International Journal of Electronic and Electrical Engineering.
ISSN 0974-2174 Volume 7, Number 10 (2014), pp. 1099-1102

http://www.irphouse.com

Design of Quantum Unitary Gates by Perturbation of Hamiltonian by PAM signals with variable Fractional Delays

Pushkar Kumar [1], Harish Parthasarthi [2], Tarun Kumar Rawat [3]

[1] ECE Division, NSIT, Sector-3, Dwarka, New Delhi-110075, Delhi India
[2] ECE Division, NSIT, Sector-3 Dwarka, New Delhi 110075, India
Room No. 130, ECE Division, NSIT, Sector-3, Dwarka, New Delhi 110075, India

Abstract

In this paper we propose an algorithm for designing a fractional delay system that generates a quantum unitary gate which matches closely a given unitary gate. In this work we are taking input signal fixed which is passes through a fractional delay system which produce an output signal that modulates a given potential which acts as a perturbation to a given Hamiltonian matrix. Unitary evolution matrix obtained as the solution to Schrodinger's equation with the perturbed Hamiltonian is computed as a linear-quadratic functional of a fractional delay filter. This computation is based on second order time independent perturbation theory. The error energy between these unitary evolution operator at T and a given unitary gate is then computed up to linear-quadratic terms in the fractional delay impulse response. Impulse response is parametarise by the set of finite filter tape weight and a set of fractional delays.

Keywords: Fractional-delay filter, quantum mechanical perturbation theory, quantum unitary gate, Frobenius norm.

Introduction

The basic idea in this paper is to modulate the perturbing potential of a quantum system with a PAM signal whose amplitudes are adjustable and whose intersymbol delays are also adjustable fractions of the sampling time interval. These amplitudes and fractional delays are to be chosen so that the unitary gate obtained by evolving the quantum system after time T is as close in distance to a given unitary gate subject to an energy constraint on the PAM signal. The idea in using adjustable fractional delays is that varying the delays does not change the energy dissipated by the PAM signal in a resistor and may yet yield a better approximation to the given unitary gate.

PROBLEM FORMULATION:

Consider the excitation of a quantum system having Hamiltonian with a PAM pulse subject to fractional delays. The exciting potential is given as

$$V(t) = f(t)V_1 \tag{1}$$

Where,

$$f(t) = \sum_{n=0}^{N-1} x[n]p(t - n\Delta - \tau_n) \tag{2}$$

τ_n are fractional delays and are subjected to condition $0 \leq \tau_n/\Delta < 1$.The pulse p(t) is $\theta(t)-\theta(t-\Delta)$, i.e., p(.) is a rectangular pulse of duration Δ. The amplitudes x[n]as well as delays τ_n are within our control subject only to the energy constraints.

$$E = \int_0^T f^2(t)dt \tag{3}$$

i.e. $\sum_{n,m} x[n]x[m]A(n\Delta + \tau_n, m\Delta + \tau_m) = E_0$.

Where $A(\tau, \tau') = \int_0^T p(t-\tau)p(t-\tau')dt$.

The Quantum evolution equation is

$$iU'(t) = (H_0 + \epsilon . f(t)V_1)U(t). \tag{4}$$

Or it can be equivalently written as

$$W'(t) = \epsilon V_1(t) f(t) W(t).$$

Where

$U(t) = \exp(-iH_0 t)\, W(t)$ and

$V_1(t) = \exp(itH_0)V_1 \exp(-itH_0)$.

Thus,

$$W(T) = I - i\epsilon \int_0^T f(t)V_1(t)dt - \epsilon^2 \int_{0<s<t<T} f(t)f(s)V_1(t)V_1(s)dtds + O(\epsilon^3)$$

The gate to be designed is U_d and hence $\{x[n], \tau_n\}$ must be selected so as to minimize the following expression

$$F(x[n], \tau_n) = \|U_d - U(T)\|^2 - \lambda\left(\sum_{n,m} x[n]x[m]A(n\Delta + \tau_n, m\Delta + \tau_m) - E_0\right) \tag{5}$$

Now,

$$F = \|U_d - U(T)\|^2 = \|W(T) - exp(iTH_o)\|^2$$
$$= \left\| i\epsilon \int_0^T f(t)V_1(t)dt + \epsilon^2 \int_{0<t<s<T} f(t)f(s)V_1(t)V_1(s)dtds \right\|^2 + O(\epsilon^3)$$

So, our task is to minimize the above expression.

Approach:

The above expression can be written as $G = F - \|W_d\|^2$

Where $W_d = \exp(iTH_0)\,U_d - I$ and

$$G = 2\epsilon \int_0^T ImT_r\left(W_d^* V_1(t)\right)f(t)dt$$
$$- 2\epsilon^2 \int_{0<t<s<T} Re(T_r\left(W_d^* V_1(t)V_1(s)\right)f(t)f(s)dtds$$
$$+ \epsilon^2 \int_{0<t<s<T} T_r\left(V_1(t)V_1(s)\right)f(t)f(s)dtds + O(\epsilon^3)$$

In the above equation defining
$h(t) = 2Im\left(T_r W_d^* V_1(t)\right), g(t,s) = -2ReT_r\left(W_d^* V_1(t)V_1(s)\right)\delta(t-s) + T_r(V_1(t)V_1(s))$

We can write the equation as
$$G = \int_0^T h(t)f(t)dt + \epsilon^2 \int_{[0,T]^2} g(t,s)f(t)f(s)dtds \tag{6}$$

