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Interval Type-2 Fuzzy Logic System

Suman Lata[1] and Mohammad Ayyub[2]

[1,2]*Department of Electrical Engineering, Aligarh Muslim University, Aligarh ,Aligarh-202002, India*

Abstract

This paper presents the theory and design of interval type-2 fuzzy logic systems (FLSs) because of computational complexity of using general type-2 fuzzy set (T2FS) in type-2 fuzzy system. We propose an efficient and simplified method to compute the input and antecedent operations for interval type-2 FLSs; one that is based on a general inference formula for them. We introduce the concept of upper and lower membership functions (MFs) and illustrate our efficient inference method for the case of Gaussian primary MFs. Output processing, which consists of type reduction and defuzzification. Type-reduction methods are extended versions of type-1 defuzzification methods. Type reduction captures more information about rule uncertainties than does the defuzzified value (a crisp number). Illustrative example is provided which demonstrated the interval type-2 reasoning with interval type-2 fuzzy inputs.

Index Terms: Gaussian type-2 set, Type-2 fuzzy logic controller, Type reduction, Uncertainties, Interval type-2 fuzzy sets, Interval type-2 fuzzy inference system.

1. Introduction

Fuzzy systems have displaced conventional technology in different scientific and system engineering applications, especially in control systems. The fuzzy sets were presented by L.A. Zadeh in 1965 [1-3] to process / manipulate data and information affected by un-probabilistic uncertainty / imprecision.

The knowledge that is used to construct the rules in a fuzzy logic system (FLS) is uncertain. Antecedent or Consequent uncertainties translate into uncertain antecedent or consequent membership functions. Type-1 FLSs, whose membership functions are type-1 fuzzy sets, are unable to directly handle rule uncertainties. Type-2 FLSs, in which antecedent or consequent membership functions are type-2 fuzzy sets, can handle rule uncertainties. The concept of type-2 fuzzy sets was introduced by Zadeh [5] as an extension of the concept of an ordinary fuzzy set, i.e., a type-1 fuzzy set. Type-2 fuzzy sets have grades of membership that are themselves fuzzy. A type-2 membership grade can be any subset in (0, 1)—the *primary membership*; and, corresponding to each primary membership, there is a *secondary membership* (which can also be in (0, 1)) that defines the possibilities for the primary membership. A type-1 fuzzy set is a special case of a type-2 fuzzy set. Mizumoto and Tanaka studied the set theoretic operations of type-2 sets [8], theoretic operations of type-2 sets are given in [7].

Similar to a type-1 FLS; a type-2 FLS includes fuzzifier, rule base, fuzzy inference engine, and output processor [6]. The output processor includes type-reducer and defuzzifier; it generates a type-1 fuzzy set output (from the type-reducer) or a crisp number (from the defuzzifier). A type-2 FLS is again characterized by IF–THEN rules, but its antecedent or consequent sets are now type-2. Things simplify a lot when secondary membership functions (MFs) are interval sets (in this case, the secondary memberships is one and we call them interval type-2 sets). When the secondary MFs are interval sets, we call the type-2 FLSs “interval type-2 FLSs”.

An Interval type-2 fuzzy set theory and an efficient method to compute the input and antecedent operations for interval type-2 FLSs are given in Section II; fuzzy inference system and its simulation implemented in MATLAB toolbox are discussed in Section III; and finally, the conclusions are given in Section IV.

2. Interval Type-2 fuzzy Set Theory

2.1 Type-2 Fuzzy Sets Concept

A type-2 fuzzy sets, denoted $\tilde{A}$, is characterized by a type-2 membership function (MF) $\mu_{\tilde{A}}(x,u)$ where $x \in X$ and $u \in J_x \subseteq (0, 1)$ i.e. $\mu_{\tilde{A}}(x,u): X \rightarrow [0, 1]$. J_x is the primary MF in (0, 1) interval and u are the primary membership values, as shown in Fig. 1 in which $0 \leq \mu_{\tilde{A}}(x,u)) \leq 1$. $\tilde{A}$ Can also be expressed as:

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} \mu_{\tilde{A}}(x,u) \big/ (x,u) \quad J_x \subseteq (0, 1)$$

