^2) \qquad (8)$$

$$y(n)=w^T(n)x(n) \qquad (9)$$

$$e(n)=d(n)-y(n) \qquad (10)$$

Where $\|e_L(n)\|^2 = \sum_{i=0}^{L-1} |e(n-i)|^2$

The parameters μ and L in the algorithm are appropriately chosen to achieve the best trade off between convergence speed and low final MSE [4].

Simulation Results

In this simulation the input signal x (n) for all algorithms is a zero mean white Gaussian random signal of unit variance. The length of the unknown System impulse response is assumed to be N=4 and the Impulse Response (h) of this Unknown System is assumed to be [1, 0.7, 0.5, -0.2]. The internal unknown system noise n(n) is assumed to be white Gaussian with mean equals zero and variance equals 0.09. The Step Size Parameter (μ) in all Algorithms is chosen to obtain the same exact value of Misadjustment (M) equals 2.5%. The value of M is estimated by averaging excess

MSE over iteration number (n) after the algorithm has reached steady state. Simulation plots are obtained by ensemble averaging of 200 independent simulations runs.

Simulation plots of MSE for different values of step size parameter (μ) for NLMS and ENSS-LMS algorithms are shown in fig. 2 and fig. 3 respectively. The comparison of plots shows that variation of MSE curves with respect to step size parameter (μ) near the steady state region is less in ENSS-LMS algorithm as that of NLMS algorithm.

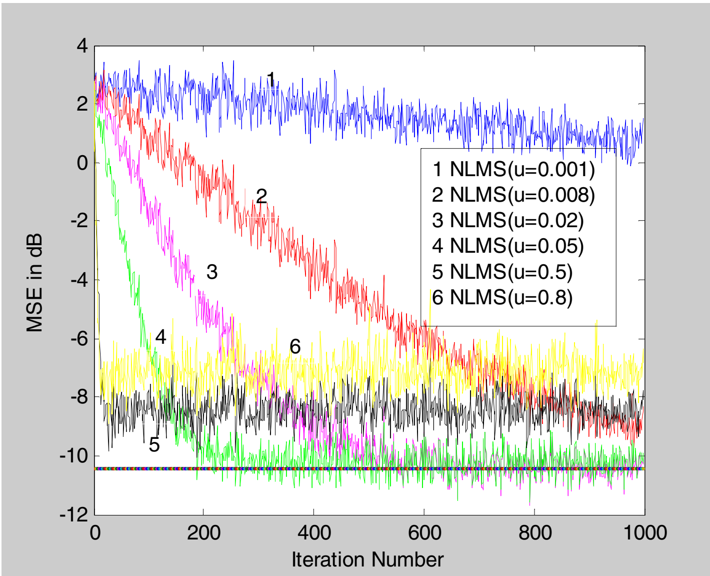

Figure 2: MSE curves for different values of step size parameter for NLMS algorithm

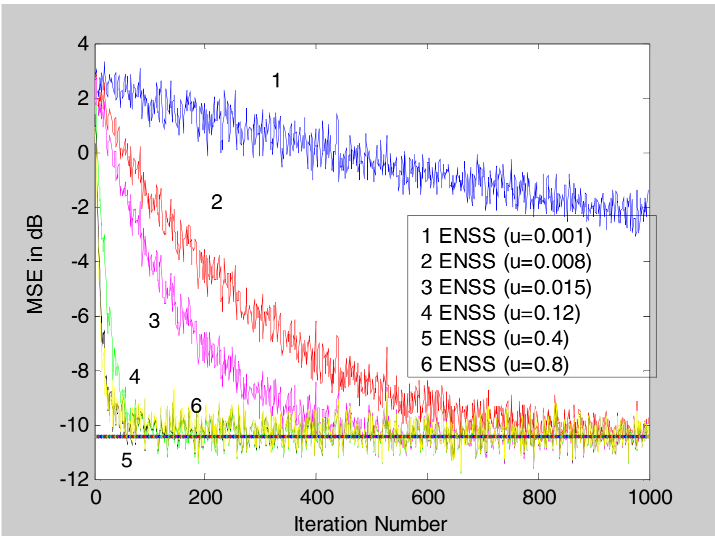

Figure 3: MSE curves for different values of step size parameter for ENSS-LMS algorithm

The simulation plots of Misadjustment for different values of step size parameter (μ) for both algorithms are shown in fig. 4 and fig. 5. These curves are used to find the step size (μ) for which Misadjustment is 2.5%.The value of step size parameter (μ) for NLMS and ENSS-LMS obtained are 0.0201 and 0.8 respectively.