Let
$h_T^\wedge(\omega) = \int_{\mathcal{R}} h(t)\exp(-j\omega t)dt$, and
$g_T^\wedge(\omega_1,\omega_2) = \int_{[0,T]^2} g(t,s)\exp(-j\omega_1 t - j\omega_2 s)dtds$

Then, $\int_0^T h(t)f(t)dt = \int_{\mathcal{R}} h_T^\wedge(\omega)f_T^\wedge(\omega)d\omega/2\pi$

Similarly,
$\int_{[0,T]^2} g(t,s)f(t)f(s)dtds = (2\pi)^{-2}\int_{\mathcal{R}^2} g_T^\wedge(\omega_1,\omega_2)f^\wedge(\omega_1)f^\wedge(\omega_2)d\omega_1 d\omega_2$

Where $f^\wedge(\omega) = p^\wedge(\omega)\sum_n x[n]\exp(-j\omega(n\Delta + \tau_n))$. We replace the integral over ω by a discrete sum as given below

$$h_T^\wedge(\omega)f^\wedge(\omega) \approx \delta \sum_n x[n] \sum_k h_T^\wedge(\omega_k)p^\wedge(\omega_k)\exp(-j\omega_k n\Delta)\exp(-j\omega_k\tau_n) \tag{7}$$

Where $\omega_k = k\delta$, $k = \pm 1, \pm 2, \pm 3, \pm 4$.Now defining the Matrix $\mathbb{H} = [\![H(n,k)]\!] \in \mathbb{C}^{N\times p}$ Then $[\![H(n,k)]\!] = \delta h_T^\wedge(\omega_k)p^\wedge(\omega_k)\exp(-j\omega_k n\Delta)$ and also defining steering Matrix $e(\tau) = (\exp(-j\omega_k\tau))\quad -q < k < q \in \mathbb{C}^p$

And defining the PAM amplitude vector $\mathbb{X} = \left[x[1], x[2], x[3] \ldots \ldots \ldots \ldots \ldots x[N-1]\right]^T$. Here p is the total number of frequency discretization point $(\omega_k, k = -q, -q+1, q-1, q, 2q+1 = p)$ is the set of discrete frequency points used to approximate the frequency domain integrals by sum. Then

$$\int h_T^{\wedge}(\omega) f^{\wedge}(\omega)\, d\omega \approx \sum_{n,k} x[n] h[n]^T e(\tau_n) \tag{8}$$

Where $h[n]^T = [H[n,-q], H[n,-q+1], H[n,q-1], H[n,q]]$ is the $n+1^{th}$ row of $\mathbb{H}$.

We further have

$$\int g_T^{\wedge *}(\omega_1,\omega_2) f^{\wedge}(\omega_1) f^{\wedge}(\omega_2)\, d\omega_1 d\omega_2 = \sum_{n,m} x[n]x[m] \int g_T^{\wedge *}(\omega_1,\omega_2) p^{\wedge}(\omega_1) p^{\wedge}(\omega_2) \exp\big(-j\Delta(n\omega_1 + m\omega_2)\big) . \exp\left(-j(\omega_1\tau_n + \omega_2\tau_m)\right) d\omega_1 d\omega_2 \approx \sum_{n,m,k,r} x[n]x[m] G(n,m;k,r) \exp\left(-j\Delta(\omega_k\tau_n + \omega_r\tau_m)\right) \tag{9}$$

Where, $G(n,m;k,r) = \delta^2 g_T^{\wedge *}(\omega_1,\omega_2) p^{\wedge}(\omega_k) p^{\wedge}(\omega_r) \exp\left(-j\Delta(n\omega_k + m\omega_r)\right)$

Result

Then $\int g_T^{\wedge *}(\omega_1,\omega_2) f^{\wedge}(\omega_1) f^{\wedge}(\omega_2)\, d\omega_1 d\omega_2 \approx \sum_{n,m} x[n]x[m] g[n,m]^T (e(\tau_n) \otimes e(\tau_m))$

Where $g[n,m] = \sum_{k,r} G(n,m;k,r)(u[k] \otimes u[r])$ where u[k] is a $p \times 1$ column vector having a one in the $(q+k+1)^{th}$ position and a zero at all the other positions with $k = -q, -q+1, q-1, q$.

References:

[1]. Chien-Cheng Tseng and Su-Ling Lee Efficient Design and Implementation of Variable Fractional Delay Filters Using Differentiators IEEE Transaction on Circuits And Systems I: Regular Papers, Vol. 58, No. 6, JUNE 2011

[2]. H. Parthasarathy, mathematical ideas for processing application, I.K. International Pub. House Pvt. Ltd. 2013.

[3]. Donny cheung, waterloo, "using generalized quantum Fourier transforms in quantum phase estimation algorithms",2003.

[4]. M. A. Nielsen and I. L.Chuang. Quantum Computation and Quantum Information. Cambridge Univ. Press, 2000.

[5]. A. M. Perelomov. Generalized coherent states and their applications. Texts and monographs in Physics. Springer-Verlag, 1986

[6]. P.A.M. Dirac, the principal of quantum mechanics, oxford university press, 4th edition, 1958.

[7]. H. Goldstein, C.P. poole, john L. safko, classical mechanics, pearson Hall, 3th edition, 2002.

[8]. Tosio Kato, Perturbation Theory for Linear Operators, Springer-Verlag Berlin Heidelberg New York, 2nd edition, 1980