Where $\iint$ denotes union over all admissible x and u. $f(u)$ is the secondary MF.

$$\tilde{A} = \{((x, u), \mu_{\tilde{A}}(x, u)) \,|\, \forall x \in X, \forall u \in J_x \subseteq (0,1)\}$$

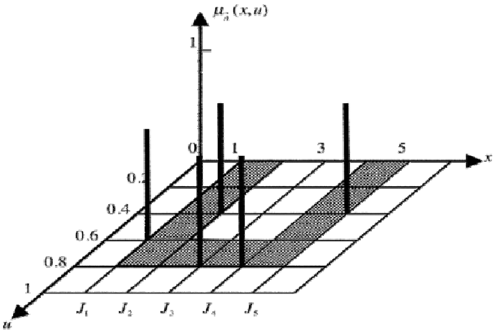

Fig. 1: Example of full type-2 MF. The shaded area is 'the foot print of uncertainty (FOU). The amplitude of sticks is the secondary membership value [11].

When all secondary MF $\mu_{\tilde{A}}(x,u)$ of the type-2 set $\tilde{A}$ is unity then $\tilde{A}$ set is an interval type-2 fuzzy set (IT2 FS).

$$\tilde{A} = \int_{x \in X} \int_{u \in J_x} 1/(x, u); \qquad J_x \subseteq (0,1)$$

The Uncertainty in the primary memberships of an IT2 FS $\tilde{A}$ consists of a bounded region that we call the *foot-print of uncertainty (FOU)*. It is the union of all primary memberships.

2.2 Gaussian type-2 fuzzy set

A Gaussian type-2 fuzzy set is one in which the membership grades of every domain point is Gaussian type-2 fuzzy set contained in (0, 1) as shown in Fig. 2 and Fig. 3 [6, 10 & 13].

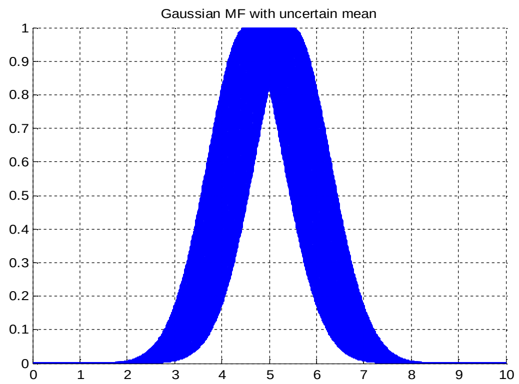

Fig. 2: ype-2 MF, the centers of Gaussian MFs with uncertain mean.

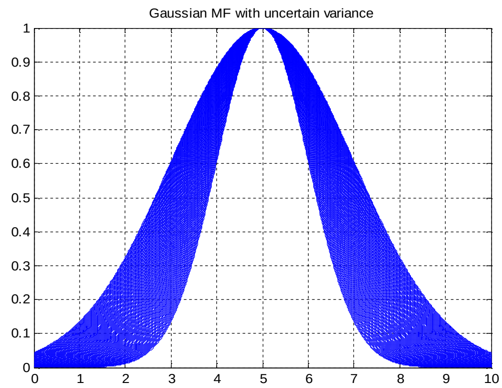

Fig. 3: The centre of the Gaussian MFs with uncertain variance.

2.3 Illustrative Examples

Input and Antecedent Operations: In Fig. 4, we plot the results of input and antecedent operations with singleton, type-1 nonsingleton, and type-2 nonsingleton fuzzification.