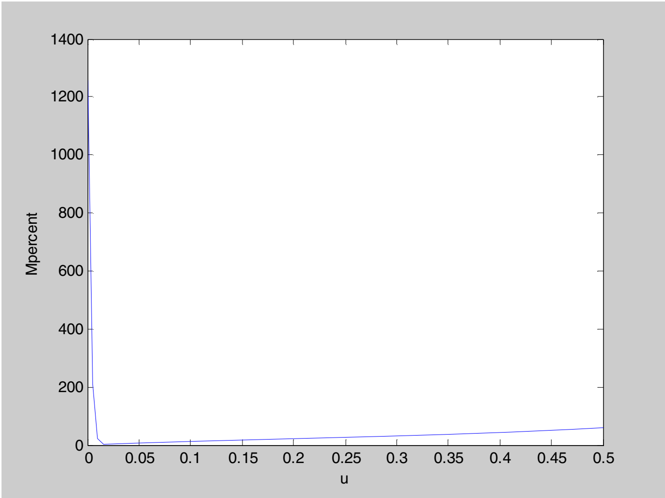

Figure 4: Misadjustment curves for different values of step size parameter for NLMS algorithm

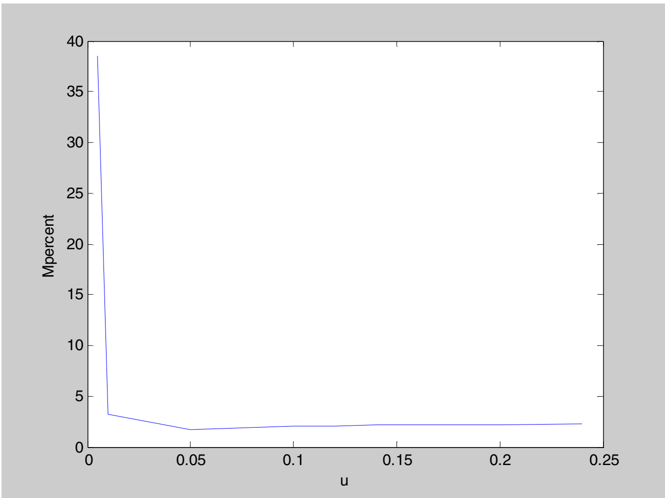

Figure 5: Misadjustment curves for different values of step size parameter for ENSS-LMS algorithm.

The individual MSE curves for NLMS and ENSS-LMS algorithm for the case when Misdjustment is 2.5% are shown in fig. 6 and fig. 7 and combined MSE curves of both algorithms is shown in fig. 8. From the fig. 8 it is found that convergence speed of ENSS-LMS is faster than NLMS algorithm.