The number of antecedents is p = 2. In all cases, the firing strength is an interval type-1 set $[\underline{f}^1, \bar{f}^1]$, where $\underline{f}^1 = \underline{f}_1^1 * \underline{f}_2^1$ and $\bar{f}^1 = \bar{f}_1^1 * \bar{f}_2^1$. For singleton fuzzification $\bar{f}_k^1$, denotes the firing strength between input x_k and $\bar{\mu}_{\tilde{F}_1}^k$, namely $\bar{\mu}_{\tilde{F}_1}^k(x_k)$ and $\underline{f}_k^1$ denotes the firing strength between input x_k and $\underline{\mu}_{\tilde{F}_1^k}$, namely $\underline{\mu}_{\tilde{F}_1^k}(x_k), (k = 1, 2)$. For non-singleton type-1 fuzzification $\bar{f}_k^1$ denotes the supremum of the firing strength between the t-norm of membership functions μ_{X_k} and $\bar{\mu}_{\tilde{F}_1}^k$; and $\underline{f}_k^1$ denotes the supremum of the firing strength between the t-norm of membership functions μ_{X_k} and $\underline{\mu}_{\tilde{F}_1^k}, (k = 1, 2)$. For non-singleton type-2 fuzzification $\bar{f}_k^1$ denotes the supremum of the firing strength between the t-norm of upper membership functions $\bar{\mu}_{\tilde{X}_k}^k$ and $\bar{\mu}_{\tilde{F}_1}^k$; and, $\underline{f}_k^1$ denotes the supremum of the firing strength between the t-norm of lower membership functions $\underline{\mu}_{\tilde{X}_k}$ and $\underline{\mu}_{\tilde{F}_1^k}, (k = 1, 2)$.

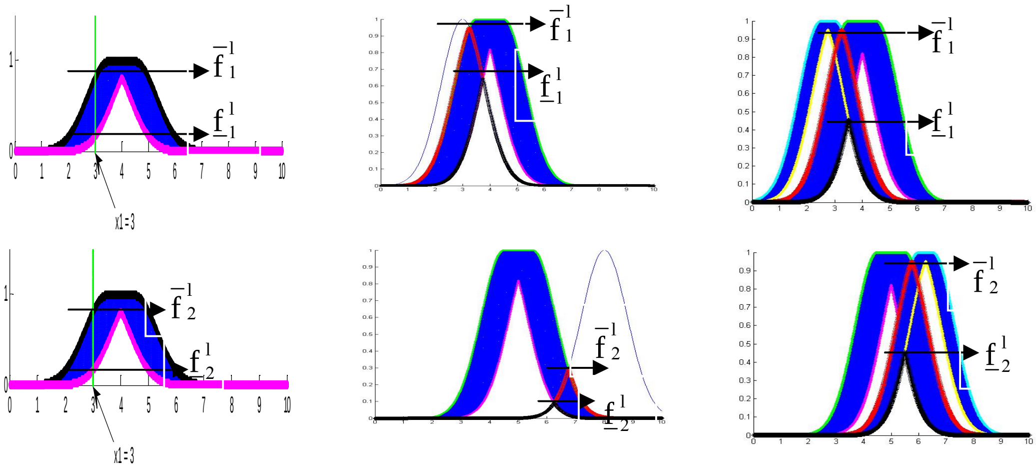

Fig. 4: Type-2 FLS: input and antecedent operations. (a) Singleton fuzzification with minimum t-norm; (b) NS type-1 fuzzification with minimum t-norm; and (c) NS type-2 fuzzification with minimum t-norm.

3. Fuzzy Inference System

An Inference Fuzzy System is rule based system that uses fuzzy logic. Its basic structure includes four components as shown in Fig. 5(a).

1. *Fuzzificator.* Translates inputs (real values) to fuzzy values.
2. *Inference System.* Applies a fuzzy reasoning mechanism to obtain a fuzzy output.
3. *Type Defuzzificator/Reductor.* The Defuzzificator reduces one output to precise values; the type reductor transforms a type-2 fuzzy set into a type-1 fuzzy set.

4. *Knowledge base.* Contains a set of fuzzy rules, and a membership functions set are known as data base.

3.1 Type reduction and Defuzzification of Interval Type-2 Fuzzy Logic System

Type-reduction was proposed by Karnik and Mendel [12]. It is an “extended version” using the extension principle of type-1 defuzzification methods and is called type-reduction because this operation takes us from the type-2 output sets of the FLS to a type-1 set that is called the “type-reduced set.” This set may then be defuzzified to obtain a single crisp number; however, in many applications, the type reduced set may be more important than a single crisp number since it conveys a measure of uncertainties that have flown through the type-2 FLS. There exist many kinds of type-reduction, such as centroid, centre-of-sets, height, and modified height, the details of which are given in [12]. We use centre-of-sets method for type-reduction of interval type-2 fuzzy logic system.

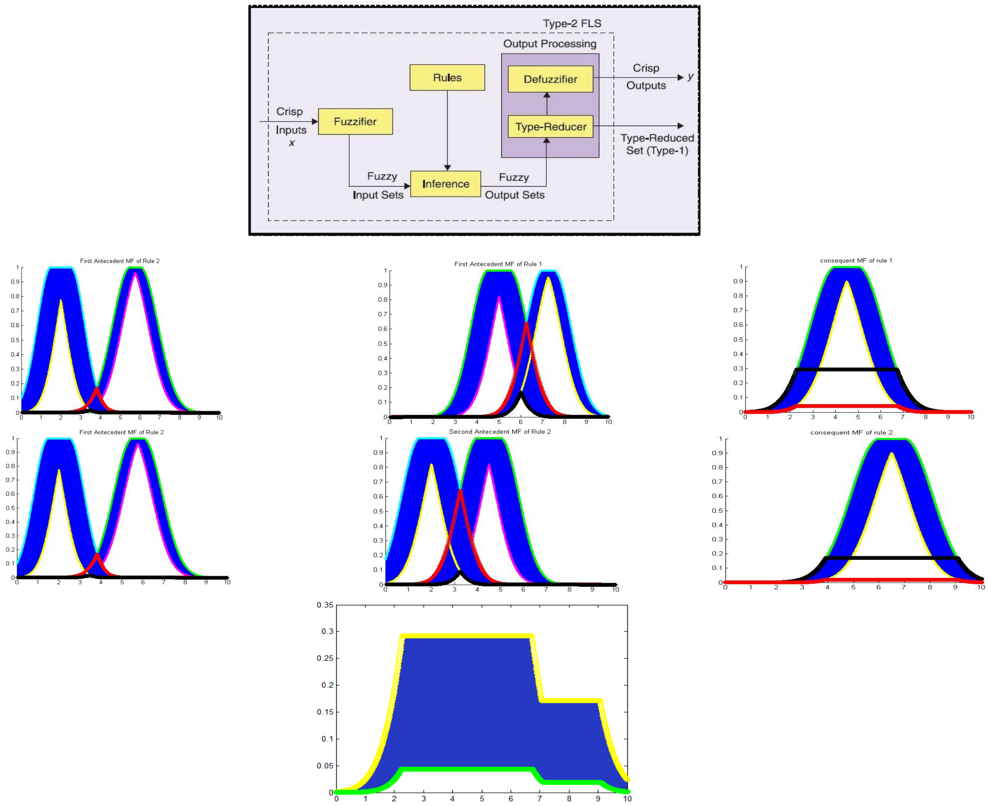

Fig. 5 (a) Type-2 inference fuzzy system structure [11] (b) Interval type-2 fuzzy reasoning with interval type-2 fuzzy inputs [13].

3.2 Simulation of Interval Type-2 Fuzzy Inference System

Fig. 5(b) depicts the interval type-2 fuzzy reasoning with interval type-2 fuzzy inputs implemented in MATLAB toolbox. The interval type-2 fuzzy reasoning with two rule model has two antecedents and one consequent for each rule. In each antecedent, we have found the supremum of firing strength between t-norm of upper membership functions and supremum of firing strength between t-norm of lower membership functions. These are called the upper and lower firing level respectively. These firing levels are then t-normed with consequent set. Same processes are done for second rule then take the maximum between these resultant consequent sets as shown in Fig. 5(b). Using type-reduction, IT2FS is converted into interval type-1 set. After type-reduction and defuzzification of IT2 FLS, the interval set is (5.1856, 5.0511) and the defuzzified output is 5.1183.

4. Conclusion

We have presented a type 2 FLS as a natural extension of a type 1 FLS. We have shown that all of the results that are needed to implement an IT2 FLS can be obtained using T1 FS mathematics. We have presented the theory and result using MATLAB toolbox for design of interval type 2 FLSs, including an efficient and simplified method to compute their input and antecedent operations, interval type 2 fuzzy reasoning, type reduction and defuzzification. So we have concluded that IT2 FLS models higher levels of uncertainties than does a T1 FLS, this opens up an efficient way of developing improved control system and for modelling human decision making.

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