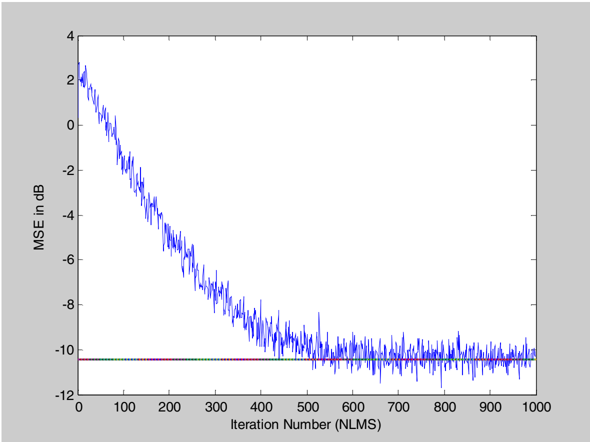

Figure 6: MSE curve for NLMS algorithm

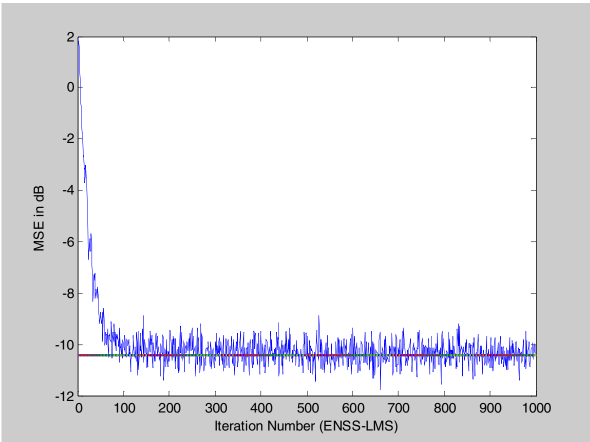

Figure 7: MSE curve for ENSS-LMS algorithm

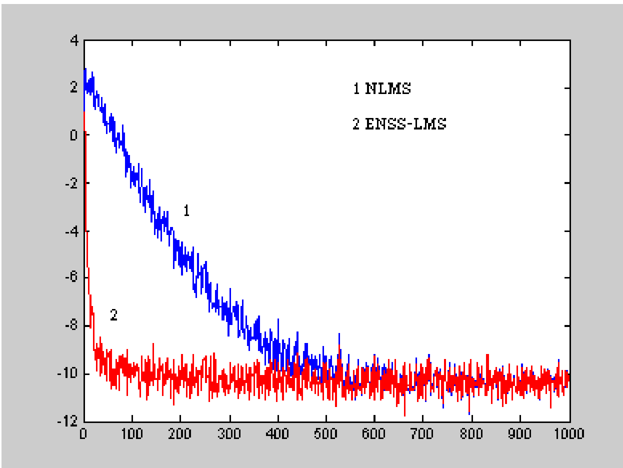

Figure 8: Combined MSE curves for NLMS and ENSS-LMS algorithms.

Fig. 9 shows the Execution Time for NLMS and ENSS-LMS algorithms. The Execution Time for LMS algorithm is 33.6406s and that of ENSS-LMS algorithm is 32.6406s.

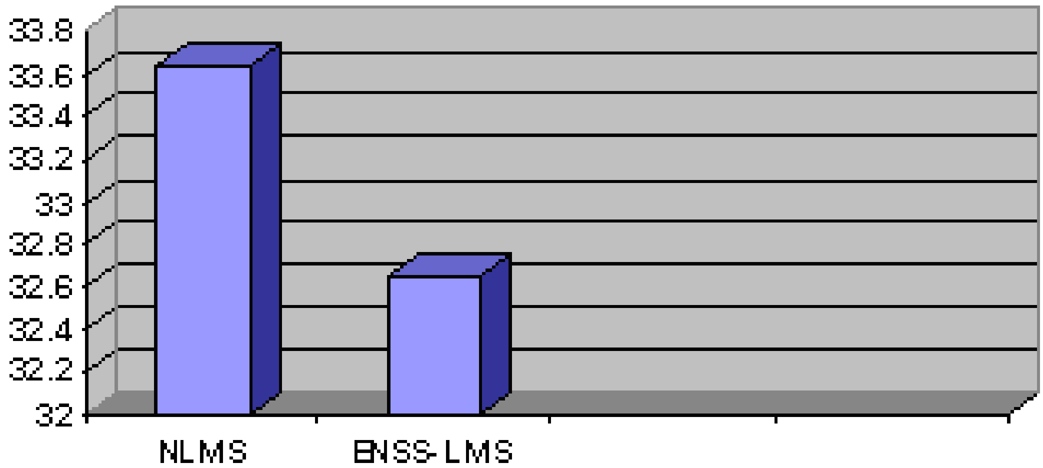

Figure 9: Execution Time for NLMS and ENSS-LMS algorithms.

Conclusion

In this paper a comprehensive comparative study between NLMS and ENSS-LMS algorithms is presented. The study is based on utilizing three performances criterion: Minimum Mean Square Error (MSE), Convergence Speed and algorithm Execution Time. Simulation results showed that MSE performance of ENSS-LMS algorithm is better than NLMS algorithm The Execution Time of ENSS-LMS algorithm is less than that of NLMS algorithm. The comparison also shows that variation of MSE curves with respect to step size parameter (μ) near the steady state region is less in ENSS-LMS algorithm as that of NLMS algorithm.

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